Proc. IMEKO-TC7 Symposium (Cracow, Poland, June 25-27, 2002)

A New Experimental Method for Correcting Dispersion in Pressure Bar Measurements of Impulsive Force and Pressure

Arkadiusz DROBNICA
and
Zdzisław KACZMAREK

Institute of Electrical Engineering and Metrology
Kielce University of Technology
Al. Tysiąclecia P. P. 7, 25-314 Kielce, Poland

ABSTRACT

A method for correcting dispersive distortion inside a pressure bar, on the basis of experimentally determining the bar's dispersive characteristic is presented in the paper. For this purpose the continuous wavelet transform together with the Gabor analyzing function was applied. In this method, a one-point measurement of the strain inside the bar was used. The results of correction of impact force waveforms, generated by longitudinal impacts of steel spheres into a steel bar, are presented.

Keywords: Dispersion Correction, Wavelet Transform, Pressure Bar.

1. INTRODUCTION

In measurements of impulsive mechanical quantities a commonly used sensor is constructed from a metal bar with strain gauges glued on it. The measured quantity, acting on one of the front faces of the bar, generates elastic strain (stress) waves, which propagate inside the bar as longitudinal waves. The strain gauges convert the strain waves into proportional resistance changes, subsequently converted into voltage changes in the electric connection circuit of the strain gauges. The recorded voltage changes are the measure of the sought quantity.

For impulsive quantities, which create strain waves of lengths far greater than the bar's diameter, the bar can be treated as a non-distorting transducer. For impulsive quantities, which in the bar produce strain waves of lengths comparable to, or smaller than the bar's diameter, their propagation has dispersive features. The dispersion is the cause of distortions in the strain waveforms generated by different phase velocities of their frequency components.

There exist analytical and experimental methods of correcting dispersive distortion in the pressure bar. In the analytical methods the bar's dispersive characteristics are used, obtained as a result of numerically solving the equation of movement of the bar, with the input being a harmonic function. On the ground of these characteristics, it is possible to reverse the effect of dispersion by applying appropriate phase shifts to each frequency component of the experimental record. This procedure was carried out by Gorham [1]. The disadvantage, however, is that this technique produces satisfying results for moderately dispersed signals and cannot correctly reconstruct the input waveform in a long bar.

In the experimental methods the dispersive characteristic of the bar, used for measurements, is determined empirically. For this purpose the one-point measurement as well as the two-point measurement of the strain inside the bar is applied [2], [3].

This work presents a new experimental method of determining the bar's dispersive characteristic, which is used for numerical correction of dispersive distortion in the pressure bar.

The dispersive characteristic of the transducer was obtained by using the wavelet transform applied to the output signal $y(t)$ of the strain gauge. As the analyzing function in the wavelet transform, the Gabor function was utilized.

2. WAVE PROPAGATION ANALYSIS BY MEANS OF WAVELET TRANSFORM

The continuous wavelet transform of a function $f(t)$ is defined by [4]:

$$(Wf)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt \qquad (1)$$

where: $a > 0$ is the scale coefficient, b is the translation coefficient, $\psi(t)$ - basic wavelet. The Gabor function is expressed as [5]:

$$\psi_g(t) = \frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\exp\left[-\frac{(\omega_0/\gamma)^2}{2}t^2 + j\omega_0 t\right] \qquad (2)$$

where ω_0 and γ are positive constants.

In this study $\gamma = \pi\sqrt{2/\ln 2}$ and $\omega_0 = 2\pi/T$, where T is the time window.

