Application of Information Theory in Measurement - A Survey

Eugen-Georg WOSCHNI
Faculty of Electrical Engineering and Information Technology
Chemnitz University of Technology, D-09107 Chemnitz, Germany

ABSTRACT

In 1948 Shannon founded the information theory. Originally developed for application in communication now it is used in various fields of science, so also in measurement and instrumentation. The paper represents as well a survey with important examples and results as new investigations in optimal systems. Last not least unsolved problems as the consideration of semantic aspects were treated.
Keywords: Information Theory; Channel Capacity; Optimal Systems.

1. INTRODUCTION

As an introduction Shannon's well-known channel capacity Q will be explained [1]: With the signal-to-noise ratio P_s / P_n and the number of symbols per second $n = 2f_c = 1/t_r$ - where f_c is the critical frequency and t_r the response time, - the number of bits per second, the channel capacity is under the supposition of optimal coding

$$C_t = f_c \text{ lb } (P_s / P_n + 1) \tag{1}$$

An intuitive approach explains the number under the binary logarithm to be the number of distinguishable power steps and the square root the number of amplitude steps [2;3;4;5;6].

2. COMPARISON OF ANALOGUE AND DIGITAL METHODS

As suitable quality criterion to compare both methods the channel capacity is used:
With analogue methods the number of distinguishable amplitude steps m is - if F is the the relative amplitude e

$$m = 1 + 1/ (2F) \tag{2a}$$

and thus the channel capacity is

$$C_{t\,an} = 1 / t_r \text{ lb } m = 2f_c \text{ lb } (1 + 1 / 2F) \tag{2b}$$

In the case of digital methods pulses with the pulse frequency f_i are counted during the time T i.e.

$$m = 1 + Tf_i = 1 + f_i / (2f_c) \tag{3a}$$

and the channel capacity is

$$C_{t\,dig} = 1/T \text{ lb } m = 2f_c \text{ lb } (1 + f_i / (2f_c)) \tag{3b}$$

Both values are equal

$$Tf_i = f_i / 2f_c = 1 / 2F = m - 1 \tag{4}$$

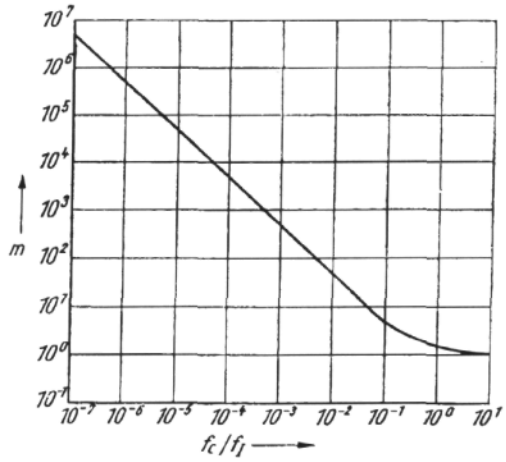

Figure 1. Values of equal channel capacities of analogue and digital methods [5]
m = number of amplitude steps; f_c / f_r = relation of critical frequency to pulse frequency

Taking into consideration the tendency of the development of microelectronics as given in Figure 2 it follows that the critical frequency will shift to higher

[7].
An additional result is the fact of the same tendency with respect to the aliasing errors [8]: Especially in digital measurement an antialiasing filtering before sampling often is not possible because sensors with direct-digital output are direct coupled to the process to be measured. It may be noted on the other hand that the

aliasing errors are depending of the signal processing after sampling [9].

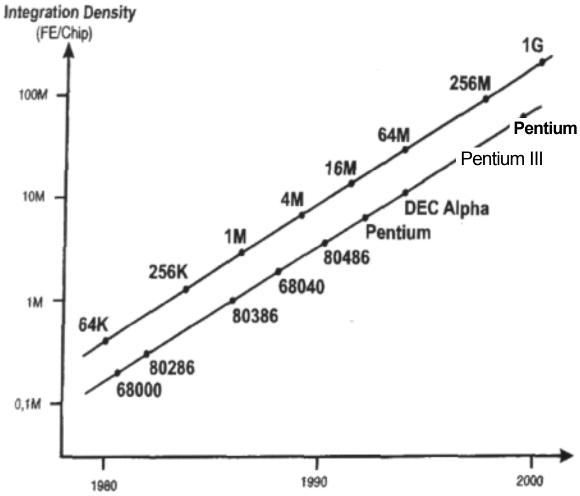

Figure 2. Development of integration degree

3. READING OF INSTRUMENTS

The next example deals with reading of instruments. A group of students was used to gain a relation between reading time and reading error. From these data the channel capacity was calculated using Eq. (1) leading to values between 10...20 bit/s. A comparison with other human activities as e.g. piano playing yields around the same results [3;5].

4. A NEW CONCEPT: OPTIMAL INFORMATION FLOW

Another application - leading to new results - concerns the correction of the dynamic behavior of a measuring system by means of a series-connected network or computer with a PD-algorithm.

