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On the Field Nature of Measuring Process

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ABSTRACT

Basing upon the philosophical assumption that it is not possible to develop theory of measurement independent from the physical nature of measured objects, we formulate the topological theory of measurement. The model of measurement is a fibre bundle space with the base space D representing topological properties of measured quantity and fibres corresponding to the algebraic properties of space values of measured quantity. This model is consistent with the field theory.

keywords: theory of measurement, topology, topological theory of measurement.

1. INTRODUCTION

Measurement is the main source of scientific knowledge, therefore the theory of measurement plays the role of the epistemology of science. Nowadays we observe a very strange situation: these two theories of measurement are developed in completely different ways:

- the quantum theory of measurement is developed by physicists
- the classical theory (based on representational approach) is developed by metrologists.

There are two reasons for seeking a new theory:

- quantum physics has a practical use for metrology: standards of basic values are of quantum origin.
- the most fundamental theory in physics is the quantum field theory.

In our opinion this situation results from unsolved fundamental problems of measurement [1] in both, classical and quantum approach. We think that the basic unsolved problem is the relation between the structure of measurement instrument, and the physical theory which is experimentally verified by means of using measurement instruments (built on the grounds of this physical theory itself).
The interaction between measuring instrument and studied physical object is a physical relation. This interaction depends on measured quantity and the structure of measuring instrument, but is linked with physical nature of measured quantity. In the standard metrological approach an assumption was made that the interaction between instrument and object can be described in the framework of lumped constants system, but real interactions prove their field character. Therefore we recognize that a field theory approach towards measurement theory is necessary, especially because the field approach to measurement theory has not been developed yet.

2. PHILOSOPHICAL ASSUMPTION: COGNITION AND CONCEPTUALIZATION

The aim of measurement is to obtain information about studied objects. If we assume that we can express this information in the language of mathematical objects, it is possible to understand the measurement process as mapping viz. measurement is evaluation [2]:

$$f_c : X \longrightarrow Y$$

where:
X - space of studied object states (bodies and phenomena),
Y - mathematical relational system (usually real numbers)
f_c - measurement function of quantity c.
Unfortunately, this model is too simple because:
1) The studied objects must be made measurable. The process of preparation has two steps: physical isolation and conceptualization. Therefore we identify the real objects through the conceptualized object [3]. The process of physical isolation employs names and sets of specific properties defining the conceptualized object.
2) The measuring instrument is a physical system which changes its state due to interactions between this measuring instrument itself, and the measured object. The read-outs (numbers or other mathematical symbols) describe the states of the instrument. The only direct result of measurement interaction is the change of state of the measuring instrument.
The process of measurement preparation runs in following stages:

1. preparation of measured object;
2. determining and isolation of the aim of the measurement;
3. selection of methods and instruments appropriate for the task.

In the first stage the real object must be made measurable. We confine the studied object to a certain class of objects defined by the class of quantities which can be studied. This process of restricted classification, named "conceptualization" can be described as mapping:

$$\kappa : X \to \Omega \quad (1)$$

Where:
X – space of real objects
Ω- space of conceptualized objects.
κ- function of conceptualization.
The conceptualized objects have a form:

$$\omega = \{x, F(q_1, q_2, ..., q_n)\} \quad (2)$$

Where:
x - object name
$q_1, q_2, ..., q_n$ - physical quantities
F - function describing the properties of conceptualized object.
During the measurement process, the interaction between the object and the measuring instrument causes the change in the state of measuring instrument:

$$m : \Omega \to Q$$

Ω - space of conceptualized objects,
Q - space states of measuring instrument (measurement states)
m - measurement function

In the classical approach of metrology, Q is a space of quantity manifestations of real objects, but in our approach all properties are defined by the measurement states (as in the quantum theory of measurement).

3. THEORY OF MEASUREMENT

We describe the measurement as the sequence of mappings [4]:

$$X \xrightarrow{\kappa} \Omega \xrightarrow{m} Q \xrightarrow{s} Y \quad (3)$$

where:
X represents the space of physical bodies states, Ω is the conceptualized physical phenomena space, Q – the space of measuring instrument states, Y – the mathematical space of measurement results (usually

real numbers or fuzzy numbers).
The subsequent functions which describe the measurement are: conceptualization - κ, quantities (physical measurement) -m, and scaling -s.