One may consider two harmonic waves of unit amplitude and frequencies ω_1 and ω_2 propagating in the x direction written as

$$u(x,t) = e^{-j(k_1x-\omega_1t)} + e^{-j(k_2x-\omega_2t)} \qquad (3)$$

where k_1 and k_2 are the wave numbers. Equation (3) can be rearranged into

$$u(x,t) = 2\cos(\Delta kx - \Delta\omega t)e^{-j(k_c x - \omega_c t)} \quad (4)$$

where

$$k_c = (k_1 + k_2)/2, \quad \omega_c = (\omega_1 + \omega_2)/2 \quad (5)$$

and

$$\Delta k = (k_1 - k_2)/2, \quad \Delta\omega = (\omega_1 - \omega_2)/2 \quad (6)$$

The wavelet transform of $u(x,t)$ is given by

$$WTu(x,a,b) = \sqrt{a}\left[e^{-j(k_1 x - \omega_1 b)}\overline{\hat\psi(a\omega_1)} + e^{-j(k_2 x - \omega_2 b)}\overline{\hat\psi(a\omega_2)}\right] \quad (7)$$

and the magnitude of the wavelet transform is

$$|WTu(x,a,b)| = \{ [\hat\psi(a\omega_1)]^2 + [\hat\psi(a\omega_2)]^2 + 2\hat\psi(a\omega_1)\hat\psi(a\omega_2)\cos(2\Delta kx - 2\Delta\omega b) \}^{1/2} \quad (8)$$

In the case of sufficiently small $\Delta\omega$ the function $u(x,t)$ may be treated as a wave with phase velocity c_p and group velocity c_g. If $\hat\psi(a\omega_1) \approx \hat\psi(a\omega_2) \approx \hat\psi(a\omega_c)$ we obtain

$$|WTu(x,a,b)| \approx \sqrt{2a}|\hat\psi(a\omega_c)|\left[1 + \cos(2\Delta kx - 2\Delta\omega b)\right]^{1/2} \quad (9)$$

If the Gabor wavelet is used, the function $\hat\psi_g(a\omega_c)$, has its maximum at $a = \omega_0/\omega_c$. For a fixed value of x , the magnitude of the wavelet transform reveals a peak at $a = \omega_0/\omega_c$ and $b = (\Delta k/\Delta\omega)x = x/c_g$ on the time-frequency plane. Therefore, the peak of the magnitude of the wavelet transform corresponds to the propagation of a wave with group velocity c_g at frequency ω_c.

3. METHOD PRINCIPLE

In the proposed method, the phase correction of each frequency component of dispersed signal is carried out on the basis of the experimentally obtained bar's dispersive characteristic. This characteristic shows the relation c_p/c_0 as a function of d/Λ, where c_p is the phase velocity for wavelength Λ, d is the diameter of the bar, c_0 is the longitudinal wave velocity expressed as:

$$c_0 = \sqrt{E/\rho} \quad (10)$$

where: E is the Young modulus and ρ is the density of the bar.

The proposed method is based on the one-point measurement of the strain inside the bar. Let us consider a finite bar with length L_P, instrumented by a bridge of strain gauges at $x = L$. Its right end at $L_P = 1.5L$ is free. The path of the pulse generated by the impact $F(t)$ is represented schematically in figure 1.