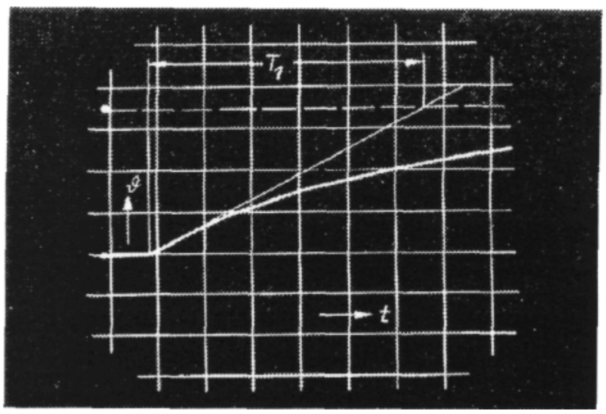

Figure 3 a) Transient function of an uncorrected (original) temperature sensor

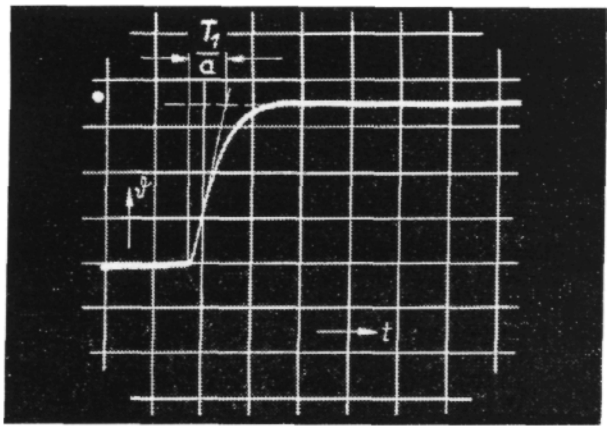

Figure 3 b) Transient function of the corrected system, degree of correction a = 10

As shown from experiment using a temperature sensor one obtains transient functions as with Figures 3a,b without or with correction. A comparison between both Figures shows that the noise is increasing (thinner lines in 3a).

Instead of minimizing the total mean-square error e^2 - as with well-known optimal filtering algorithm -now an optimal information flow I should be realized. Figure 4 shows as well the course of the error components and the total error as the information flow as a function of the degree of correction $a = f_c / f_0$ with the critical frequency of the corrected system f_c and of the original system f_0. The investigations show that because of the factor f_c in Eq. (1) the minimal error does not lay at the same value of the degree of correction a_{opt1} as with optimal filtering but at higher values a_{opt2}. Furthermore it may be emphasized that in this case also - as with optimal filtering - the advantages must be paid by increasing parameter sensitivity [3;5].

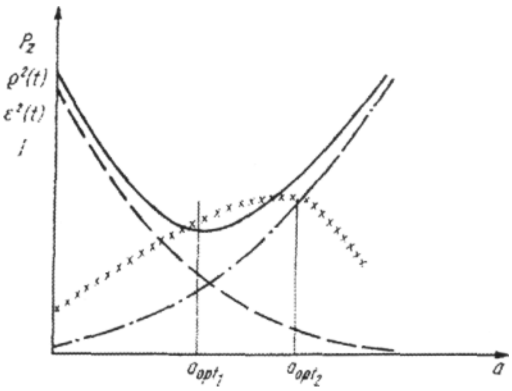

Figure 4. Errors and information flow as a function of the degree of correction
----dynamic error: -.-.- noise error; — total error;
xxxxx information flow

5. A GENERALIZATION: INCLUSION OF SEMANTIC ERRORS

The last problem treated concerns the regard of semantic aspects. At first it is shown how Shannon's information theory combines two different quality criterions - the static or statistic behavior and the dynamic behavior - into one criterion, the channel capacity: If the equation

$$Q = \lambda\ (P_s / P_n\ ;\ f_c) = \lambda_1\ (P_s / P_n + \lambda_2\ (f_c) \qquad (5)$$

is used as general quality criterion one obtains with

$$\lambda_1 = \log f_c$$

$$\lambda_2 = \log [\ \mathrm{lb}\ (P_s / P_n + 1)] \qquad (6a,b)$$

$$\begin{aligned} Q &= \log f_c + \log [\ \mathrm{lb}\ [(P_s / P_n + 1)] \\ &= \log [\ f_c\ \mathrm{lb}\ (P_s / P_n + 1)] \end{aligned} \qquad (6c)$$

especially for

$$C_t = 10^Q = f_c\ \mathrm{lb}\ (P_s / P_n + 1) \qquad (7)$$

the channel capacity.

It was hoped that it would be possible to include also nontechnical aspects - especially semantic aspects - into a generalized information theory using the same method with weighting functions. This would be very important because in this case an optimization using an unified information theory with respect to economy would be possible. The hope was not fulfilled, the problem only is shifted to the next level - the choice of the weighting function.

The consequences are demonstrated with an obvious example: Building a tunnel through a hill by two groups coming from both sides. The optimum is not minimizing the error by means of a very good measurement (tracing) but an allowable error with less costs for tracing.

6. CONCLUSIONS

After a short introduction to the information theory Shannon founded in 1948 applications in measurement and instrumentation were treated:

- The comparison of both analogue and digital methods leading to the result, that due to the development of microelectronics the field of digital methods will be extended to higher critical frequencies one order of magnitude every 7 years.
- Reading of instruments showing that the channel capacity is 10...20 bit/s.
- A new concept of optimal filtering with an optimum information flow.
- Last not least the problem of inclusion of nontechnical - especially semantic problems with the result, that an extension of the information theory to solve these problems is not possible.

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