In the framework of traditional metrological theory of measurement, the space Q is identified with the space of physical quality manifestations, and the process of measurement is described as the mapping from the manifestation of physical properties to mathematical relational structure Y (often real numbers):

$$s : Q \to Y \tag{4}$$

In our approach, Q is the space of measuring instruments states (measurement space), and $Y-$ is the functional space describing the results of measurement of complex measurement systems.
The properties of scaling function s depend on topological properties of measurement space Q. The physical nature of this space defines the nature of measured quantity. During the measurement process, the measured objects interact with the measuring instrument, and the instrument changes its state. This process of measurement is described by measuring function m (eq.3). This function defines the measured quantity, and describes the physical aspects of measurement. We identify the quantity with the measurement function.

The following thesis in topological theory of measurement is assumed:

a) the essence of measurement is based on the process of mapping states of physical objects into states of measuring instruments,

b) the theory of measurement is strictly connected with the physical theory of measured phenomena,

c) the topological properties of measuring instruments define the fundamental physical quantities [5].

4. MEASUREMENT AND PHYSICAL THEORY OF MEASURED PHENOMENA

Every quality of an object can be observed by varying states of measuring instruments; these states are designed based on physical theory describing the measured quantity. The aim of our theory is to give a mathematical model of designing principles of measuring instrument and its measuring states. This process has two stages:

a) construction of the measuring instrument D as an element of physical space;

b) creation of the permissible states – measuring states Q of measuring instrument D.

For the sake of developing a mathematical model of measurement it is necessary to design the mathematical model of X space - the space of measured object states, and the space of conceptualized objects Ω. The X space is unknown, but the space of conceptualized objects states Ω is described by the same language of expression (the same physical terms of quantity manifestations) as the space of measuring states Q. A relationship between the observed states and measurement instrument suggests following:
States of the observed (conceptualized) reality are spanned on the measuring states
The operation of spanning is a mathematical construction that allows to build a greater space by the use of a single set of elements, and operation of stretching of these elements. The simplest mathematical span is a linear combination. Such construction is used in quantum mechanics where the eigenstates determine the base of states of objects [5]. In our model we will use the method of constructing fibre spaces [1]. We describe the measuring space Q as the space of fibre bundles over the space D characterizing the topological properties of the measuring instrument D. In our mathematical model we identify the space D with measuring instrument, and topological properties of D is a basis of topological classification of physical quantities [5]. The result of measurement is the section of this fibre bundle.

5. TOPOLOGICAL THEORY OF MEASUREMENT: THE FIELD APPROACH

In the framework of standard metrological approach, the scaling functions are simply real value

functions. In our approach, we use the field theoretical model of measurement. Physical field is described as a function of space and time with values given as real numbers or vectors. In modern approach this function is described as the cross-section of fibre bundles over space and time [7]. Therefore we propose the model in which the measurement is described as a choice of fibre bundle section over topological space D representing measuring instrument D. Total space of this bundle is the space of events, and phenomena are the sections of this bundle. In the framework of this theory we can topologically classify the fundamental physical quantities, and unify the measurement theory with field theory.
In the measuring process two stages are distinguished:

a) the mapping of states of the examined objects X into the states of measuring instruments

b) the mapping of states of measuring instruments into (real) numbers. This is achieved by defining the scale.

In this moment of our analysis we do not distinguish between the space of real objects X and the conceptualized space Ω due to choosing only those objects which can be uniquely conceptualized for the sake of the measurement process. To describe these elements according both to the field theory and dimensional analysis we will create the measuring space (space of all the possible measurement results) using the mathematical construction of fibre bundle.
Let D be the space describing topological properties of measuring instrument, and R – a space describing group properties of measures of these states, hence the bundle $\xi = \{Q, \pi, D\}$ over the space D with its fibre $\Re = \pi^{-1}(d)$, $d \in D$ is a space of all possible results of measurement. Space $\Re$ is defined by algebraic properties (composition operation). Elements of the total space Q are expressed with pairs (d, α), where $d \in \Re$. Certain result of measurement is the section of such a bundle because it is based upon the choice of a certain value on a fibre $\Re$ therefore this section defines the function on the base space.