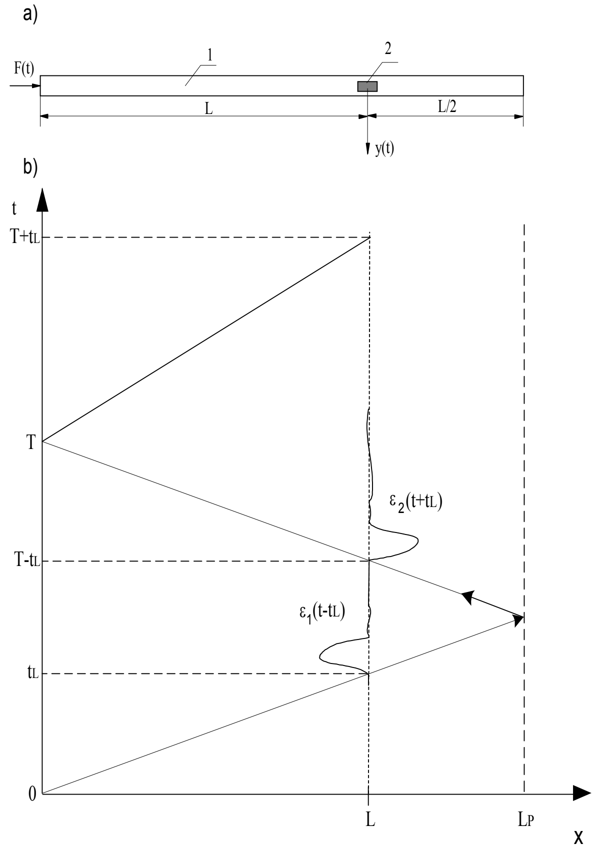

Fig. 1 Lagrangian diagram for longitudinal waves in cylindrical bar: 1 - cylindrical bar, 2 - strain gauges

The strain waveform at $x = L$ can be written as:

$$\varepsilon_L(t) = \varepsilon_1(t - t_L) + \varepsilon_1(t + t_L) \quad (11)$$

where: $\varepsilon_L(t) = \varepsilon(L,t)$, $t_L = L/c_0$, ε_1 and ε_2 are longitudinal strain waveforms due to respectively: the first pulse generated by the impact and the reflection of the pulse at the free nonimpacted end at $x = L_P$. The pressure bar is chosen to satisfy two conditions: $L \geq 20d$ and $2(L_P - L)/c_0 > \alpha$, where α is the duration of the impact pulse $F(t)$.

For $t < T - t_L$ the reflected waveform is not propagating in the bar, which means $\varepsilon_2(t + t_L) = 0$ and we obtain:

$$\varepsilon_L(t) = \varepsilon_1(t - t_L) \quad (12)$$

For $T - t_L < t < T + t_L$ at $\alpha < T - 2t_L$ the first waveform is not propagating in the bar, which means $\varepsilon_1(t - t_L) = 0$ and we obtain:

$$\varepsilon_L(t) = \varepsilon_2(t + t_L) \quad (13)$$

In the presented method of measurement only the $\varepsilon_1(t - t_L)$ and $\varepsilon_2(t + t_L)$ strain waves at $t < T + t_L$ are used.

The result of wavelet transform applied to the signal $y(t)$ is the propagation delay $\Delta t_g(f)$ of the various frequencies f. This propagation delay can be obtained directly from the magnitude of the wavelet transform like the one in figure 2.

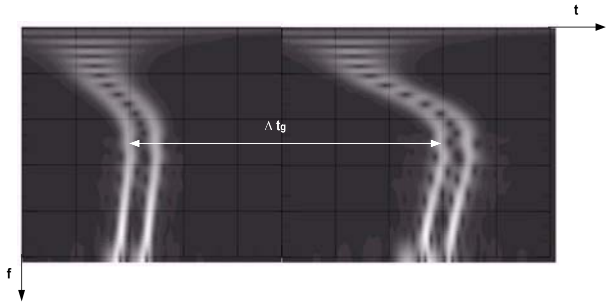

Fig. 2 Example of wavelet transform magnitude of the force sensor output signal for a rectangular pulse impact waveform.

The propagation delay $\Delta t_g(f)$ is related to the group velocity $c_g(f)$ by

$$c_g(f) = \frac{L}{\Delta t_g(f)} \tag{14}$$

The group velocity can be written in a more useful relation expressed as:

$$\frac{c_g}{c_0} = g\left(\frac{d}{\Lambda}\right) \tag{15}$$

The relation between the phase velocity c_p and the group velocity c_g is given by

$$c_g(\Lambda) = c_f(\Lambda) - \Lambda \frac{dc_f}{d\Lambda} \tag{16}$$

Converting into the notation which is more useful for the purposes of computation, we find

$$\frac{c_g}{c_0}\left(\frac{d}{\Lambda}\right) = \frac{c_f}{c_0}\left(\frac{d}{\Lambda}\right) + \frac{d}{\Lambda}\frac{d\dfrac{c_f}{c_0}}{d\dfrac{d}{\Lambda}} \tag{17}$$

