Shapes of Input Signals for Calibration of Measuring Systems Intended for Dynamic Signals Measurement

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ABSTRACT

Calibrating signals for measuring systems intended for measurement of dynamic signals are generally limited to standard signals. The unit step input in time domain and sinusoidal signal in frequency domain are the most often applied here. However dynamic signals have in the practice quite arbitrary shapes which significantly differ from the standard signals. In consequence, the results of a calibration process realised by means of standard input signals are in practice useless, except of some very particular cases. On the other hand the process of calibration can not be realised by means of any dynamic signal because it is undefined and the set of such signals is infinite. In such a situation assumption that the calibration results have to ensure the mutual comparability of measuring systems is possible to attain only if the errors estimated in process of calibration receive maximum values. For this purpose the special class of calibrating signals maximising the assumed error criterion have to be applied. The results received then are similar to the class indexes resulting from the maximum static error determined for measuring instruments for static measurements. The paper presents the possible shapes of input signals which maximise integral-square-error criterion on the output of linear time invariant measuring system and its standard. In such a way a situation in which all the possible dynamic signals which might occur at the input of a real system are taken into consideration at the same time. The advantage of the presented method is the possibility of making the calibration process independent of the input signal shape through the application of signals which maximize the chosen error criterion.

Index terms – dynamics of signals, integral-square-error criterion, matching of input

1. INTRODUCTION

The problem of calibration of measuring systems intended for dynamic measurement is not work out up to now. Generally it is limited to determination of frequency characteristic for such systems. A good example can be here commonly applied frequency characteristics such systems like e.g. shock and vibration measuring systems with seismic transducers, pressure transducers systems, strain gauge amplifiers, measuring condenser microphones, filtering circuits or other measuring systems operating together with piezoelectric or capacitive transducers. The usefulness of frequency characteristics is widely known and for signals with known spectral distribution it is easy on the base of them to determine the errors which burden the measurement. When the spectral distribution is not known, which usually occurs in the case of dynamic signals, then it is in principle not possible to determine such errors. This is the result of the fact that errors are defined in the time domain by means of a chosen error criterion, whose value depends both on its form as well as on the shape of the input signal. Thus, the problem is: by means of what signals should the calibration be conducted for systems which have at their input undetermined signals of unknown shape and unknown spectral distribution, thus being basically different from standard signals? The application of signals which maximise the error criterion solves this problem to a significant degree. It is worth to point out here that the values of errors received by means of such signals are very similar to the maximum errors determined in static calibration process and which are then applied for establishing the hierarchy of accuracy of measuring systems for static measurement.

2. CONSTRAINTS OF CALIBRATING SIGNALS

The searching of signals maximising the chosen error criterion can be made on the analytical way or by means of numerical computation. In all cases however the primary problem to be solved is to define the space, in which such signals can exist. Such a space depends generally on the type of constrains imposed on signals and are conditioned by the dynamic properties of calibrated systems. Let us consider as an example the low-pass systems and the constraints resulting from their dynamic properties. For such system we can deal with two constraints. The first one refers to the constraint of magnitude a of calibrating signal $u(t)$, while the second one refers to the rate of change ϑ of this signal. For the obvious reason constraint of magnitude a must always be imposed on signal, hence

$$|u(t)| \le a \qquad 0 < a \le 1 \tag{1}$$

The rate of change constraint ϑ of signal represents the matching of its dynamics to the dynamics of the system. One can say that the dynamics of the signal is properly matched to the dynamics of system if it is transmitted by the system without deformation. In order to determine the relationships describing constraint ϑ we will consider below two different points of view. The first one refers to the frequency domain. Let us consider the frequency characteristic of the calibrated system and let the maximum angular frequency corresponding to the cut-off frequency of this characteristic by denoted by ω_m. This means from the assumption, that the series of harmonics

$$u(t) = A \sin \omega t \quad \omega = 1, 2, \ldots \ \omega_m \tag{2}$$

up to the frequency ω_m inclusive, should be transmitted by this system without deformations. The maximum velocity of the signal corresponding to the highest harmonic of this series is thus

