Synthesis of the Measuring Scale for Shape of Electrical Signals

Tadeusz PIWOWARCZYK
Cracow University of Technology
Department of Electrical and Computer Engineering
Institute of Electrical Metrology E-1
24 Warszawska st., 31-155 Cracow, Poland

ABSTRACT

The paper presents an application of the linear algebra in the creation of a measuring scale for some class of multidimensional quantities. It shows how important the following two elements are: the domain of objects (signals), and the method of their comparison. In most cases when we change one of those elements or both, the sense of the measuring quantity which is to be defined also changes. The paper shows how to select a set of primary standards and how to generate a set of secondary standards. The standards are modelled by a set of vector bases, and the changed set of standards is modelled by a linear transformation by using a transient matrix. The measuring process is modelled by a linear combination, and the "zero method" by a certain linear dependence. Two scales are created in order to illustrate the above process. One for the "shape" of a steady solution, and the other for the "shape" of a transient solution, both in the RLC linear electrical circuit .

Keywords: Multidimensional quantity, domain, comparison, standards, linear combination, linear dependence, measuring scale, basis change.

1. INTRODUCTION

For some reason, the shape[1] of electrical responses in the RLC linear circuits has been taken as a certain measuring property of electrical signal.

The first objective is to consider the limitation of the domain of the subspace V_h which contains exclusively electrical responses which are described by the m harmonic functions i.e. the steady solutions. The

"natural" approach which forces itself here is to use the 2m samples according to the Kotelnikov – Shannon theorem, and to create a 2m dimensional scale. In case of some steady responses it seems however more reasonable to give up the 2m samples and to introduce some other equivalent method. Such a method as will be shown below, has definite practical advantages. The proposed method is created around the terms belonging to the linear algebra.

The second objective is to consider the space V which contains all the electrical responses described by a certain composition of the sine wave functions, both exponent and polynomial functions. In other words, we turn here to the transient solutions. The "natural" approach which becomes obvious is to use 2m samples according to some generalization Kotelnikov – Shannon theorem, and to create a 2m dimensional scale.

The transient responses can be modelled by the so called finite numerical sequences [2]. They create a certain vector space V which has some interesting properties, sometimes quite unexpected. One of them is the above mentioned Kotelnikov – Shannon generalized theorem. The theory of finite numerical sequences provides a highly abstract algebraic method of solving, in a very simple and fast way, the transient state in the RLC electrical circuits. In the measurement science this theory works as a theoretical basis for the creation of the measuring scale for the "shape" of transients and steady responses.

The shape of electrical responses can be treated as a measuring quantity similar to other quantities like mass, electric charge, electric current and so on. It can be compared to the elected standards. The comparison bears all the stigma of the "zero method". The measuring scale is multidimensional and the result of the measuring process is the set of numerical values (vectors) each with a given error evaluation. Finally the shapes can be combined (concatenated) in a similar way to the mass concatenation, the charge concatenation, and so on.

2. MEASURING SCALE FOR THE STEADY RESPONSE IN THE RLC LINEAR ELECTRICAL CIRCUITS

This chapter presents the synthesis of the measuring scale for the shape of a certain class of steady responses. What needs to be done at first is the definition

---

[1] It is important to underline, that in a general case, when we try to analyze a new measuring quantity we do not have a word to talk about it. It has to be made up and put forward as a proposition. The absence of a name for the quantity, especially in the first stage of the investigation, is a serious obstacle for a researcher who cannot communicate the result of his work. It is amazing that the greatest inventors and scientists have been able to cope with this problem. In the example presented above we are in a much better situation since we know the name which is " the shape of an electrical transient response". We have the intuition of what the shape is. This intuition has been developed in all natural languages and it belongs to the realm of culture. Thus, due to this fact, we can communicate to others the results of our research although only approximately.

of the scope of our objects i.e. steady electrical responses.

2.1. LIMITATIONS OF THE STEADY RESPONSES (THEORY)

One of the most important activities when creating a new measuring quantity is to define the exact area of objects or signals with clearly given restrictions and constraints.

Steady electrical responses that we have to do with in practice exist in an   enormous, infinite variety of shapes. Creating just one theory covering all possible cases seems to be of little importance. What seems to be of most interest of all   and not without   practical implications, is to elaborate a new measuring scale for some restricted area. Thus we can consider an instance of the steady shapes occurring in the RLC linear electrical circuits when the generators produce the   sine wave functions with the limited number of m harmonics.
We also have to do with an analogous limitation in case of one dimensional quantity, like mass, for example. When measuring the mass of the atomic particles we follow a specific procedure which differs from the procedures applied when measuring the mass   in everyday life. And the   masses of cosmic objects are measured (or evaluated) in still some other, sometimes quite a sophisticated way.
Let us consider that the RLC linear circuit contains a certain number of voltage and current generators realizing the functions with only m harmonics.

$$f(t)=C_1\sin(\omega t+\varphi_1)+ C_2\sin(2\omega t+\varphi_2)+...+ C_m\sin(m\omega t+\varphi_m) \quad (1)$$

We know that with certain restrictions any electrical current and any voltage drop in such a circuit (in a steady state) can be described by this function. We know the function if we know ω and 2m parameters i.e. $C_1, C_2 ,..., C_m, \varphi_1, \varphi_2 ,..., \varphi_m$ which assume real values.

