IMEKO 22nd TC3, 12th TC5 and 3rd TC22 International Conferences
3 to 5 February, 2014, Cape Town, Republic of South Africa

DETERMINING THE UNCERTAINTY OF MEASUREMENT WHEN TRANSFERRING A RANGE OF A MEASUREMENT SCALE

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Abstract: Each recording of measurement values postulates the consistency of the realization and mapping of the measurement scale by the measuring device. The transfer of a measurement scale to a measuring or testing device at two or more data points is done in a linear space. A functional relationship between measurement scale and measured value realization is derived from regression analysis. A differentiation can be made between systematic, random and symmetric deviations between measurement realization and measurement scale. The systematic and symmetric deviations are compensated by using a correction function. All remaining deviations become part of the measurement uncertainty. The functional description of the uncertainty of measurement can be modelled using different models with symmetric and asymmetric limits of the interval.

Keywords: calibration, transfer of measurand scale, uncertainty of measurement

1. INTRODUCTION

A measurand is characterised by definition (verbal description of physical relations), realization (technical implementation of the definition to a measuring device), presentation (subset of real numbers and physical unit) and transfer (metrological chain, transfer standards). The best possible physical and technological implementation of the definition in a measurement standard or a measuring device takes place at the level of national metrology institutes. The transfer of the measurement scale happens via calibration standards, which realise a scale point or a scale range. When performing the calibration, the presentation of the measured value by the measuring device is compared with the measuring scale at the calibration standard. This comparison is made at a point on the scale or over a scale range. When measuring, a value on the measuring scale is assigned to the measurement object (sample). In case the sample measurement value does not comply with the point of scale verification, as regards the realization of the measured value in the specified scale range, continuity must be preconditioned.

2. MODEL FOR THE CONTINUITY OF THE SCALE DEPICTION

The proof of the consistency as regards the presentation of the measurement scale always requires a multipoint calibration, since consistency is mathematically unverifiable at a single point calibration. The calibration of the range of a measuring scale is realised by verification at multiple data points. In order to achieve continuity in the scale range when calibrating on the basis of a multipoint calibration, the measurement function is implemented. To express the function mathematically, a vector space over the elements of the measuring scale (closed interval on a subset of real numbers) is defined. In this vector space the expected values of the measurement realization and the calibration values of the standards are functionally linked as ordered points [(x_1,y_1), … (x_n,y_n)] depending on the measurement scale (**fig. 1**). The functional connection of these points, defined as a measurement function, is a unique and continuous description of the realization of measured values over the measurement scale. Thereby the type of functional composition is unknown. The ideal functional composition (Figure) when transferring scale is the straight line $y = f(x) = x$ as the objective function of the calibration. Thereby the objective function is the mirror axis (symmetry axis) of the scale depiction on a deviation-free presentation of the realization of the measured value. Since usually at each data point multiple measurements are performed at the standard, are the resulting measurement values to be evaluated according the stochastic rules. The experiential mean (1) of the measurement series is an estimate for the expected value of the measurand realization at the data point.

$$\bar{y}_s = \frac{1}{n}\cdot\sum\left(y_{s,i}\right) \qquad (1)$$

The empirical standard deviation is an estimate of the expected value of the standard deviation (2) at the particular scale data point.

$$s_{y,s} = \sqrt{\frac{1}{n-1}\cdot\sum\left(y_{s,i}-\bar{y}_s\right)^2} \qquad (2)$$

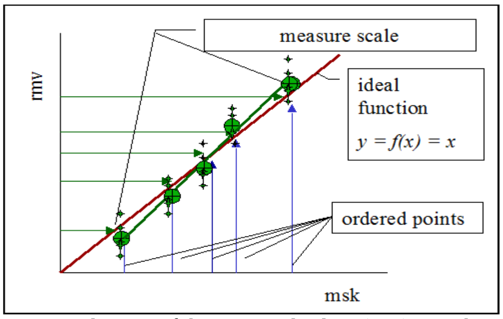

Fig. 1: Realization of the measured values (rmv) over the measure scale (msk)

The interval for the expected value „$E_y = \mu_y$“(confidence level) from the sample character when determining realization of the measured value at the data point is estimated by the T-distribution (student's distribution) (3) [2] assuming a selected probability „α“ and a number of measurements „n“.