The solution of the above equation for the initial condition

$$\frac{c_f}{c_0}(0) = 1 \tag{18}$$

is

$$\frac{c_f}{c_0}\left(\frac{d}{\Lambda}\right) = \frac{1}{d/\Lambda}\int \frac{c_g}{c_0}\left(\frac{d}{\Lambda}\right) d\,\frac{d}{\Lambda} \tag{19}$$

Thus, by integration of function $\frac{c_g}{c_0}\left(\frac{d}{\Lambda}\right)$, the relation $\frac{c_f}{c_0}\left(\frac{d}{\Lambda}\right)$ can be obtained.

The process of dispersive distortion correction is carried out in the frequency domain making use of the FFT techniques. The recorded output signal $y_1(t)$ can be written as

$$y_1(t) = \sum_{k=0}^{K}\left[a_k \cos(2\pi k f_1 t) + b_k \sin(2\pi k f_1 t)\right] \tag{20}$$

where a_k and b_k are the coefficients of FFT calculation, $f_1 = 1/T$ denotes the fundamental frequency related to the processed record length T.

The estimator $\widetilde{F}(t)$ of the input signal at the front end of the bar can be expressed as :

$$\widetilde{F}(t) = \sum_{k=0}^{K}\left[a_k \cos(2\pi k f_1 t + \varphi_k) + b_k \sin(2\pi k f_1 t + \varphi_k)\right] \tag{21}$$

where $\varphi_k = \varphi(kf_1)$ denotes the phase shift calculated at frequency kf_1 by formula

$$\varphi_k = 2\pi \cdot kf_1 \cdot L\left(\frac{1}{c_f(kf_1)} - \frac{1}{c_0}\right) \tag{22}$$

In the above formula $c_f(kf_1)$ is the quantity obtained from the dispersion characteristic $\frac{c_f}{c_0}\left(\frac{d}{\Lambda}\right)$ at the specified value d, c_0 and $\Lambda = \frac{c_f}{kf_1}$.

4. EXPERIMENT RESULTS

The method of correction discussed in the previous section was utilized to reconstruct the waveforms of impulsive force produced by impact of small spheres.

The work was carried out with a vertically mounted pressure bar (figure 3). Steel spheres in the range of 3mm to 10mm diameter were dropped from a height of 200mm onto the centre of the front face of the bar. A pressure bar of 22mm diameter and 1.5m length was used. The bar was instrumented with silicon strain gauges located 1m from the impact end. Digitization and storage of the output signal were carried out with an 8-bit resolution 2MS/s digital oscilloscope.

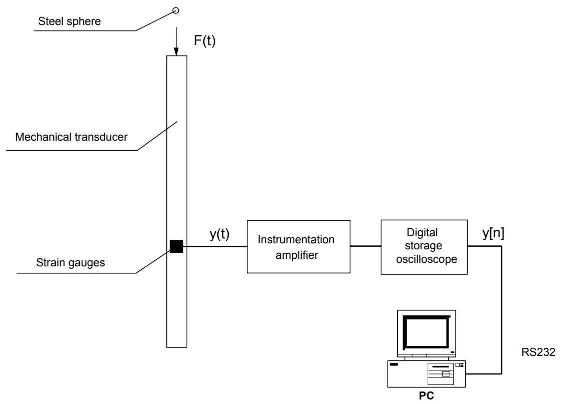

Fig. 3 The instrument to measure the impulsive force produced by impact of small spheres.

A series of signals recorded from impacts of steel spheres of 3mm, 4.762mm, 5.556mm and 10mm, is shown in figures 4 - 7.