The set of all functions (1) constitutes a vector space V_h of the dimension 2m. The zero function denoted by the symbol "0" belongs to the vector space V.

The concatenation of signals is, of course, just their summation and it is denoted by a symbol "+". The multiplication of signals by a real number is just the amplification of the signals. In practice we need to have the amplifications of the coefficient k which would satisfy

$$- 1 \leq k \leq 1 \quad (2)$$

This means that we can just operate with simple voltage dividers of good precision.

2.2. COMPARATION OF SHAPE OF SIGNALS (THEORY)

In order to define any measuring quantity we must have a laboratory method to compare any two objects, any two signals $f_1(t)$ and $f_2(t)$ in this case. To find such a method it is enough to check if there is a difference between the   two signals. If the difference is

zero for all t then we can conclude that two signals are identical

$$f_1(t) - f_2(t) = 0 \quad (3)$$

We can use a sensitive oscilloscope or a sensitive RMS device. The high sensitiveness of the indicator is its key property. On the other hand the fidelity of the indicator is of little importance. The zero indicator in this case has the   properties analogous to those of the galvanometer.

2.3. STANDARDS OF SHAPE (THEORY)

Once we know the exact set of selected electrical signals (1), and once we have the lab method to compare those signals (3) we have all the necessary information needed in order to construct in our mind a physical notion of a measuring quantity. We can call this quantity a "shape of electrical signals". The sense of "the shape" is a function of the selected electrical signals and of the laboratory comparison. If we change any one of the two above mentioned components we will obtain a new meaning of the "shape".

From the available electrical signals we select a set named STAND of 2m signals which are independent in the vector space V.

$$STAND_1(t), \ STAND_2(t), ..., STAND_{2m}(t) \quad (4)$$

The selected standards should satisfy a long list of necessary properties   like stability, fidelity, availability, facility in operation, maintainability and many others.

It means that any standard signal $STAND_i(t)$ cannot be expressed as a linear combination of the others.

2.4. "AUTOCRACY" IN MEASUREMENT

It is important to stress that the primary standard signals (4) as the physical phenomena realized in a measurement lab remain unknown, and their shapes especially. Being the primary standards of shape they should be distributed in all labs in form of the secondary standards of shape. The results of the measurement of shapes can therefore be communicated between labs. Using secondary standards it is easy to create any shape and we can communicate it by giving its measuring data.
  An analogous situation occurs in case of   primary standards. For example the standard of mass (1 kg object in Sevre) also remains unknown. The number of protons, neutrons and other particles in it remain unknown! By definition we assume with infinite precision that its value is 1 kg . There is no room for any discussion on this matter! We have to do here with some kind of autocracy in metrology! This autocracy however is very helpful for science. Sometimes a revolution   of some sorts may affect the definition of standards . The same may also happen as a result of some slow evolution.

2.5. MULTIDIMENSIONAL PRIMARY STANDARD (THEORY)

One-dimensional measuring quantity, like the mass for example, has only one primary standard. There is nothing to analyse in such a case.

For a multidimensional measuring quantity, like the electrical signal shape, we may discuss some aspects of its multidimensional standard. There is no violation involved here of the principle that the basic idea of a standard is beyond discussion.

The independence of signals can be tested in the following procedure: any signal (1) can be written in form of

$$f(t)=A_1\sin(\omega t)+A_2\sin(2\omega t)+...+A_m\sin(m\omega t)+...$$
$$+B_1\cos(\omega t)+B_2\cos(2\omega t)+...+B_m\cos(m\omega t) \qquad (5)$$

Let us assume, that the selected 2m standards are denoted by

$$STAND_1(t)=A_{11}\sin(\omega t)+A_{12}\sin(2\omega t)+...+A_{1m}\sin(m\omega t)+...$$
$$+B_{11}\cos(\omega t)+B_{12}\cos(2\omega t)+...+B_{1m}\cos(m\omega t) \qquad (6)$$

$$STAND_2(t)=A_{21}\sin(\omega t)+A_{22}\sin(2\omega t)+...+A_{2m}\sin(m\omega t)+...$$
$$+B_{21}\cos(\omega t)+B_{22}\cos(2\omega t)+...+B_{2m}\cos(m\omega t) \qquad (7)$$
....