$$\bar{y}_s - \frac{t_{(n-1,1-\alpha)}\cdot s_{y,s}}{\sqrt{n}} \ge E_{y,s} \le \bar{y}_s + \frac{t_{(n-1,1-\alpha)}\cdot s_{y,s}}{\sqrt{n}} \quad \Rightarrow$$

$$U_{Ey,s} = \frac{t_{(n-1,1-\alpha)}\cdot s_{y,s}}{\sqrt{n}} \qquad (3)$$

Input variables for the regression analysis are the calibration values of the transfer standards and the expected values (empirical mean values) of the realization of measured values at the data points. The regression analysis provides the estimating function for the measuring function „$f_M(x)$“. The estimator depends on the calibration values of the transfer standards, the number of readings

„i“ at the data points of the calibration and the number of data points „s“. The number and location of data points, and the randomness of the mean values cause the variation and the randomness of the type and the coefficients of the estimator „$\hat{y}$“ ($\hat{y} = f_M(x) \Rightarrow f_M(x) = S(x_s, \bar{y}_s) \in (x, y)$).

3. MEASUREMENT DEVIATION AND UNCERTAINTY AT THE DATA POINTS

The determination of the deviation between measurement scale and measured value occurs in the vector space. The location of the true value of the material measure on the measuring scale is stated in the interval of uncertainty „$U_{TN,s}$“ by assuming a certain probability [1]. If applicable components of measurement uncertainty from the drift „$U_{ND,s}$“ and differing ambient conditions „$U_{NU,s}$“ of the transfer standard have to be taken into consideration. For the uncertainty of the transfer standard „$U_{NA,s}$“ when performing calibration follows:

$$U_{NA,s} = \sqrt{U_{TN,s}^2 + U_{ND,s}^2 + U_{NU,s}^2} \qquad (4)$$

The interval of the measurement uncertainty of the expected value is modeled from the components of the random sample character of the calibration „$U_{Ey,s}$“ (confidence interval), the minimum measuring step „$U_{MRms,s}$“, the drift „$U_{MRD,s}$ and differing ambient conditions of the measurement realisaton „$U_{MRU,s}$“. This results in the uncertainty of the expected value „$U_{MRA,s}$“ (5). For $n \to \infty$ (in practice for $n > 50$) it arises an unbiased estimator (minimum interval) for the expected value ($\bar{y}_s \to Ey_s = \mu_s$) [4].

$$U_{MRA,s} = \sqrt{U_{Ey,s}^2 + U_{MRms,s}^2 + U_{MRD,s}^2 + U_{MRU,s}^2}$$

$$= \sqrt{\frac{\left(t_{(n-1,1-\alpha)} \cdot s_{\bar{y},s}\right)^2}{n} + \left(\frac{ms}{\sqrt{3}}\right)^2 + U_{MRD,s}^2 + U_{MRU,s}^2} \qquad (5)$$

The calibration value and the expected value of the realization of the measured value are the symmetry points of their associated measurement uncertainty intervals. To determine the deviations, the expected value of the realization of the measured value including the interval limits is projected in the intersection of the objective function and the expected value **(Fig. 2)**.

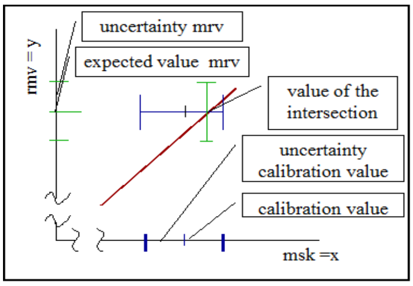

Fig. 2: Deviation, including the intervals of measurement uncertainty

The uncertainty interval of the calibration value is projected onto the line of the expected value and as the case may be intersects the projection of the uncertainty interval of the realization of the measured value.