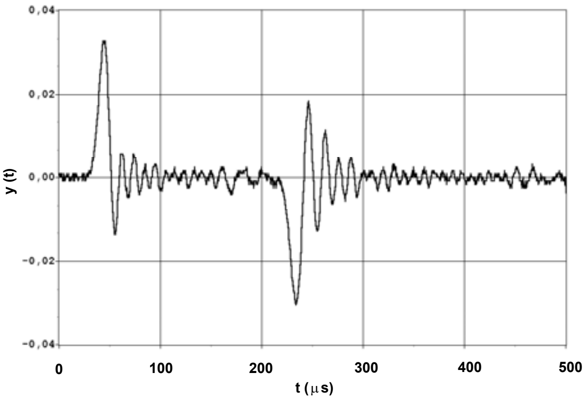

Fig. 4 Recorded stress waveform arising from the impact of 3mm steel sphere

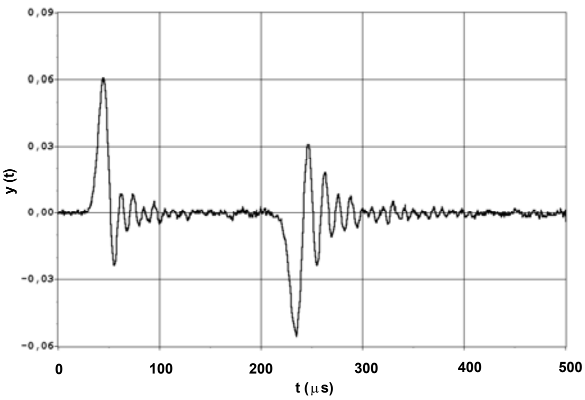

Fig. 5 Recorded stress waveform arising from the impact of 4.762mm steel sphere

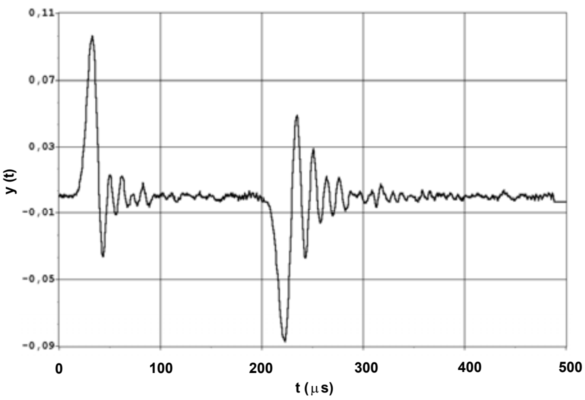

Fig. 6 Recorded stress waveform arising from the impact of 5.556mm steel sphere

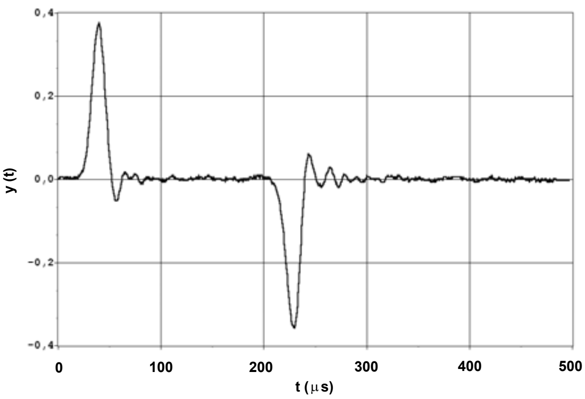

Fig. 7 Recorded stress waveform arising from the impact of 10mm steel sphere

The wavelet transformation was applied to the signal $y(t)$ from the impact of 3mm sphere. The magnitude of the wavelet transform is shown in figure 8.