$$STAND_{2m}(t)=A_{2m1}\sin(\omega t)+A_{2m2}\sin(2\omega t)+...+A_{2mm}\sin(m\omega t)+...$$
$$+B_{2m1}\cos(\omega t)+B_{2m2}\cos(2\omega t)+...+B_{2mm}\cos(m\omega t) \qquad (8)$$

In this transformation we can compose the following square matrix S

$$S=\begin{bmatrix} A_{11} & A_{12} & \cdot & A_{1m} & B_{11} & B_{12} & \cdot & B_{1m} \\ A_{21} & A_{22} & \cdot & A_{2m} & B_{21} & B_{22} & \cdot & B_{2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{2m1} & A_{2m2} & \cdot & A_{2mm} & B_{2m1} & B_{2m2} & \cdot & B_{2mm} \end{bmatrix} \qquad (9)$$

The chosen standards are independent if the matrix S is non singular. It is the first condition which should be satisfied by the chosen standards.

The second condition is also obvious. The determinant of the matrix S should be clearly different from zero i.e. $\det(S) >> 0$ or $\det(S) << 0$. It means that any two standards should be clearly different. The best situation is when the scalar product of two different functions is zero (orthogonal case)

$$\int_0^T STAND_i(t)\cdot STAND_j(t)\,dt=0 \qquad i\neq j \qquad (10)$$

2.6. MEASURING SCALE OF SHAPE (THEORY)

The set of electrical signals and the method of comparing their shapes constitute the logical basis of the notion of shape. If the set of signals and/or the method of comparing change, the meaning of the shape changes in a general case.

The above described method of comparing shapes, the compensation method in this case, may serve

as a basis to design special measuring devices. Of course such devices must be doted with selected standards. There also must exist a logical procedure describing how to use the measuring devices and how to operate with standards of shape. The block diagram presented below shows an apparatus which has got chosen standards. This apparatus is a certain kind of the voltage compensator. The galvanometer is replaced by the oscilloscope or by a sensitive RMS device or by the 2m vibration galvanometers sensitive to the pulsation ω, 2ω,...,$m\omega$. To avoid the amplification greater than 1 in the apparatus there are 2m + 1 voltage dividers.

It is a practical problem how to compensate the measured shape if we have a rather large number of regulation points. The Monte Carlo method can be helpful in this case. We can also use other methods to diminish the search area. When the strip to be searched is narrow for each regulation point we can start with a proposed apparatus to compensate the unknown shape of the measuring electrical signal. As the result of the measurement we get the following linear combination

$$x(t)-[a_1 STAND_1(t)+a_2 STAND_2(t)+...+a_{2m}STAND_{2m}(t)]=0 \qquad (11)$$

or

$$x(t)=a_1 STAND_1(t)+a_2 STAND_2(t)+...+a_{2m}STAND_{2m}(t) \qquad (12)$$

or a vector composed of coefficients

$$\begin{pmatrix} a_1 & a_2 & . & . & . & a_{2m} \end{pmatrix} \qquad (13)$$

with obligatory maximum errors of each value

$$\begin{pmatrix} \Delta a_1 & \Delta a_2 & . & . & . & \Delta a_{2m} \end{pmatrix} \qquad (14)$$

In conclusion we can say that the scale for the shape of electrical signals is multidimensional. The newly created notion of "shape" is a measurable quantity. It would be of limited importance if it were used in one lab only. Its importance increases to enormous levels when it is used in different labs. In such a case a large number of scientists can have identical intuition associated with a new notion. They can communicate with one another, they can compare their results. The new notion may become an international term. Its existence is not subjective. A particular observer has the impression that this term exists objectively, independently of any human being.

2.7. MULTIDIMENSIONAL SECONDARY STANDARDS (THEORY)

The acceptance of a new measuring quantity by a large milieu of scientists (perhaps the international one) means that this milieu claims to have the secondary standards of the new quantity. The multiplication of the standards refers to the measurement procedure described above.

In case of one dimensional measuring quantity one primary standard STAND is duplicated in a set of secondary standards $stand_1$, $stand_2$, ..., $stand_k$. Each secondary standard $stand_i$ is very similar to the primary standard STAND.