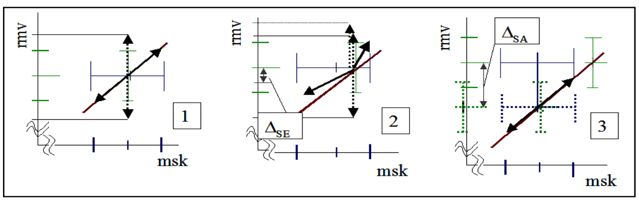

Fig. 3: Position of intersections of measurement uncertainty intervals

In case the values of the intersection points at all data points are located within the measurement uncertainty intervals (calibration value and measurement value realization), the deviations can be explained by coincidence **(Fig. 3-1)** and there are no systematic deviations (6). The measured value indication represents the measurement scale. Is the value of the intersection outside the projected interval limits at least at one data point, the deviations can't be explained by chance and there is a systematic deviation (7).

$$x_s - U_{NA,s} \geq (f(x) = \hat{y}_s) \leq x_s + U_{NA,s} \qquad (6)$$

$$x_s - U_{NA,s} \leq (f(x) = \hat{y}_s) \text{ or } x_s + U_{NA,s} \geq (f(x) = \hat{y}_s) \qquad (7)$$

The measured value indication does not represent the measurement scale (**Fig. 3-2**). If so a new adjustment of the measuring device is needed or a correction of the measuring value indication. The simplest form of the correction function is the inverse function ($f_M(x) \to f_Z(x) = x \Rightarrow f_{kor}(x) = f^{-1}{}_M$) of the measurement function (8). Via correction function a new value is assigned to each measured value. Mean value, standard deviation and measurement uncertainty are recalculated for the corrected values. If the intersections, after correction of all values at all data points, are within the measurement uncertainty interval of the corrected expected values of the realization of the measured values and the measurement uncertainty interval of the calibration values, the correction function will represent the measurement scale. Is the intersection not within the intervals of uncertainty at least at one data point „s“, the correction function does not represent the measurement scale. With this correction formula no better adjustment is possible. Thus, the measurement uncertainty is expanded by the uncorrectable residual deviation, ie by the difference between the calculated value from the correction function and the calibration value of the standard (8).

$$\Delta_{kor,s} = f_{kor}(y_s) - x_s \qquad (8)$$

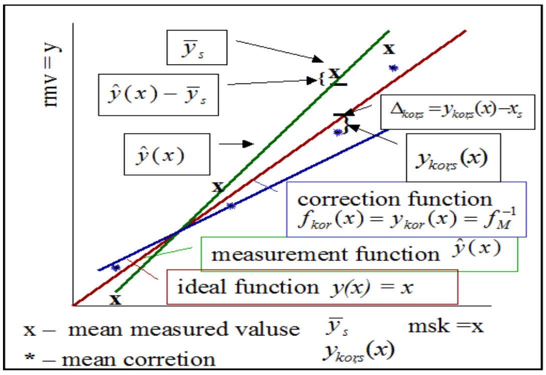

Fig. 4: Measuring function and correction function

For linear independence, the uncertainty of the calibration „$U_{AK,s}$“ results from the quadratic addition (vector addition of orthogonal vectors) of the measurement uncertainty of the calibration standard and the measurement uncertainty of the realization of the measuring value (table 1). Symmetric interval limits of the measurement uncertainty of the calibration will only arise if the symmetry points (calibration value and expected value) are at the intersection **(Fig. 5-1)**. Since usually the points of symmetry are not at the intersection, this results in an asymmetric interval for the measurement uncertainty of the calibration (**Fig. 5-2)**. The asymmetric interval limits are calculated from the intersection point, in each case as a resultant vector of the two orthogonal vectors (direction similar to the objective function). The symmetry deviation arises at the measurement uncertainty interval of the material measure and is based on the difference „Δ_{SE}“ between intersection and point of symmetry (9) (Fig. 5-2).

$$\Delta_{SE,s} = \bar{y}_s - x_s \Rightarrow U_{NAU,s} = U_{NA,s} + \Delta_{SE,s} \text{ and}$$

$$U_{NAO,s} = U_{NA,s} - \Delta_{SE,s} \qquad (9)$$

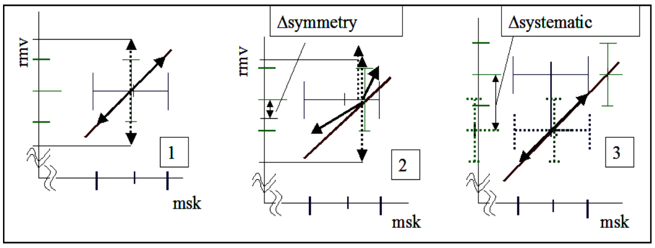

Fig. 5: Modelling the measurement uncertainty of calibration

Is there no intersection point (systematic deviation) within the interval limits, a correction is required, ie a shift of the expected value onto the calibration value by use of the correction function (**Fig. 5-3**). Do deviations „$\Delta_{kor,s}$“ between the intersection and the points of symmetry still exist after correction, this results in an asymmetric interval of measurement uncertainty (10) **(Fig. 5-2)**.