$$\vartheta = \max |u'(t)| = A\omega_m \tag{3}$$

where the limit ω_m is arbitrary and can, but does not need to, correspond to a 3dB reduction in signal magnitude. The second point of view refers to the time domain. Let $h(t)$ and $k(t)$ denote the step and impulse responses of the system. Then the proper matching can be obtained by restricting the maximum rate of change of the input signal $\max|u'(t)|$ to a value less or at most equal to the maximum velocity of the step response of the system. Hence we can write in this case that

$$\vartheta = \max_{t\in[0,T]} |u'(t)| \le \max_{t\in[0,\infty]} |h'(t)| = \max_{t\in[0,\infty]} |k(t)| \tag{4}$$

3. SHAPES OF SIGNALS

Assuming the integral-square-error criterion on the output of the calibrated system and its standard we will prove below, that input signal with a and ϑ constraints that maximise this criterion, must over the interval [0,T], always reaches one of the above constraints. Let us, for this purpose, express our error criterion as follows

$$I_2(u) = \int_0^T [\varepsilon(t)]^2 dt \tag{5}$$

where $\varepsilon(t)$ denotes the difference between the responses of the calibrated system $y_c(t)$ and its standard $y_s(t)$

$$\varepsilon(t) = y_c(t) - y_s(t) \tag{6}$$

Suppose U is a set of functions $u(t)$ of the class C^1 in sections, over the interval $[0,T]$. Let $I_2(u)$ (5) be the positive square functional on U which means that bilinear symmetrical transformation $\exists\, \tilde{I}_2$ exists [L4]

$$I_2(u) = \tilde{I}_2(Ku, Ku) \qquad u \in U \tag{7}$$

where Ku presents convolution integral

$$Ku(t) = \varepsilon(t) = \int_0^t k(t-\tau)\, u(\tau) d\tau \tag{8}$$

and $k(t)$ denotes the difference between impulse responses of the calibrated system $k_c(t)$ and its standard $k_s(t)$

$$k(t) = k_c(t) - k_s(t) \tag{9}$$

Let the condition be fulfilled

$$\forall\ 0 < b < c < T \quad \exists\ h \in U : supp h \subset [b, c] \tag{10}$$

such that

$$I_2(h) \ne 0 \tag{11}$$

which automatically means that $I_2(h) > 0$
Let us define the following set A

$$A : \{u(t) \in U : |u(t)| \le a \quad |u'_+(t)| \le \vartheta \quad |u'_-(t)| \le \vartheta \quad t \in [0,T] \tag{12}$$

If $u_0(t) \in A$ fulfils the condition

$$I_2(u_0) = sup\{I_2(u) : \ u \in A\} \tag{13}$$

then:

Theorem

$$\forall\, t \in [0,T] \quad |u_0(t)| = a \quad \text{or} \quad |u'_+(t)| = \vartheta \quad \text{or} \quad |u'_-(t)| = \vartheta \tag{14}$$

Proof

Suppose that (14) is not true. Then

$$\exists \varepsilon > 0 \quad \exists\ 0 < b < c < T \tag{15}$$

such that

$$|u(t)| \le a - \varepsilon, \quad |u'_{0+}(t)| \le \vartheta - \varepsilon, \quad |u'_{0-}(t)| \le \vartheta - \varepsilon \tag{16}$$

Let us choose h according to (10)

$$supp\, h \subset [b, c] \quad I_2(h) > 0 \tag{17}$$

Then for the small $d \in R$, say $d \in (-\delta, \delta)$ is

$$u_0 + dh \in A \qquad \forall d \in (-\delta, \delta) \tag{18}$$

and from the optimum condition in $u_0(t)$ it results that

$$I_2(u_0) \geq I_2(u_0 + dh) = I_2(u_0) + 2d\widetilde{I}_2(u_0, h) + d^2 I_2(h) \qquad d \in (-\delta, \delta) \tag{19}$$

hence

$$0 \geq 2d\widetilde{I}_2(u_0, h) + d^2 I_2(h) \quad d \in (-\delta, \delta) \tag{20}$$

However, it can be easy seen that solution (20) represents a parabola crossing zero and directed upwards, so that the last inequality can never be fulfilled for $I_2(h) > 0$, $d \in (-\delta, \delta)$.