In case of multidimensional measuring quantity the above may not be true. Any particular secondary standard may differ in a significant way from all primary standards. However, the whole set of secondary standards is in some way similar to the whole set of primary standards. It means that we can choose 2m secondary standards denoted by the following symbols

$$\text{stand}_1(t), \ \text{stand}_2(t), \ ..., \ \text{stand}_{2m}(t) \qquad (15)$$

The measuring process leads to the following relation

$$\begin{bmatrix} \text{stand}_1(t) \\ \text{stand}_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{stand}_{2m}(t) \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdot \cdot \cdot \cdot \cdot & c_{12m} \\ c_{21} & c_{22} & \cdot \cdot \cdot \cdot \cdot & c_{22m} \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ c_{2m1} & c_{2m2} & \cdot \cdot \cdot \cdot \cdot & c_{2m2m} \end{bmatrix} \cdot \begin{bmatrix} \text{STAND}_1(t) \\ \text{STAND}_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{STAND}_{2m}(t) \end{bmatrix}$$

$$(16)$$

The square matrix in the above formula has the dimension 2mx2m and it will be denoted by C. In the terms of the vector space this matrix is called a transient matrix (properly speaking the transpose of this matrix denoted by C^T). When transferring a secondary standard to any lab "NR 2" it is obligatory to join the matrix C with the evaluated precision organized also in a matrix form and denoted by ΔC.

Let us suppose that in the lab "NR 2" the shape of the signal x(t) is measured. It means that the following linear combination is found

$$x(t) = c_1 \cdot \text{stand}_1(t) + c_2 \cdot \text{stand}_2(t) + ... + c_{2m} \cdot \text{stand}_{2m}(t)$$

$$(17)$$

Of course, each value c_i is known with the appropriate maximum error Δc_i. Now the lab NR 2 wants to express the result of the measurement in terms of primary standards. When substituting (15) in (16) we obtain the following formula

$$x(t) = \begin{pmatrix} c_1 & c_2 & \cdot \cdot \cdot \cdot \cdot & c_{2m} \end{pmatrix} \cdot \begin{bmatrix} c_{11} & c_{12} & \cdot \cdot \cdot \cdot \cdot & c_{12m} \\ c_{21} & c_{22} & \cdot \cdot \cdot \cdot \cdot & c_{22m} \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ c_{2m1} & c_{2m2} & \cdot \cdot \cdot \cdot \cdot & c_{2m2m} \end{bmatrix} \cdot \begin{bmatrix} \text{STAND}_1(t) \\ \text{STAND}_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{STAND}_{2m}(t) \end{bmatrix}$$

$$(18)$$

Similarly in the lab "NR 3" operating with the secondary standards the same shape of the signal x(t) has the form

$$x(t) = \begin{pmatrix} d_1 & d_2 & \cdot \cdot \cdot \cdot \cdot & d_{2m} \end{pmatrix} \cdot \begin{bmatrix} d_{11} & d_{12} & \cdot \cdot \cdot \cdot \cdot & d_{12m} \\ d_{21} & d_{22} & \cdot \cdot \cdot \cdot \cdot & d_{22m} \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ d_{2m1} & d_{2m2} & \cdot \cdot \cdot \cdot \cdot & d_{2m2m} \end{bmatrix} \cdot \begin{bmatrix} \text{STAND}_1(t) \\ \text{STAND}_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \text{STAND}_{2m}(t) \end{bmatrix}$$

$$(19)$$

Finally we can write the following identity $[c_i][c_{ij}] = [d_i][d_{ij}]$. When transforming it we obtain the following two formulas

$$[c_i] = [d_i][d_{ij}][c_{ij}]^{-1} \quad \text{or} \quad [d_i] = [c_i][c_{ij}][d_{ij}]^{-1} \qquad (20)$$

Both formulas are useful when it comes to comparing the results of measuring of the same shape performed in two labs, lab "NR 2" and lab "NR 3". From the economy point of view of the notation it is better to write both formulas in a transposed form

$$\begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ c_{2m} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{21} & \cdot \cdot \cdot \cdot \cdot & c_{2m1} \\ c_{12} & c_{22} & \cdot \cdot \cdot \cdot \cdot & c_{2m2} \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ c_{12m} & c_{22m} & \cdot \cdot \cdot \cdot \cdot & c_{2m2m} \end{bmatrix}^{-1} \begin{bmatrix} d_{11} & d_{21} & \cdot \cdot \cdot \cdot \cdot & d_{2m1} \\ d_{12} & d_{22} & \cdot \cdot \cdot \cdot \cdot & d_{2m2} \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ \cdot & \cdot & \cdot \cdot \cdot \cdot \cdot & \cdot \\ d_{12m} & d_{22m} & \cdot \cdot \cdot \cdot \cdot & d_{2m2m} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ d_{2m} \end{bmatrix}$$

$$(21)$$

Any numerical value in the formula right above is known with a certain maximum error Δc_i, Δc_{ij}, Δd_{ij}, Δd_i. The full Δc_i error analysis of the left column might be quite an interesting task, however it would take a separate paper to discuss it.

