# A New Approach to Generation of Measurement Methods

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## ABSTRACT

In this paper two logical levels of the measurement procedure are discussed: physical level, on which the magnitude of a measurand is equated with the magnitude of a material measure; formal level, on which a value is assigned to the measurand. There are three types of logical functions involved in the measurement procedure: introduction of measurand and metric; change of quantity. Depending on the number of particular quantities, two models are proposed: two quantities are involved - measurand and material measure; three quantities are involved - measurand, material measure and one auxiliary constant quantity. According to SI all possible operations are described by an additive algebra. Based on possible physical interactions between quantities, common variants have been synthesized for creation of measurement methods. The second model allows proposing a class of methods based on using an inversion. The proposed approach allows coverage of all possible measurement methods applicable to the measurement of extensive quantities.

**Keywords**: Measurement procedure, Model of measurement procedure, Method of measurement, Method with inversion, Method classification.

## 1. INTRODUCTION

The essence of measurement is explained by the representational theory of measurement [1] as a process of receiving, in an accepted formal system, of an image of a certain property of an object from a certain real system. The process structure comprises the following stages:
1) choice of the unit of measurement or the respective scale
2) creation of material measures to be used in measurement
3) the measurement procedure of assigning numerical value to the measurand by using previous created material measures.

The subject of the present research is the measurement procedure for extensive quantities. Measurement of these quantities can be accomplished only based on chosen unit. Development of a scale (ratio scale) and using of parts thereof as material measures, offers more convenience of measurement.

The measurement procedure [2] is characterized by a definite method of measurement and corresponding principle, ensuring the physical realization of the method. Only measurement methods (for assigning a number to the measurand magnitude) are discussed here. These are discussed under the conditions of measurement practice. According to VIM [2], the method is perceived as a “logical sequence of operations”. For measurement procedure research, it is appropriate to split it into two levels : physical one, where the operations are carried out with real objects and their properties, and a formal one, where operations are carried out with object and their properties images in a chosen formal system.

In measurement practice, various measurement methods have been developed in connection to specific tasks, most often related to measurand specifics or to requirements for increase of measuring accuracy. Their description and naming is usually connected with a formal characteristic, such as type of manipulation – transposition, complementing, substitution, etc.

Ornatskiy [3] proposes an approach to formation and classification of measurement methods based on the logic of measurement. He perceives measuring instruments as constructive components of structures. Thus, utilization of instruments is mostly discussed rather than the sequence of operations needed for achieving the final goal.

Muravyov and Savolainen [4] formulate the “general measurement problem” (GMP), which describes the logic of measurement procedure from the point of view of operations and could serve as a starting point for generation of measurement methods.

In this paper is accepted an approach based on GMP, making it more specific. To the highest degree this approach follows the logic of the measurement procedure and specific measurement practice.

## 2. LOGIC OF THE MEASUREMENT PROCEDURE

On the physical level of the measurement procedure we operate with two sets – set *M* of measurands, and set *MM* of quantities of the same kind with known numerical values (material measures).

The set *MM* is structured on a physical (empirical) and on a formal level - each element having characteristics on both levels, magnitude and numerical value, respectively. The magnitude is the material carrier of its image (number) in the formal system as well.

The set *M* is defined only on physical level.

The measurement procedure consists of performing of a physical experiment, where an element from the set *MM* is selected equal in magnitude to a certain element from *M*. Equal magnitudes have equal numerical values (within the prescribed accuracy), which is why the numerical value of material measure is assigned to the measurand. If by $a_1$ we denote measurand magnitude, and by $a_2$ – that of the material measure, the following expression may be written:

$$a_1 = a_2, \quad (1)$$

which must be realized and solved on a physical level by selection of the respective $a_2$. The expression (1) is called a basic equation. It describes the aim of the procedure. If (1) is divided by the magnitude $a_e$ of the unit, it is translated into the language of the formal level and looks, as follows:

$$N_1 = N_2\,, \quad (2)$$

where *N* denotes numerical values of $a_1$ and $a_2$. Thus, the numerical value of $a_2$ is assigning to $a_1$, i.e. the image of $a_1$ is introduced into the formal system.