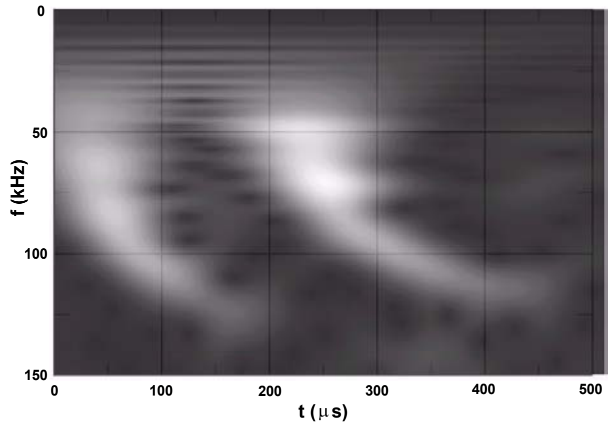

Fig. 8 The magnitude of the wavelet transform applied to the signal $y(t)$ generated by the impact of 3mm steel sphere

According to this plot, the propagation time delay $\Delta t_g(f)$ was obtained for frequencies in the range of up to 120 kHz. The relation $\Delta t_g(f)$ extracted by using the wavelet transform is presented in figure 9.

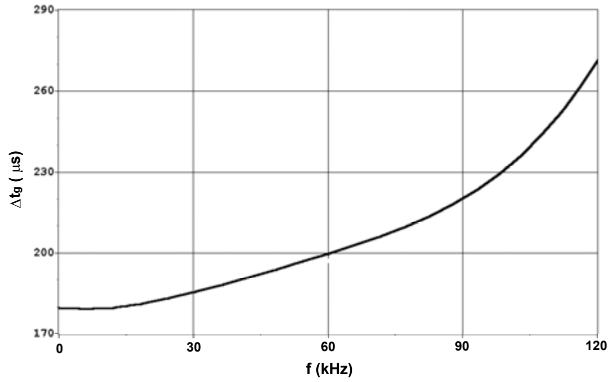

Fig. 9 The spreading of propagation time delay $\Delta t_g(f)$ over the signal spectrum

Using the relations described in previous section the dispersion characteristic $\frac{c_f}{c_0}\left(\frac{d}{\Lambda}\right)$ of the pressure bar was obtained. It is shown in figure 10.

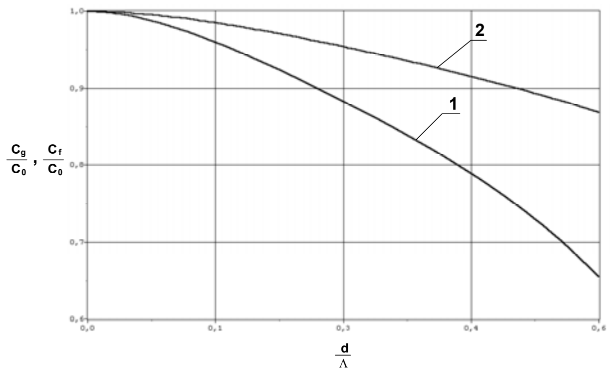

Fig. 10 The experimentally obtained characteristics of the pressure bar: (1) $\frac{c_g}{c_0}\left(\frac{d}{\Lambda}\right)$ and (2) $\frac{c_f}{c_0}\left(\frac{d}{\Lambda}\right)$

On the basis of this characteristic the correction of dispersion for all four spheres was carried out. The results of correction are presented in figures 11 - 14.