2.8. EXAMPLE - COMPUTER SIMULATION

In this example we analyse the shapes of the electrical voltage drop described by the function (1) for m =3. We select 6 different electrical signals which are taken as primary standards of shape. It is not necessary to know their exact functions. However we can visualize their shapes using a simple oscilloscope.

$$(22)$$

$$(23)$$

(24)

(25)

(26)

(27)
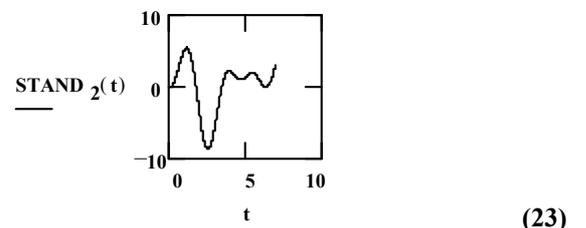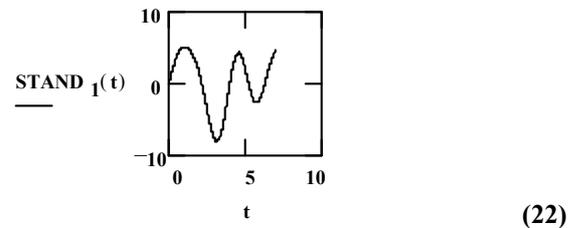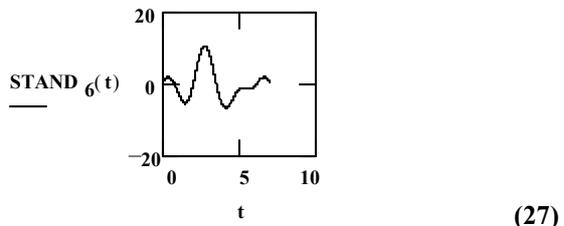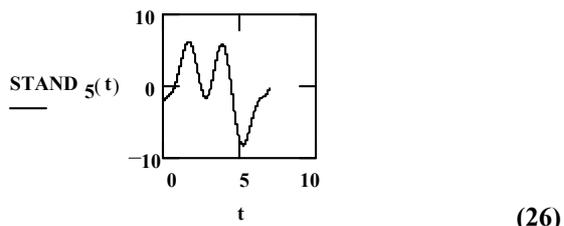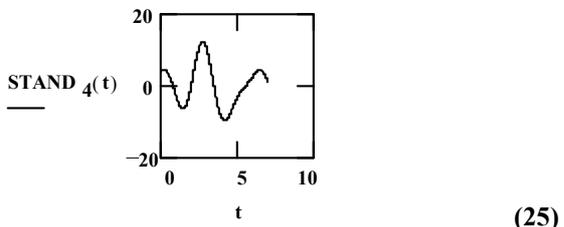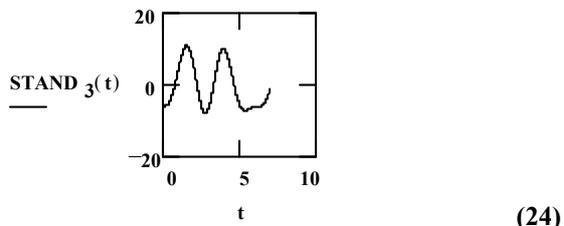
The above oscilloscope visualization is about 10% accurate. However it is of little importance for the compensation measurement with zero method. This visualization can be helpful in acceleration of the compensation procedure.

Let us suppose that we have to measure the following shape x(t). We can also visualize its shape with same remarks as above.

(28)
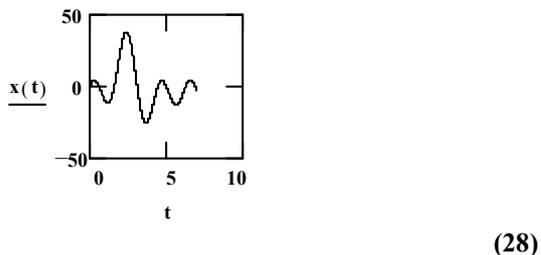
Now we can perform the measuring activities by using the measuring compensation device described above. When searching for the equilibrium - zero indicated by a "generalized galvanometer" we find the following linear decomposition

x(t)=3.172STAND$_1$(t)-5.463STAND$_1$(t)+2.821STAND$_1$(t)+...
3.116STAND$_1$(t)-3.332STAND$_1$(t)-1.751STAND$_1$(t)

(29)

As a result of measuring we get the coefficients of the linear combination. From the voltage dividers and the sensibility of the "generalized galvanometer" we are able to read the maximum error.
The result of measuring is the following

$$(\ 3.172 \quad -5.463 \quad 2.821 \quad 3.116 \quad -3.332 \quad -1.751 \ ) \ \textbf{(30)}$$

The vector of maximum errors is as follows

$$(\ 0.001 \quad 0.001 \quad 0.001 \quad 0.001 \quad 0.01 \quad 0.001 \ ) \qquad \textbf{(31)}$$

2.9. CONCLUSION

The paper shows how to create a new scale for the measuring of the shape of electrical signals. But there are also other possible ways to go about this problem.. For example we can draw from the Kotelnikov - Shannon theorem where 2m samples are analysed. Whatever the approach, the unknown shape is decomposed in standard shapes (a linear combination). It is advisable, if possible, to select the standards whose shapes are similar to the measuring shapes of signals (similia similibus curantur). If this is not possible any set of 2m independent shapes can be chosen.