$$\Delta_{kor,s} = f_{kor}(x) - x_s$$

$$\Rightarrow U_{NAU,s} = U_{NA,s} + \Delta_{kor,s} \text{ and } U_{NAO,s} = U_{NA,s} - \Delta_{kor,s} \qquad (10)$$

For modelling symmetric interval limits the measurement uncertainty interval of the calibration value is unilaterally expanded by the difference between the expected value and the calibration value (the larger part of the asymmetric interval is taken into consideration). However, in this modeling the probability of the values in the interval is not equal.

Table 1: Models for the uncertainty of measurement
(A) Deviation without correction; (B) Deviation after correction;
(C) Deviation with interval expansion without correction;
(D) Deviation with interval expansion after correction

	Intersect		
	Points of symmetry	Random deviation	Systematic deviation
A	$U_{AK,s}=\sqrt{U^2_{NA,s}+U^2_{MRA,s}}$ (11)	$U_{AKU,s}=\sqrt{U^2_{NAU,s}+U^2_{MRA,s}}$ $U_{AKO,s}=\sqrt{U^2_{NAO,s}+U^2_{MRA,s}}$ (12)	–
B	$U_{AK,s}=\sqrt{U^2_{NA,s}+U^2_{MRA,s}}$ (11)	$U_{AKU,s}=\sqrt{U^2_{NAU,s}+U^2_{MRA,s}}$ $U_{AKO,s}=\sqrt{U^2_{NAO,s}+U^2_{MRA,s}}$	(12)
C	-	$U_{AK,s}=\sqrt{(U_{NA,s}+\bar{y}_s-x_s)^2+U^2_{MRA,s}}$	(12)
D	-	$U_{AK,s}=\sqrt{(U_{NA,s}+f_{kor}(x)-x_s)^2+U^2_{MRA,s}}$	(12)

4. FUNCTIONAL DESCRIPTION OF MEASUREMENT UNCERTAINTY (CALIBRATION)

The modeling of the measurement uncertainty in accordance with GUM [1] currently only describes the determination on the basis of a scale-point transfer.In section 2, the continuity of the mapping of the measurement scale on the realization of the measured values as a subset of the real numbers was modeled. Since probability spaces are defined on subsets of the real numbers, e.g. by normal distribution [3], the continuity of the measurement uncertainty can be modelled on the basis of the functional combination of the measurement uncertainty at the data points. The measurement uncertainties of the data points, the number of data points and the deviation between the expected value of the realization of reading and the calibration value have to be taken into account. The functional dependence of the measurement uncertainty over the realization of reading can be modelled within the transfer range according to different models on symmetric and asymmetric intervals (**Fig. 6**). The modelling occurs on the interval limits of the measurement uncertainty of the data points. The functional derivation over the interval boundaries is the same for asymmetric and symmetric interval limits, but in case of asymmetric limits it leads to two different functional descriptions relating to the upper and lower limits of the interval.

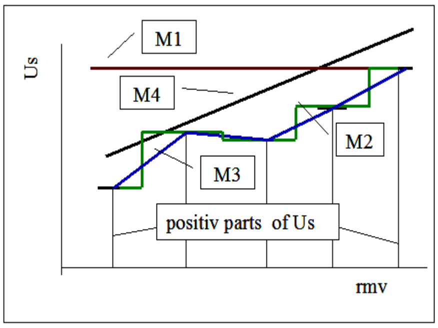

Fig. 6: Models for the functional description of measurement uncertainty

For symmetric intervals only the positive portion is considered for the functional description of the measurement uncertainty. The estimator of the measurement uncertainty fU(x) with regard to the upper or lower interval limit is obtained by regression analysis. The type of estimating functions (upper or lower interval limit) and the type of coefficients vary depending upon the number and location of the data points. The degree of the polynomials of the selected estimators, each for the upper and lower limit, should be at least by two less than the number of data points. The estimator of the measurement uncertainty „$\hat{U}(x)$“ (16) from regression analysis is based on the expected values of the realization of reading and the positive and negative portions of the measurement uncertainty at the data points„s“:

M1: Maximum measurement uncertainty at data point (13) (**Fig. 6-M1**).

$$U(x) = \max(U_{s=1},\ldots,U_{s=k}) \qquad (13)$$

M2: Step function with jump at half the interval width between data points (14) (**Fig. 6-M2**) (non-recommended modelling).

$$U(x) = \begin{cases} \left[x_1 > x \le \frac{x_1 + x_2}{2}; \max(U_{s=1}, U_{s=2}) \right] \\ \vdots \\ \left[\frac{x_{n-1} + x_n}{2} > x \le x_n; \max(U_{s=n-1}, U_{s=n}) \right] \end{cases} \qquad (14)$$

M3: Best-fit line between the data points (slope„$m_{Un,Un-1}$“, point of intersection „b“) (15) (**Fig. 6-M3**).

$$U(x) = \begin{cases} \left[x_1 > x \le x_2; m_{U1,U2} \cdot x + b_{U1,U2} \right] \\ \vdots \\ \left[x_{n-1} > x \le x_x; m_{Un-1,Un} \cdot x + b_{Un-1,Un} \right] \end{cases} \qquad (15)$$

M4: Functional description by regression analysis at interval limits of measurement uncertainty at data points (**Fig. 6-M3**).

$$\hat{U}(x) = fU(x) \rightarrow [U_{s=1}, y_1; \ldots, U_{s=k}, y_{s=k}] \qquad (16)$$

Since coefficients vary depending on the number and location of data points, groups of functions can be generated, which calculate different function values for the same input values. The deviations between the values calculated with the estimator and the calibration values at the data points are normally distributed. The variations of the deviation of the measurement uncertainties minus functionally calculated measurement uncertainty at the data point (17, 18) are also normally distributed (**Fig. 7**).

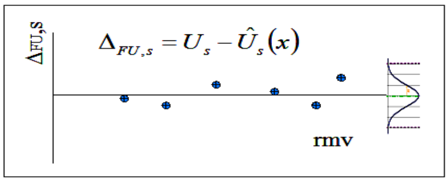

Fig. 7: Uncertainty of measurement from the functional non-conformance

The experiential standard deviation „$s_{\Delta FU}$“ (19) is a random variable and depends on the number of data points. From the sample character arises the uncertainty of the functional non-conformance „U_{FU}“ (20) via T-distribution (if so for of the upper and lower limit each). Is the degree of the estimator is smaller than the number of data points, the regression analysis through the least squares method causes, that at least at one data point the value of measurement uncertainty, calculated by estimator, is lower than the dertermined measurement uncertainty at this data point (i.e. lower probability at this point).

$$\Delta_{FUO,s} = U_s - \hat{U}_s(x) \quad (17)$$

$$\Delta_{FUU,s} = \hat{U}_s(x) - U_s \quad (18)$$

$$s_{\Delta FU} = \sqrt{\frac{1}{k-1}\sum \Delta^2_{FU,s}} \quad (19)$$

$$U_{FU} = \frac{t_{(k-1,1-\alpha)} \cdot s_{\Delta FU}}{\sqrt{k}} \quad (20)$$

This is compensated by linear transformation as measurement uncertainty from interval boundary adjustment „U_{IA}“ (when indicated the upper (21) and lower (22) interval limit).

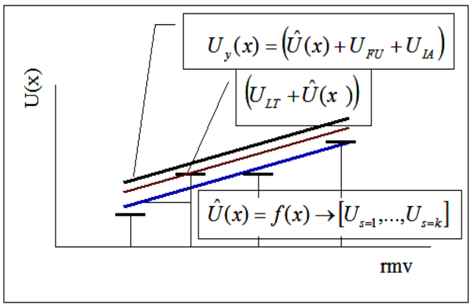

Fig. 8: Components of the functional view of the measurement uncertainty

$$U_{IAO} = \max\left(U_s - \hat{U}(x_s)\right) \quad (21)$$

$$U_{IAU} = \max\left(\hat{U}(x_s) - U_s\right) \quad (22)$$

$$U_y(x) = (\hat{U}(x) + U_{IA} + U_{FU}) \quad (23)$$

The uncertainty function „*Uy(x)*“ (23) arises from the additive expansion of the estimator to include the measurement uncertainty components from the functional nonconformance and the interval boundary adjustment **(Fig. 8)**. Asymmetric interval limits result in two functional presentations of the measurement uncertainty for the lower interval limit „$U_{yU}(x)$“ and the upper interval limit „$U_{yO}(x)$“ (24).