In compliance with SI, permitted operation, when working with magnitudes of a quantity of certain kind, are only additive ones. On the other hand, extensive quantities can be presented by an equivalent sum of a certain number of component quantities. That is why both sides of the basic equation (1) can be presented as sums of *n* and *m* members, respectively, as follows:

$$a_{11} + a_{12} + \ldots + a_{1n} = a_{11} + a_{12} + \ldots + a_{1m}, \quad (3)$$

where *n* and *m* are integer positive numbers, and their sum defines the number of quantities involved in the procedure. The elements of set *M* and set *MM* are uniform (set *MM* can be defined as a sub-set to *M*), and in the physical experiment (physical level) they are equivalent. That is why, for specification and realization of (3), transposition of addends is permitted between both sides of the equation.

The proposed dependence (3) present more opportunities for realization of the measurement procedure as compared to the simple assignment of numerical values, described by (1). The problem is reduced to finding of images in *MM* of random combinations of addends in (3) and transformation of (3) in the formal system in a form permitting unambiguous solution in relation to the sought numerical value $N_1$.

From the point of view of measurement procedure logic, the basic equation should include elements, which implement the following logical functions:
1. Introduction of measurand magnitude into the measurement procedure.
2. Introduction of metric defined by the chosen unit. At least one of the magnitude involved in the procedure must have know numerical value in order to represent the unit.
3. Change of one side of the basic equation. For successful solving of (1) or (3), it is necessary for at least one of participating magnitudes to assume the function of a variable. The measurand cannot assume this function. It must remain constant until the end of procedure.

The description shows that the essence of a certain measurement method depends on the specific decision of the following problems:
A. Composing of a basic equation, interrelating elements from both sets *M* and *MM*, i.e. specific form of (3). In the sense of SI, the dependence should fallow the rules of the additive algebra.
B. Realization and solving of the equation on a physical level.
C. Translation of the basic equation into the language of the formal system (dependence of numbers)
D. Calculation or assigning of numerical value to the measurand magnitude, i.e. solving of the basic equation in terms of the unknown $N_1$.

In order to realize equation (3), an appropriate physical phenomenon must be selected. When there are difficulties in this respect, transformation into another type of quantity is required.

Solving of equation (3) on a physical level is reduced to the use of algorithm for searching an element of *MM*-set equal in magnitude to the measurand. The possible algorithms are:
- sequential search;
- binary search (division by two). In this case it is necessary to choose an initial state, which can be defined by the interval of the measuring instrument or the initial uncertainty of the measurand.

For solving of problem B, an element in the structured set *MM* must be found. Obviously, it would be appropriate to use suitable coherence of the searching algorithm and structuring rule.

Problem D is solved based on mathematical rules for solving of equations. When there is more than one unknown value, it is necessary to repeat the procedure on a physical level in order to obtain new linearly independent equations in terms of $N_1$.

The specifics of a certain method depend on the specific solving of the four problems. The essence of the method is defined by solving of problem A, and the remaining three only detail it.

A convenient approach to general review of the logic of the measurement procedure is provided by the implementation of categorical method [5], which deals with population of sets and respective morphisms. The measurement procedure involves both sets – *M* and *MM*, and the respective mutual mapping (morphism). In this case the morphism means finding of elements or a combination of elements with equal magnitude from both sets. In this sense, equations (1) and (3) may depict different morphisms or different rules of the same morphism. In both cases the sequence of operations is defined as a measurement method.

## 3. GENERATION OF MEASUREMENT METHODS

Taking into account the above-said, we could propose the following two models for implementation of the measurement procedure (hereinafter referred to as "measurement model") – a model of two quantities (*model 2*) [6] and model of many quantities. *Model 2* has the minimum number of participants (quantities) in the measurement procedure. From the multiple – quantity models, the model of three quantities (*model 3*) has a practical significance. The grounds for implementation of this model could be the fact that the third quantity is actually always present in the measurement procedure – this is the measuring interval or the initial uncertainty of measurand magnitude in a certain measurement. A model with more than three quantities is possible in principle, but complicating the procedure needs practical justification. A single quantity model could be discussed but it does not have any practical sense. Acceptance of such a model would mean that the measurand implements the functions of introduction into the metric and thus resulting numerical value always would be 1. In this case, equation (1) is always true, as the measurand is equal to itself. This model makes sense within the procedure for choice of unit for certain quantities – for instance, mass.

The models described so far refer to possible interactions between particular quantities on a physical level in the following two cases:
a) there are no formal limitations on interacting quantities;
b) there are certain limitations, resulting from the measurement procedure.