The shape of electrical signals is measured on a multidimensional scale. Such a scale produces a large number of properties unknown in a one dimensional scale. Some of them are presented in the paper.

The full introduction into science of the new property called "the shape" with its measuring scale would take place at the moment when this new property would be implemented in physical laws and when it would be combined with other existing quantities. But this process is rather a long one and it belongs to the evolutionary process in science.

It should be underlined that the method of measuring the shape as a quantity developed in this paper is only an example which serves to illustrate the fundamentals of the theory of measurement. The simulation presented in this paper should be treated as an introductory step in the process of the preparation for the most serious task which is the creation of the measurement scale for a newly recognized measuring quantity, a multidimensional one in particular.

2.10. POST SCRIPTUM

Now we can reveal the functions of the primary standards which are hidden in a computer simulation. Using a computer we can simulate a real situation in a lab where the primary standards remain unrecognized in their shapes and are undisputable

STAND$_1$(t)= 1sin(t)+2sin(2t)+1sin(3t)+
3cos(t)-4cos(2t)+1cos(3t) **(32)**

STAND$_2$(t)=-1sin(t)+3sin(2t)-2sin(3t)+
3cos(t)-2cos(2t)-1cos(3t) **(33)**

STAND$_3$(t)= 2sin(t)+5sin(2t)-4sin(3t)+
-3cos(t)-5cos(2t)+2cos(3t) **(34)**

STAND$_4$(t)= 3sin(t)-4sin(2t)+3sin(3t)+
-1cos(t)+6cos(2t)-1cos(3t) **(35)**

$STAND_5(t)= 3\sin(t)+3\sin(2t)-2\sin(3t)+$
$\qquad -3\cos(t)-1\cos(2t)+2\cos(3t)$ (36)

$STAND_6(t)= 2\sin(t)-3\sin(2t)+3\sin(3t)+$
$\qquad -2\cos(t)+4\cos(2t)-1\cos(3t)$ (37)

Now, knowing the above functions, we can check their linear independence.

$$\left\| \begin{vmatrix} 1 & 2 & 1 & 3 & -4 & 1 \\ -1 & 3 & -2 & 3 & -2 & -1 \\ 2 & 5 & -4 & -3 & -5 & 2 \\ 3 & -4 & 3 & -1 & 6 & -1 \\ 3 & 3 & -2 & -3 & -1 & 2 \\ 2 & -3 & 3 & -2 & 4 & -1 \end{vmatrix} \right\| = 776 \qquad (38)$$

Thus the determinant differs from zero and assumes a rather large value. It means that the primary standards have been well chosen.

Now we can check if the result of measuring (30) is correct.

3. MEASURING SCALE FOR THE TRANSIENT RESPONSE IN RLC LINEAR ELECTRICAL CIRCUITS (OUTLINE OF THEORY)

Any transient response in the RLC linear electrical circuits can be described by a function composed of sin wave functions, both exponent and polynomial functions. In other words any response can be described by the Laplace transform $F_k(s)$ which is a rational function of the given order denoted by "k". For k=1,2,3 the analyzed Laplace transforms have the following shape

$$F_1(s) = \frac{b_0}{s + a_0} \qquad (39)$$

$$F_2(s) = \frac{b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0} \qquad (40)$$

$$F_3(s) = \frac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{s^3 + a_2 \cdot s^2 + a_1 \cdot s + a_0} \qquad (41)$$

etc.

It is evident, that for the order k there are 2k coefficients in the numerator and denominator of the Laplace transform. This obvious fact is a clue to how to create a measuring scale. All we have to know is how to measure those coefficients and how to evaluate the corresponding error.

A much better way is to transform the above coefficients using the following transformations

$$\begin{pmatrix} A_0 \\ A_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ a_0 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} b_0 \\ 0 \end{pmatrix} \qquad (42)$$

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_1 & 1 & 0 & 0 \\ a_0 & a_1 & 1 & 0 \\ 0 & a_0 & a_1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_0 \\ 0 \\ 0 \end{bmatrix} \qquad (43)$$

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_2 & 1 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 1 & 0 & 0 & 0 \\ a_0 & a_1 & a_2 & 1 & 0 & 0 \\ 0 & a_0 & a_1 & a_2 & 1 & 0 \\ 0 & 0 & a_0 & a_1 & a_2 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} b_2 \\ b_1 \\ b_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (44)$$

etc.