$y : D \to \Re$ such, that $\pi \circ y = id_D$ id_D – identity function on D

π is the function of mapping into D while the graph of the function y is the distinguished subspace stratum of D; the space of all functions y forms the space Y of measurement results. In the case of field approach Y is a functional space, only when D is equivalent to one point (is contractible) and Y is a space of real numbers.
The fibre structure determines algebraic properties of measures of that quantity; $\Re$ is a space with set of relations r,for instance: order, multiplication by a number, addition, etc. The space D is a topological space that reflects the design of the measuring instrument. The simplest example of such an instrument is zero-dimensional sphere S^0 in case of the distance measurement (two marks on scale), or one-dimensional sphere S^1 for electromagnetic measurements (measurement circuit). The measuring instrument is mapped (physically placed) into space X in which the observed phenomena occur. It represents all the possible physical facts, hence it is the space of all possible physical states of the examined objects; simultaneously a certain bundle Q^* over X describes all the possible measurements over X (the physical field).
The mapping of the measuring instrument D into the space X determines the structure of bundle over X together with the structure of bundle over D using the operation called "pullback"[8]:

$$\begin{array}{ccc} f^*Q^* = Q & \xrightarrow{f^*} & Q^* \\ \pi \downarrow & & \downarrow \pi^* \\ D & \xrightarrow{f} & X \end{array} \tag{5}$$

The induced bundle $\xi^* = \{D^*, \pi^*, X\}$ describes the process of measurement in the space X and defines the function y^* of mapping elements of X into their quality measures (values of fields).The mapping f moves the structure from the bundle E^* to E,

$$Q = f^*Q^* = \{(d, x); \pi^*(x) = f(d), x \in X, d \in D\} \tag{6}$$

as well as defines the section y of bundle Q (the measurement results) for a given section y^* in Q^*:

$$y = f^{*} y^{*}$$

So, if a physical object $x \in X$ has a certain quality, which induces a corresponding section in the bundle Q, it means that the state of the measurement instrument is related to the value of measured quantity.

6. ALGEBRA OF MEASURING INSTRUMENTS

Let us consider simultaneous measurement of two (or more) physical quantities using instruments D_1 and D_2. In the standard measurement theory the measurement space of these measured properties is constructed as a Cartesian product of results of particular quantity measurement space.

For example: if we measure time and position of some bodies, the measurement space is the space-time pair $(t, X) \in T \times X$ T – real axis represents all time moments, X – real axis represent all positions, $T \times X$ – Cartesian product of spaces T and X). In topological theory presented here, we must design an instrument measuring simultaneously time and space (time-space measurement instrument), and then using the above construction of fibre bundle space, we can build a space-time. This construction is consistent with the fact that we have one gauge (caesium watch) for measurement of time and distance.

We propose the following construction of measurement space of two (or more) physical quantities Q_1 and Q_2 measured with instruments D_1 and D_2 in the steps:

1. establishing topological structure of instruments D_1 and D_2
2. construction of topological structure of instrument D (joined instrument) for simultaneous measurement of Q_1 and Q_2 as a join structure of D_1 and D_2:

   $$D = D_1 * D_2 \qquad (7)$$

   Join operation $*$ is defined as:

   $$D_1 * D_2 = \{D_1 \times D_2 \times [0,1]\} / \sim,$$

   where $\sim$ is equivalence relation defined by: $(p_1, D_2, 0) \sim (p_1, D_2, 0)$ and $(D_1, p_2, 1) \sim (D_1, p_2, 1)$ and $[0,1]$ denotes interval.
   This equivalence relation contracts each of the bottoms, higher and lower, of the $D_1 \times D_2 \times [0,1]$ cylinder into one point.
3. measurement space Q of join measurement Q_1 and Q_2 is a fibre bundle over "joined" instrument D.

   $$\xi_Q = (Q, \pi_D, D)$$

The definition of "join" was considered by Milnor [9] in order to classify fibre bundles. The main properties of space $D = D_1 * D_2$ are such that a section over subspace D_1 (or D_2) of fibre bundles over $D_1 * D_2$ by choosing the parameter $\lambda \in [0,1]$ equal 0 (or 1) gives the fibre bundles over space D_1 (or D_2). There is a parameter λ which deforms sections over D_1 into sections over D_2.

For time and distance measurements $D_1 = D_2 = S^0$, the join $D = D_1 * D_2 = S^0 * S^0$ forms a circle S^1.

The fibre bundle over S^1 is a simple sum of a trivial bundle (cylinder) and nontrivial bundle (spanned on Moebius strip). The trivial part relates to distance measurement, and nontrivial part to the time measurement. This fibre bundle is a time-space for space-time measurements with unified time-space meter. This theory of time-space is relativistically immanent (the Lorentz transformation is immanent property of this fibre bundle).

CONCLUSIONS

Modern measurement techniques give possibilities of unifying measurements of many quantities. This gives us a new instrument to study the physical reality. These new possibilities show us the reality in a new perspective. In order to understand this possibility, the new mathematical theory of measurement is necessary. Our proposition is only the first step in this direction.

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