Possible algorithms for solving of the basic equation on a physical level require change of at least one of participating magnitudes in the basic equation.
In conducting research, it was necessary to account for the following peculiarities:
1. Measurement variants involving two or three quantities are defined on a physical and on a formal level of the measurement procedure. They refer to extensive and scalar quantities.
2. For convenience, when describing variants on both levels, mathematical symbols are uses, and on physical level, it is necessary to account for the specifics of the measurement procedure.
3. When discussing variants on a physical level, some limiting conditions are required to account for the specifics of undergoing processes.
4. The algebra of the sets of the magnitudes of a certain quantity is additive, pursuant to SI.

When using mathematical symbols to describe the measurement process, it is necessary to consider the specifics, related to measurement. These are, as follows:
a) the symbols for presentation of the physical process should differentiate not only between various magnitudes but also between their carriers. A variant is possible where the same magnitude is represented by different carriers. Then the two particular quantities are not equivalent;
b) the symbol "-" is one of subtraction (difference). It does not possess a physical sense as a sign of the respective magnitude. By subtraction, the operation of comparison is implemented (comparator – *C*);
c) the sign "=" is the one of real physical equalization. The equalization is the process target. The sign does not possess the sense of an operator for value assignment. The equalization (comparison and equalizing operations) contains comparator (*C*) and a feedback, which includes the operation of management of quantity magnitude (*MQM*);
d) the sign "+" denotes summation (adder $\Sigma$);
e) the brackets, used for process description, define the sequence of operations. Both sides of the basic equation are separately formed. Each side is reduced to one quantity via appropriate sequence of operations;
f) the result of the comparison (subtraction) is always a positive quantity.

Based on the adopted conditions (specifics) the formal records

$$a_1 + a_2 = a_3 \quad \text{and} \quad a_1 + a_2 - a_3 = 0 \qquad (4)$$

are not equivalent and have different sense on a physical level. Both records represent different sets of operations and different structures, respectively.

The common variants (without limitations), which could be used to solve the basic equation on a physical level, are listed below:

- for the model of two quantities, these are:
$a_1 + a_2 = 0$ – cannot be used for measurement as participating quantities are scalar and positive;
$a_1 - a_2 = 0$ – the two quantities are compared and their difference is equalized with zero;
$a_1 = a_2$ - the two quantities are equalized.

- for the model of three quantities, these are:
$a_1 + a_2 + a_3 = 0$ - cannot be used for measurement as participating quantities are scalar and positive (not zero);
$a_1 + a_2 = a_3$ - the two quantities are added and then are equalized with a third quantity;
$(a_1 + a_2) - a_3 = 0$ – the two quantities are added and then compared to a third quantity. The result is equalized with zero;
$a_1 - a_2 = a_3$ - the two quantities are compared. The difference is equalized with a third quantity;

$|(a_1 - a_2)| - a_3 = 0$ - the two quantities are compared. The difference is compared with a third quantity. The result is equalized with zero;
$|(a_1 - a_2)| + a_3 = 0$ – cannot be used for measurement as participating quantities are scalar and positive (not zero).

In all cases the equalization is achieved via changing of at least one participating quantity. The third quantity may be with a known or unknown numerical value.

The above shown variants of the basic equation present the most general description of measurement methods. These are made more specific by assigning to each quantity of a certain function related to the logic of measurement.

One of the variants of the measurement model with three quantities will be discussed in detail. It is expressed by the equation:

$$a_1 + a_2 = a_3 \,. \qquad (5)$$

Initially it is presumed that the functions, characteristic of the measurement procedure, have not been assigned to the three quantities.

The structural scheme is shown on fig.1. It is a basic one for realization of various measurement methods.

The action of the structure is as follows: the two quantities on the left side of the equation (5) $a_1$ and $a_2$ are added ($\Sigma$) and then their sum is equalized with the third quantity $a_3$. The elementary operation of comparison is accomplished in the comparator *C*. The output quantity of comparator $\Delta$ is used to manage the variation of one (or more) quantities participating in the process so that the difference $\Delta$ becomes zero.