$$U_{yU}(x) = (\hat{U}_U(x) + U_{IAU} + U_{FUU}) \quad \text{and}$$

$$U_{yO}(x) = (\hat{U}_O(x) + U_{IAO} + U_{FUO}) \quad (24)$$

5. SUMMARY

When transferring a measurement scale, error of measurement and measurement certainty always need to be considered jointly. The reference for determining the uncertainty of measurement is always the expected value of the realization of the measurand. In case expected value and calibration value don’t match, the result is – without correction or extension – an asymmetric interval for the measurement uncertainty. Functionally the measurement uncertainty can be mathematically described over the measurand realization using different models.

6. EXAMPLE

Example 1: Scale transfer (scale verification) with an object micrometer at one point of the length scale.

The calibration of the optical system at the point of the scale will be done with 25 measurements (table 2). Following datas are given:

- nominal value of the scale: 100 µm
- uncertainty of measurement: ± 0.5 µm
- estimate value at the realisation of measurements: 100,3 µm
- standard deviation: 0,25 µm
- number of measurements : 25

The estimate value at the realisation of measurements is not at equal to the nominal value from the transfer standard. The estimate value is into the limits of the uncertainty of the measurement at the transfer standard.

Table 2: Uncertainty of measurement at the calibration

Calibration standard at the referring calibration			**data point of measuring scale**	
		formula	1	
value of the calibration standard (data point)			100	µm
measurement uncertainty	U_N		0,5	µm
measurement uncertainty drift	U_ND		0	µm
measurement uncertainty environment	U_NU		0	µm
measurement uncertainty standard	**U_NA**	*4*	**0,5**	**µm**
realisation of the measured value at the data point				
number of measurements	**n=**		25	
arithmetical mean	**yq=Ey**	***1***	**100,3**	**µm**
standard deviation	**s=**	***2***	0,25	µm
T factor (n-1 = 24,1-a = 95%)			2,06	
confidence interval expected value (xq)	U_Ey	***3***	0,10	µm
minimal step	ms		0,10	µm
measurement uncertainty minimal step	U_ms		0,06	µm
measurement uncertainty drift	U_MRD		0,00	µm
measurement uncertainty environment	U_MRU		0,00	µm
measurement uncertainty expected value	**U_MRA**	***5***	**0,12**	**µm**
measurement uncertainty according the calibration at the data point				
intersection not at the point of symmetry; but no systematic deviation				
deviation of the symmetry	ΔSE		0,30	µm
measurement uncertainty standard lower limit	**U_NAU**	***9***	0,80	µm
measurement uncertainty standard upper limit	**U_NAO**	***9***	0,20	µm
measurement uncertainty at the calibration without correction asysmmetric interval				
measurement uncertainty lower limit	**U_AKU**	***12***	**0,81**	**µm**
measurement uncertainty upper limit	**U_AK0**	***12***	**0,23**	**µm**
measurement uncertainty at the calibration without correction asysmmetric interval				
maximum measurement uncertainty limit			**0,81**	**µm**
measurement uncertainty upper limit = lower limit	**U_AK**	***12***	**0,81**	**µm**

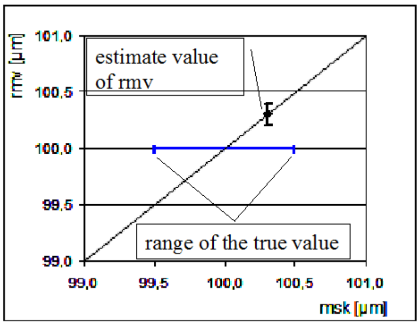

Fig. 9: Estimate value at the realisation of measurements is asymmetric to the limits of the uncertainty of the measurement at the transfer standard

The minium of the interval of uncertainty of measurement will be given by using of asymmetric limits. For symmetric limits the maximum uncertainty limits will be used at both limits.

Example 2: Indirect calibration of a hardness tester.

Using the example of scale transfer (scale verification) when performing indirect calibration of hardness testing devices in accordance with DIN EN ISO 6507-2 (Vickers HV1), the determination of the measurement uncertainty (table 3) according to model 4 is shown.