Corollary

As relationship (20) represents a contradiction, so the functional $I_2(u_0)$ can fulfil condition (13) iff the input signal $u_0(t)$ reaches one of the constraints given in (14).
It is worth to point out that carrying out the proof in an identical way to that above, it can be shown that if only one of the constraints is applied to the signal, either of magnitude a or of the rate of change ϑ, then the functional $I_2(u_0)$ reaches maximum iff the signal reaches this constraint over the interval [0, T].
The proof presented above shows that in the case of one constraint (1) the input signal maximising criterion (5) has the form of a "bang-bang" signal with the magnitude a = 1 whereas there are two simultaneous constraints (12), input signal can be only in the form of triangles with the slope inclination ϑ_+ or ϑ_- or of trapezoids with the slopes ϑ_+ and ϑ_- and a magnitude of a = 1. In consequence the space of correct solution in which searching signal maximising the integral-square-error criterion exist is significantly increased.

4. EXAPMLE

Below, as an example of the presented theory we will show the results, of calibration of the fifth, fourth and third order Butterworth filters with constituting the reference six order filter, assuming the case in which the dynamic input signal is of unknown shape and which cannot be anticipated in advance. Le us assume also that the result of calibration will be presented by means of the maximum value of the integral-square-error criterion. In the example we will assume that the time T, which occurs in the upper limit of the integration in the integral (5) will be correspond to the steady state value of the impulse responses of the filters. This time is T = 20s. and can be read out from the diagrams shown in Fig.1., which present impulse responses of the following Butterworth filter [6]:

$$K_6(s) = \frac{1}{(s^2 + a_1 s + 1)(s^2 + a_2 s + 1)(s^2 + a_3 s + 1)} \tag{21}$$

$a_1 = 1.9319,\ a_2 = 1.4142,\ a_3 = 0.5176$

$$K_5(s) = \frac{1}{(s^2 + a_1 s + 1)(s^2 + a_2 s + 1)(s + 1)} \tag{22}$$

$a_1 = 1.8478 \quad a_2 = 0.7654$

$$K_4(s) = \frac{1}{(s^2 + a_1 s + 1)(s^2 + a_2 s + 1)} \tag{23}$$

$a_1 = 1.8478 \quad a_2 = 0.7654$

$$K_3(s) = \frac{1}{(s^2 + a_1 s + 1)(a_2 s + 1)} \tag{24}$$

$a_1 = a_2 = 1$

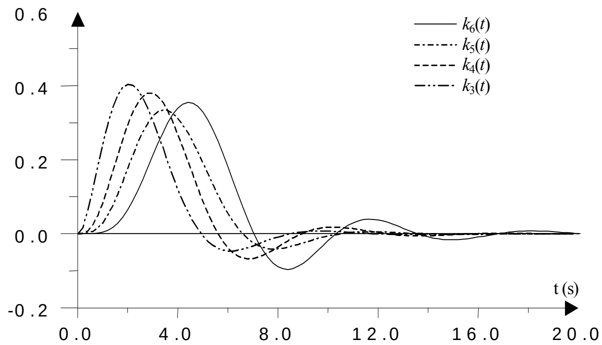

Fig.1. Impulse responses $k_6(t)$, $k_5(t)$ $k_4(t)$ and $k_3(t)$ of Butterworth filters of the VI, V, IV and III order respectively

Let us impose the constraints a (1) and ϑ (4) resulting from the model of filter (21). Then we have:
- constraint of magnitude

$$\left| u_0(t) \right|_{t \in [0,20]} = 1 \tag{25}$$

and
- constraint of rate of change ϑ, which corresponds to the maximum value of impulse response of the $k_6(t)$, and equals

$$\vartheta = \max_{t \in [0,\infty]} \left| k_6(t) \right| = 0.355 \tag{23}$$

and is reached for t = 4.44s. – Fig.1.
In order to determine the shapes of signals the program based on genetic algorithms [1] was applied. The program searched for a signal $u(t)$ = $u_0(t)$ which maximised the functional (1) where $\varepsilon(t)$ was calculated by means of the convolution integral (8). The procedure of searching for the optimum number n of signal $u_0(t)$ switchings (from ϑ_+ to ϑ_- or ϑ_+ to +1 or +1 to ϑ_- or ϑ_- to ϑ_+ or ϑ_- to –1 or –1 to ϑ_+) commenced with the assumption $n = 1$. Next the procedure was repeated for