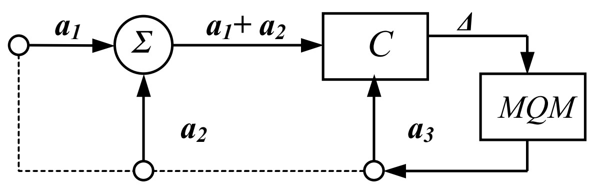


Figure 1. Structural scheme on a physical level without limitations

At this stage, the process goal is to solve the basic equation (3). This is accomplished by using feedback, which includes the operation of management of quantity magnitude (*MQM*). The effect can be directed to any of the participating quantities. It is imperative for at least one quantity to vary. When $\Delta=0$, the equalization is completed.

Based on the discussed variant, various measurement methods are synthesized. This is possible when the functions of measurand, of quantity reproducing magnitude of known value, and of variable, respectively are assigned to the participating quantities. It is possible for the same value to implement more than one function, for example a quantity reproducing magnitude of known value could be variable at the same time.

Let us look at the following distribution of functions in the measurement process:
- $a_1$ assumes the function of measurand. During the measurement its magnitude should not change, i.e. $a_1=const$;
- $a_2$ is a quantity reproducing magnitude of known value. It assumes the functions of variable as well, i.e. $a_2 = var$.
- $a_3$ is also quantity reproducing magnitude of known value and $a_3 = const$.

With this distribution of roles the structural scheme of the measurement procedure is of the type shown on fig.2

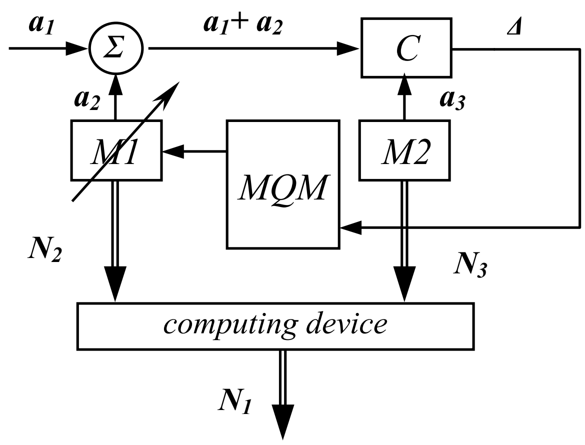


Figure 2. Structural scheme of the measurement procedure (supplementary method of measurement).

The specific basic equation (5), expressed in numerical values is, as follows

$$N_1 + N_2 = N_3 \,. \qquad (6)$$

Solving of this equation is possible if $a_2$ and $a_3$ are quantities of known numerical values. That is why the scheme includes the two material measures *M1* and *M2*, which realize these quantities. Since $a_2$ is *var*, the material measure *M1* is adjustable. The value of $N_1$ is obtained on solving the equation (6) in the computing device.

The synthesized structure corresponds to the complementary method of measurement.

## 4. MEASUREMENT METHODS WITH INVERSION

So far variants were discussed, where quantities participate only once in the measurement procedure. It is possible one or more quantities to be introduced twice or more for a single measurement procedure. Repeated use of one quantity could be implemented if it has been stored in memory. A characteristic variant is shown below of *model 3*, where one quantity is introduced twice in the measurement procedure and describes the measurement methods with inversion [7].

The inversion is a measurement operation accomplished with the magnitudes of particular physical quantities. It represent in essence an operation of subtraction between elements of the set, on the one hand, and a pre-defined particular quantity of constant magnitude of the same type (constant of inversion).

The constant in relation to which inversion is done, could random, but most often, for convenience, it is chosen according to one of the parameters of the measurement procedure. The inversion constant should not change during the measurement procedure, but for different measurements can be different. The constant may be bigger or smaller that the measured quantity, or zero (vector quantities). In the measurement procedure, the role of constant can be assigned to: measurement interval, measurand or other participating quantity of constant magnitude, which could be without know numerical value.

Implementation of inversion allows comparison and equalization of two quantities by their summation. In this sense, the methods result from adapting to measuring instruments of the known algorithm for reducing subtraction of two numbers to their summation via introduction of complement (inversion) of a binary numbers [8]. Application of this algorithm is shown on fig.3.

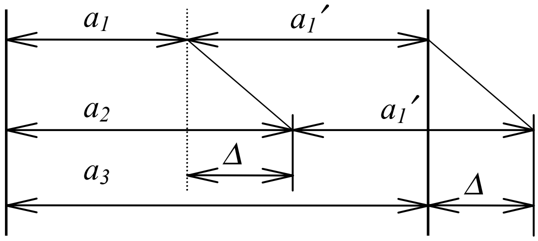


Figure 3. Comparison via inversion

The magnitude $a_1$, where $a_1<a_2$, is pre-inverted in relation to the constant $a_3$ into $a_1'$ and is added to $a_2$. From the resultant sum $a_3$ is subtracted resulting in

$\Delta = a_1 - a_2$.