Table 3: Uncertainty of measurement at the calibration

Calibration standard at the referring calibration			**data point of measuring scale**			
		formula	1	2	3	
value of the calibration standard (data point)			237	534	735	**HV1**
measurement uncertainty	U_N		3,5	10,1	14,0	HV1
measurement uncertainty drift	U_ND		0	0	0	HV1
measurement uncertainty environment	U_NU		0	0	0	HV1
measurement uncertainty standard	**U_NA**	*4*	**3,5**	**10,1**	**14**	**HV1**
realisation of the measured value at the data point						
number of measurements	**n=**		5			
arithmetical mean	**yq=Ey**	***1***	**239,9**	**535,6**	**739,6**	**HV1**
standard deviation	**s=**	***2***	2,06	4,55	5,63	HV1
T factor (n-1 = 14,1-a = 95%)			2,78	2,78	2,78	
confidence interval expected value (xq)	U_Ey	***3***	1,47	3,27	4,04	HV1
minimal step	ms		1,00	2,00	3,50	HV1
measurement uncertainty minimal step	U_ms		0,58	1,15	2,02	HV1
measurement uncertainty drift	U_MRD		0,50	0,50	0,50	HV1
measurement uncertainty environment	U_MRU		0,50	0,50	0,50	HV1
measurement uncertainty expected value	**U_MRA**	*5*	**1,73**	**3,53**	**4,57**	**HV1**
measurement uncertainty according the calibration at the data point						
intersection not at the point of symmetry; but no systematic deviation						
deviation of the symmetry	ΔSE		2,90	1,60	4,60	HV1
measurement uncertainty standard lower limit	U_NAU	***9***	6,40	11,70	18,60	HV1
measurement uncertainty standard upper limit	U_NAO	***9***	0,60	8,50	9,40	HV1
measurement uncertainty without correction						
measurement uncertainty lower limit of the interval	U_AKU	***12***	-6,63	-12,22	-19,15	HV1
measurement uncertainty upper limit of the interval	U_AK0	***12***	0,85	12,02	13,29	HV1
measurement uncertainty at the data point without correction expanded to symmetric interval limits	**U_AK**	***11***	**0,85**	**12,02**	**13,29**	**HV1**
measurement uncertainty function at the calibration without correction						
number of data points	k =		3			
estimator function lower limit UG	$\hat{U}_{UG}(x)$		-0,024619	* x +	-0,235	
estimator function upper limit OG	$\hat{U}_{OG}(x)$		0,025827	* x +	-4,323	
values estimator function data points (lower limit) U_UG(x)	$\hat{U}_{UG}(x)$	***16***	-6,1	-13,4	-18,4	HV1
values estimator function data points (upper limit) U_OG(x)	$\hat{U}_{OG}(x)$	***16***	1,9	9,5	14,8	HV1
functional non-conformance						
deviation U_UG_data p. - U_UG_estimator function	ΔU_UGs	***17***	-0,49	1,20	-0,71	HV1
deviation U_OG_data p. - U_OG_estimator function	ΔU_OGs	***17***	-1,02	2,51	-1,49	HV1
standard deviation of lower limit uncertainty of measurement UG	sU_UG	***18***	1,04			HV1
standard deviation of upper limit uncertainty of measurement OG	sU_OG	***18***	2,19			HV1
t factor (s-1 = 2, a-1 = 95%)			4,30			
functional non-conformance lower limit U_UFU	U_UFU	***19***	-2,593			HV1
functional non-conformance upper limit U_OFU	U_OFU	***19***	5,431			HV1
adjustment of the interval limit (lower limit)	U_UIA	***20***	-0,71			HV1
adjustment of the interval limit (upper limit)	U_OIA	***20***	2,51			HV1
measurement uncertainty function in the overall view						
measurement uncertainty function lower limit	**U_UG(x)**	***22***	**-0,024619**	*** x +**	**-3,538**	**HV1**
measurement uncertainty function upper limit	**U_OG(x)**	***22***	**0,025827**	*** x +**	**3,619**	**HV1**
values measurement uncertainty function lower limit without correction	U_UG(x)	***22***	-9,44	-16,72	-21,75	HV1
values measurement uncertainty function upper limit without correction	U_OG(x)	***22***	9,81	17,45	22,72	HV1
measurement uncertainty function at the calibration with correction		*formula*				
regression function			1,0028238	* X +	1,6157	