The above transformation results form the theory of symmetric polynomials of multiple variables in multipower notation [1]. It is proved in [2] that the coefficients $A_0,A_1,A_2,…,A_{2k-1}$ have very simple physical interpretation. They are the derivatives in zero of the measuring transients. It means that it is sufficient to know 2k first derivatives with corresponding errors in order to accomplish the measuring process of the shape of the transient response.

Once, we know the following measured coefficients $A_0,A_1,A_2,…,A_{2k-1}$ we can recover the coefficients of the Laplace transform using the following inverse transformations

$$\begin{pmatrix} b_0 \\ a_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -A_0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} A_0 \\ A_1 \end{pmatrix} \qquad (45)$$

$$\begin{bmatrix} b_1 \\ b_0 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -A_0 & 0 \\ 0 & 0 & -A_1 & -A_0 \\ 0 & 0 & -A_2 & -A_1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} \qquad (46)$$

$$\begin{bmatrix} b_2 \\ b_1 \\ b_0 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -A_0 & 0 & 0 \\ 0 & 0 & 1 & -A_1 & -A_0 & 0 \\ 0 & 0 & 0 & -A_2 & -A_1 & -A_0 \\ 0 & 0 & 0 & -A_3 & -A_2 & -A_1 \\ 0 & 0 & 0 & -A_4 & -A_3 & -A_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} \qquad (47)$$

etc.

The above transformations also result from the theory of symmetric polynomials of multiple variables in a multipower notation [1].

The coefficients $A_0, A_1, A_2, \ldots, A_{2k-1}$ can be expanded to the infinite series $A_0, A_1, A_2, \ldots, A_{2k-1}, A_{2k}, \ldots$ by using the following recurrence written in the matrix form for k=1,2,3.

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \cdot \\ A_k \\ \cdot \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -A_0 \\ 0 & -A_1 \\ 0 & -A_2 \\ \cdot & \cdot \\ 0 & -A_{k-1} \\ \cdot & \cdot \end{bmatrix} \cdot \begin{pmatrix} b_0 \\ a_0 \end{pmatrix} \qquad (48)$$

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \cdot \\ A_k \\ \cdot \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -A_0 & 0 \\ 0 & 0 & -A_1 & -A_0 \\ 0 & 0 & -A_2 & -A_1 \\ 0 & 0 & -A_3 & -A_2 \\ 0 & 0 & \cdot & \cdot \\ 0 & 0 & -A_{k-1} & -A_{k-2} \\ 0 & 0 & \cdot & \cdot \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_0 \\ a_1 \\ a_0 \end{bmatrix} \qquad (49)$$

$$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \cdot \\ A_k \\ \cdot \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -A_0 & 0 & 0 \\ 0 & 0 & 1 & -A_1 & -A_0 & 0 \\ 0 & 0 & 0 & -A_2 & -A_1 & -A_0 \\ 0 & 0 & 0 & -A_3 & -A_2 & -A_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & -A_{k-1} & -A_{k-2} & -A_{k-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \cdot \begin{bmatrix} b_2 \\ b_1 \\ b_0 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \qquad (50)$$

etc.

The infinite sequence $A_0, A_1, A_2, \ldots, A_{2k-1}, A_{2k}, \ldots$ can be given a multiple interpretation. First of all it can be viewed as the Markov chain or a generalized Fibonacci sequence. Also its elements divided by subsequent k! may be interpreted as the coefficients of the Maclaurin series of the function f(t).

$$f(t) = A_0 + \frac{A_1}{1!} \cdot t + \frac{A_2}{2!} \cdot t^2 + \ldots + \frac{A_k}{k!} \cdot t^k + \ldots \qquad (51)$$

From the above it follows that a certain polynomial (52) of n terms

$$p(t) = A_0 + \frac{1}{1!} \cdot A_1 \cdot t + \frac{1}{2!} \cdot A_2 \cdot t^2 + \ldots + \frac{1}{n!} \cdot A_n \cdot t^n \qquad (52)$$

satisfying

$$k \le n \le 2 \cdot k \qquad (53)$$

has a twofold meaning: it is an approximation of the function f(t), and at the same time it stores full information about the exact shape of the function f(t).

This effect is called a "fractal" or a "hologram" effect in a transient response. More on this topic can be found in [2].

3.1. EXAMPLE OF A MEASURING SCALE FOR THE TRANSIENT RESPONSE

As an example we consider a certain transient response described by the following function f(t)

$$f(t) = 3 \cdot e^{-5 \cdot t} \cdot \sin(2 \cdot t) \qquad (54)$$

Using the above theory of finite number sequences we can measure the appropriate values of the function f(t) and its derivative for t=0. As the result we get the sequence $A_0, A_1, \ldots$ Now we look for the biggest, non singular, square matrix which is in (45),(46),(47) and so on. If the biggest matrix has the degree 2nx2n, the sequence $A_0, A_1, \ldots, A_{2n-1}$ can be treated as an element of multidimensional scale for the shape of transients appearing in the RLC linear electrical circuits. Of course we must define the error Δ. In case of the shape (54) the following element of a 4 dimensional scale occurs

$$[0, 6, -60, 426], \quad [\Delta_1, \Delta_2, \Delta_3, \Delta_4] \qquad (55)$$

Full information about the transient response (54) is coded in 4 numbers in the sequence (55).