If $a_1 = a_2$, then $\Delta = 0$. The scheme acts as comparator. In order to achieve equality of $a_1$ and $a_2$, it is necessary for one of the two quantities to vary until $\Delta$ becomes zero ($\Delta = 0$). Then $a_1 = a_2$ and $N_1 = N_2$.

The basic equation is written, as follows:

$$(a_3 - a_1) + a_2 - a_3 = 0 \quad (7)$$

In order to generate a measurement method based on equation (7) it is necessary to assign roles to quantities, for example: $a_1$ – measurand, $a_2$ – variable and material measure, $a_3$ – constant. The outcome ($a_1$=$a_2$) shows that the morphism here is the same as described by (1) but realized by another rule, expressed by equation (7).

Realization of the method so described requires pre-inversion and the respective measuring instrument – inverter, as an element of the structural scheme. The general scheme is shown on fig.4. The inverter (*Inv*) can be included in the material measure chain or that of the measurand. The magnitude $a_3$ can be without known value or to be realized by the material measure *M1*. The output signal of the comparing devise controls variation of the material measure *M*. Prior to comparison the two magnitudes $a_1'$ and $a_2$ are added in the adder.

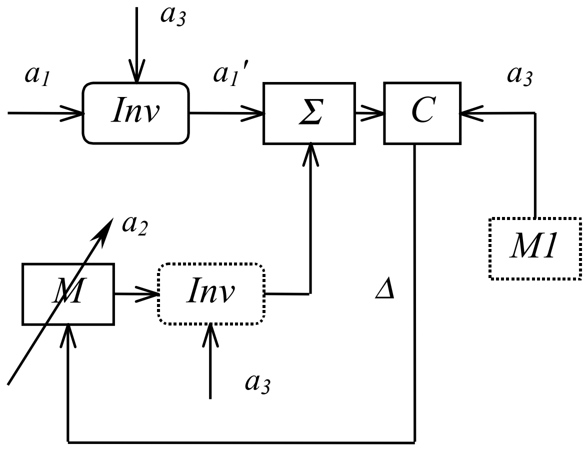


Figure 4. Structural scheme of a method with inversion.

Specific cases are possible for this general structure as well as variants with conditional inverter.

A number of existing methods could be regarded as methods with inversion: substitution method, Borda method, complementary method, method of measurement utilizing Vernier scale, etc.

The specific case is interesting where the magnitude $a_2$ is accepted to be zero. After its inversion the result is $a_2' = a_3 = const$. Then the comparison of a1 and $a_2$ by the algorithm shown on fig.3 gives the equation:

$$(a_3 - a_1) + a_2 - a_3 = \Delta \text{ or } a_1 = \Delta. \quad (8)$$

If $a_2$ is a material measure, for example a ruler as shown on fig.5, $\Delta$ is a magnitude of known value. This means that the goal of measurement (defining of $N_1$) has been achieved in the comparison stage. The described algorithm is known as a substitution method of measurement.

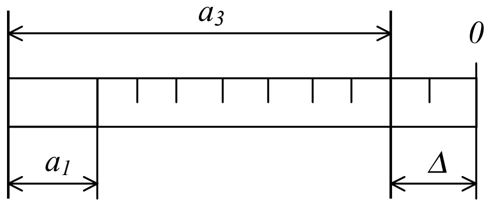


Figure 5. Substitution method of measurement

## 5. CONCLUSION

The report discusses an approach to generation of measurement methods following by the logic of specific measurement, i.e. best oriented to measurement practice.

The measurement procedure is divided into two parts: physical (empirical) experiment, and formal – computing experiment.

Based on possible physical interactions between quantities participating in the first part, common variants have been synthesized for creation of measurement methods. The final formation of methods follows the specifics in solving individual measurement problems.

The proposed approach allows coverage of all possible measurement methods applicable to the measurement of extensive quantities, i.e. in using ratio scale. The presentation of the basic equation (3) in one of the discussed variants could be used as classification attribute of measurement methods.

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