correction function MR (symmetric or systematic deviation)	f_korr(x=s)		0,9971842	* Y +	1,61115	
values at the data points with f_korr(x)	f_korr(x=s)		237,6	532,5	735,9	HV1
symmetry deviation Xs - f_korr(x=s)	**Δf_korr,s**		0,61	-1,52	0,91	HV1
measurement uncertainty standard lower limit	U_NAU	***9***	4,11	8,58	14,91	HV1
measurement uncertainty standard upper limit	U_NAO	***9***	2,89	11,62	13,09	HV1
measurement uncertainty without correction						
measurement uncertainty lower limit of the interval	U_AKU	***12***	-4,46	-9,28	-15,59	HV1
measurement uncertainty upper limit of the interval	U_AK0	***12***	3,37	12,15	13,87	HV1
symmetry is neither possible with correction of the values						
measurement uncertainty without correction of the symmetry limits of interval	**U_AK**	***11***	**3,37**	**12,15**	**13,87**	HV1
measurement uncertainty function according the calibration with correction function						
number of data points	k =		3			
estimator function (lower limit) UG	$\hat{U}_{UG}(x)$		-0,021839	* X +	1,251	
estimator function (upper limit) OG	$\hat{U}_{OG}(x)$		0,021635	* X +	-1,329	
measurement uncertainty at the estimator function data points (low) U_UG(x)	$\hat{U}_{UG}(x)$	***16***	-3,9	-10,4	-14,8	HV1
measurement uncertainty at the estimator function data points (up) U_OG(x)	$\hat{U}_{OG}(x)$	***16***	3,8	10,2	14,6	HV1
non-conformance measurement uncertainty of measurement function						
deviation U_data point lower limit (values calculated) fU_UG(x=s)	**ΔU_UGs**	*17*	-0,53	1,10	-0,77	HV1
deviation U_data point upper limit (values calculated) fU_OG(x=s)	**ΔU_OGs**	*17*	-0,44	1,95	-0,72	HV1
standard deviation non-conformance of the function lower limit	sU_UG	***18***	1,02			**HV1**
standard deviation non-conformance of the function upper limit	sU_OG	***18***	1,47			**HV1**
t factor (s-1 = 2, a-1 = 95%)			4,303			
measurement uncertainty functional non-conformance lower limit	U_UFU	*19*	-2,522			HV1
measurement uncertainty functional non-conformance upper limit	U_OFU	*19*	3,657			HV1
measurement uncertainty limit of the interval (lower limit)	U_UIA	***20***	-0,77			HV1
measurement uncertainty limit of the interval (upper limit)	U_OIA	***20***	1,95			HV1
measurement uncertainty function in the overall view						
measurement uncertainty function lower limit	**U_UG(x)**	***22***	**-0,021839**	*** X +**	**-2,041**	**HV1**
measurement uncertainty function upper limit	**U_OG(x)**	***22***	**0,021635**	*** X +**	**4,282**	**HV1**
values measurement uncertainty function lower limit	U_UG(x)	***22***	-7,23	-13,67	-18,11	HV1
values measurement uncertainty function upper limit	U_OG(x)	***22***	9,42	15,80	20,20	HV1

Regarding the functional presentation of the measurement uncertainty in the example, this yields asymmetric interval limits with and without correction over the functional presentation of the measurement uncertainty.

Measurement uncertainty functions without correction:

- Measurement uncertainty, lower limit: $U_{UG}(x) = -0{,}025 * x - 3{,}54$ (basis: measured values)
- Measurement uncertainty, upper limit: $U_{OG}(x) = 0{,}026 * x + 3{,}62$ (basis: measured values).

Measurement uncertainty functions after correction of symmetric deviations:

- Measurement uncertainty, lower limit: $U_{UG}(x) = -0{,}022 * x - 2{,}04$ (basis: corrected values)
- Measurement uncertainty, upper limit: $U_{OG}(x) = 0{,}022 * x + 4{,}82$ (basis: corrected values).

The modelling of the interval limits through a function is done by combining

$U(x) = max[abs(U_{UG}(x); abs(U_{OG}(x)]$ for each x, if so the result is a new function.

7. LITERATURE

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