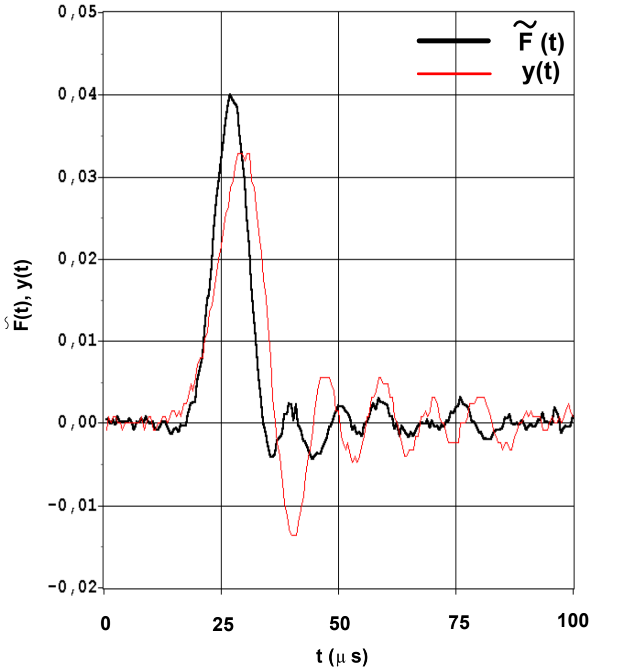

Fig. 11 Stress waveform arising from the impact of 3mm sphere before the correction $y(t)$ *and after the correction* $\widetilde{F}(t)$

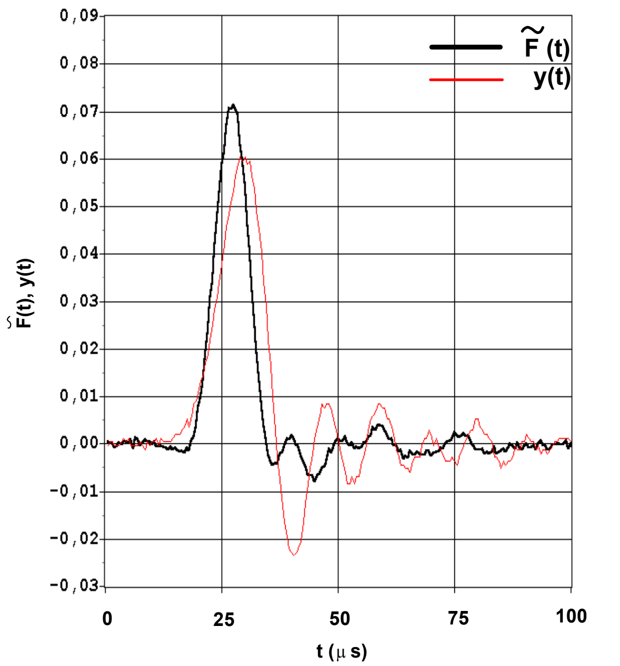

Fig. 12 Stress waveform arising from the impact of 4.762mm sphere before the correction $y(t)$ *and after the correction* $\widetilde{F}(t)$

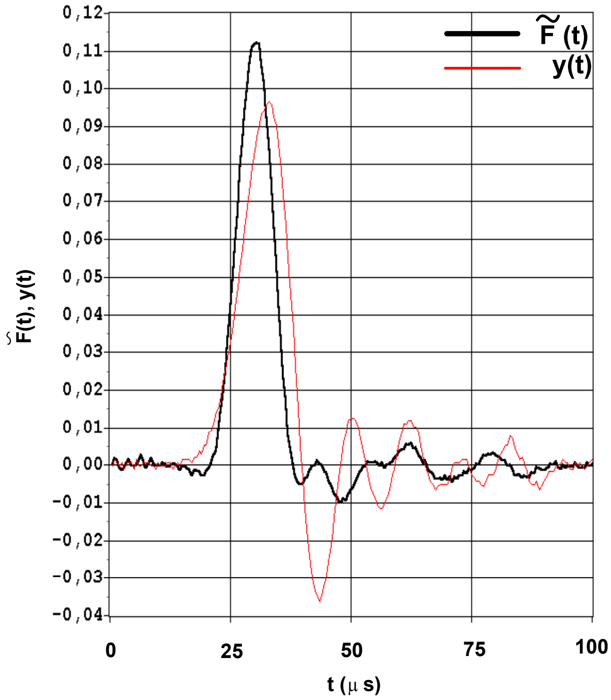

Fig. 13 Stress waveform arising from the impact of 5.556mm sphere before the correction $y(t)$ *and after the correction* $\widetilde{F}(t)$