4. COMPENSATION AND "ZERO" DETECTOR – BLOCK DIAGRAM

When applying a linear combination and a linear dependence we can create a difference between the measuring signal and the linear combination of standards (primary or secondary). The following function $\delta(t, c_1, c_2, c_3, c_4, c_5, c_6)$ represents the difference between the measuring shape and the shape composed of the linear combination of 6 elected shapes of standards.

$$\delta(t, c_1, c_2, c_3, c_4, c_5, c_6) := x(t) - \begin{pmatrix} c_1 \cdot STAND_1(t) + c_2 \cdot STAND_2(t) + c_3 \cdot STAND_3(t) \ldots \\ + c_4 \cdot STAND_4(t) + c_5 \cdot STAND_5(t) + c_6 \cdot STAND_6(t) \end{pmatrix}$$
$$(56)$$

As the criteria of "zero" we assume the RMS value of the function $\delta(t, c_1, c_2, c_3, c_4, c_5, c_6)$. For the sake of simplicity we compute a square root of the RMS multiplied by 2π.

$$RMS(c_1, c_2, c_3, c_4, c_5, c_6) := \int_0^{2 \cdot \pi} \delta(t, c_1, c_2, c_3, c_4, c_5, c_6)^2 \, dt$$
$$(57)$$

Now, it is sufficient to find the coefficients $c_1, c_2, c_3, c_4, c_5, c_6$ of the linear combination when the function $RMS(c_1, c_2, c_3, c_4, c_5, c_6) = 0$. To get this in the laboratory we can use a special device.

The idea of what it is like is presented in the following block diagram.
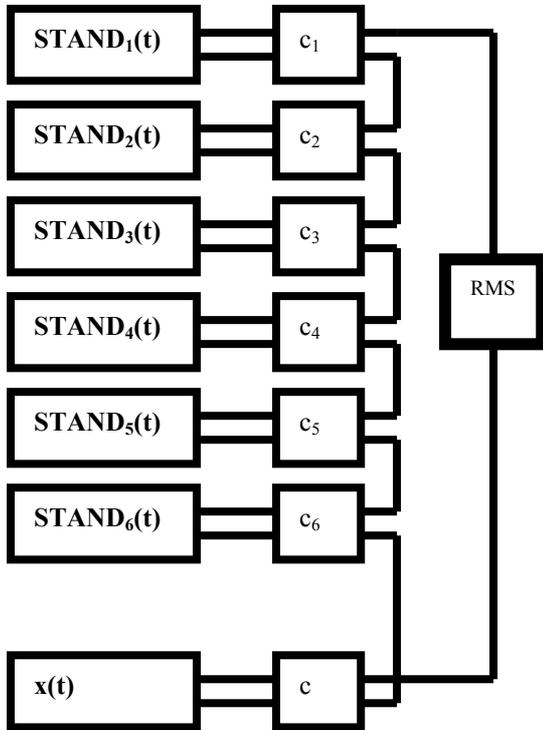
The blocks $STAND_i(t)$ for $i = 1, 2, 3, 4, 5, 6$ represent the standards of shape of the voltage of electrical signals in the selected domain. The block $x(t)$ represents an unknown shape of the voltage of an electrical signal. The blocks denoted by $c_1, c_2, c_3, c_4, c_5, c_6$ and c represent the calibrated, high resistance voltage dividers. The block RMS represents a highly sensitive RMS indicator.

5. COMPUTER PROGRAM FOR COMPENSATION SIMULATION

Below there is a special computer program which illustrates the idea of the lab procedures to follow when measuring the shape of an electrical signal (28) and using the measuring device of the block diagram shown above.

The values N_0 and n are to be initialized. The result of the simulation of the measurement becomes available in the column when we type "=". The accuracy of the measuring results $c_1, c_2, c_3, c_4, c_5, c_6$ depends of the sensitiveness of RMS indicator.

The accuracy of the measurement can be revealed by a special subprogram added to the one presented above.

6. BIBLIOGRAPHY

1. Piwowarczyk T., Multipower Notation of Symmetric Polynomials in Engineering Calculus, PAN, WGISMiE, Cracow, 2000
2. Piwowarczyk T., Theory of Finite Numerical Sequences in Electrical Circuits Analysis, CCNS, Cracow, 2002