$n = 2, 3, \ldots$ etc. In this way, the optimum number of switchings was not given in advance, but it was being consecutively increased until the value of $I_2(u_0)$ (5) reached its maximum. During the calculation the shape of signal corresponding to each particular n switchings were checked 200 times as such number of generations have been set-up in the program applied. For 866 MHz PC computational burden was within 18 to 20 s. for each one generation.

The results obtained are as follows:

Comparing of the six order model (21) with the fifth order model (22) gives the maximum value of $I_2(u_0)$ = 0.632 [V^2s] if the signal $u_0(t)$ over [0,20s] is in the form

$$\begin{aligned} u_{05}(t) \Rightarrow\ & \vartheta_+[0.00, 0.556],\ \vartheta_-[0.556, 3.130], \\ & \vartheta_+[3.130, 7.871], \vartheta_-[7.871, 11.792], \\ & \vartheta_+[11.792, 15.809] + 1[15.809, 17.390], \\ & \vartheta_-[17.390, 20.00] \end{aligned} \qquad (24)$$

where the tangent of the rise angle ϑ_+ [.] is equal to the tangent of the fall angle ϑ_- [.] with a value of 0.355

Comparing six order model (21) with the fourth order model (23) gives the maximum value of $I_2(u_0)$ = 2.649 [V^2s] if the signal $u_0(t)$ over [0,20s] is in the form

$$\begin{aligned} u_{04}(t) \Rightarrow\ & \vartheta_+[0.00, 2.807],\ \vartheta_-[2.807, 7.570], \\ & \vartheta_+[7.570, 12.332], \vartheta_-[12.332, 17.956], \\ & -1[17.956, 20.00] \end{aligned} \qquad (25)$$

Comparing six order model (21) with the third order model (24) gives the maximum value of $I_2(u_0)$ = 5.498 [V^2s] if the signal $u_0(t)$ over [0,20s] is in the form

$$\begin{aligned} u_{03}(t) \Rightarrow\ & \vartheta_+[0.00, 2.817],\ +1[2.817, 7.711], \\ & \vartheta_-[7.7711, 12.594], \vartheta_+[12.594, 17.477], \\ & +1[17.477, 20.00] \end{aligned} \qquad (26)$$

Exemplarity diagrams referring the shapes of signals in this latter case are presented in Fig. 2.

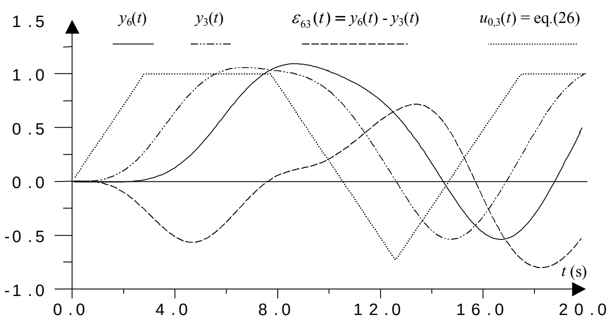

Fig.2. Responses $y_6(t)$, $y_3(t)$ and the error $\varepsilon_{6,3}(t) = y_6(t) - y_3(t)$ of the models K_6(s) and K_3(s) to the signal $u_{0,3}(t)$ (26)

5. CONCLUSION

The method of the evaluation of the dynamic properties of the systems intended for dynamic measurements by means of maximum errors is very similar to the evaluation of the instruments intended for static measurements by means of class index resulting from the maximum static errors. The application in the process of calibration the signals which maximise the chosen error criterion is particularly useful in the case of systems for which, as originally assumed, the input signals are unknown and varying in time. It is worth to point out also that the results of calibration determined in such a way are invariant to shapes of dynamic signals, in such a sense that the received errors are valid for any signals which could appear at the input of the calibrated system.

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