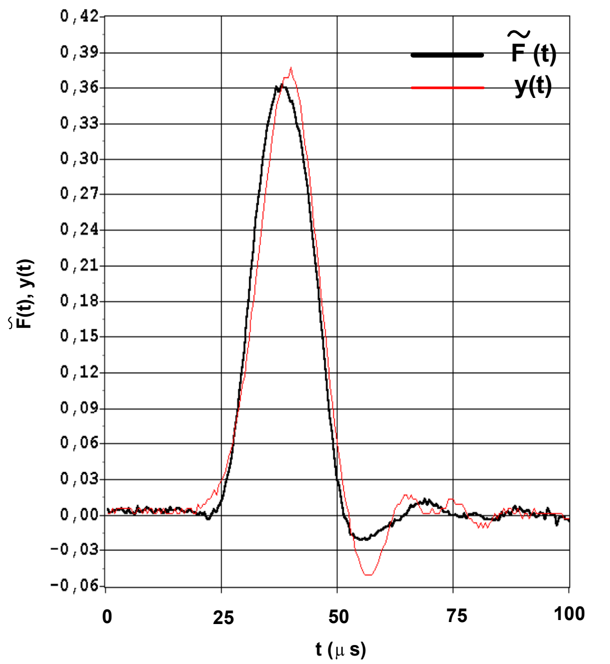

Fig. 14 Stress waveform arising from the impact of 10mm sphere before the correction $y(t)$ *and after the correction* $\widetilde{F}(t)$

5. CONCLUSION

The correction results in figures 11 - 14 illustrate a very useful method of obtaining information on the peak amplitude and duration of stress pulses arising from impulsive impact of steel spheres. The method does not depend upon analytical or numerical models of wave propagation.

Our experiments indicate that the wavelet transform with the Gabor function is an effective tool for analyzing the wave propagation phenomena in the pressure bar and that the result of this analysis can be used for dispersion correction process of impulsive force and pressure waveforms.

6. REFERENCES

[1] Gorham, "A Numerical Method for the Correction of Dispersion in Pressure Bar Signal", *J. Phys. E. Sci. Instrum.*, Vol. 16, 1983, pp. 477-479.

[2] Gorham, X. I. Wu, "An Empirical Method for Correcting Dispersion in Pressure Bar Measurements of Impact Stress", *Meas. Sci. Technol.*, No. 7, 1996, pp. 1227-1232.

[3] Z. Kaczmarek, A. Drobnica, " Waveform Reconstruction of Impulsive Force and Pressure by Means of Deconvolution ", *Studies in Applied Electromagnetics and Mechanics, Non-Linear Electromagnetic Systems*, ISBN: 90 5199 381 1, IOS Press, Amsterdam, 1998, pp. 438-441.

[4] K. Chui, *An Introduction to Wavelets*, ISBN: 90 5199 381 1, Academic Press, San Diego, CA, 1992.

[5] K. Kishimoto, H. Inoue, M. Hamada, T. Shibuya, "Time Frequency Analysis of Dispersive Waves by Means of Wavelet Transform", Trans. of the ASME, Vol. 62, 1995, pp. 841-846.

[6] A. Drobnica, Z. Kaczmarek, "A Numerical Correction of Dispersive Distortion in Mechanical Transducer by Means of Wavelet Transform", *International Conference on Modern Problems of Telecomunications, Computer Science and Engineers Training*, Lviv-Slavsko, Ukraine, 2000, pp. 5-6.

[7] A. Drobnica, Z. Kaczmarek, "A Reconstruction Method of Impulsive Pressure and Force Waveform by Means of Wavelet Transform", *9-th International Symposium on Non-Linear Electromagnetic Systems*, Pavia, Italy, May 1999