Synthesis of Optimised Digital Filters Used to Signal Correction in Measurement Systems

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ABSTRACT

An approach to the problem of designing a stable corrective digital filter is presented. Approach based on an assumption of a stabilisation level and minimisation of an approximation or an assumption of the approximation level and minimisation a stabilisation level of an ideal corrective filter. The correction is performed by a cascade connection of both systems. The obtained solution is an anticipative system.

Keywords

Correction, inverse system.

1. INTRODUCTION

In spite of more and more usage of the digital signal processing in measurement instruments, the analogue systems, like analogue input circuits which cannot be replaced by the discrete systems, are still being used. Quite often, before the processed signal is changed into the digital form, it passes through the aforementioned analogue systems, which cause signal distortions. If the analogue system has its own mathematical model in the discrete systems domain, then a chance exists of the correction of these distortions on the digital side. It is assumed that the full correction will be obtained when the cascade connection of the analogue system, henceforth the corrected system, and the digital filter, henceforth the corrective system, will be the identity system.

Let $H(z)$ be the transfer function of a corrected system and $X(z)$ the transfer function of a corrective digital filter cascaded with the corrected system. To obtain the resultant of the identity system, the filter should have the transfer function $X(z)$, to satisfying the equation

$$H(z)\cdot X(z)=1 \tag{1}$$

However, an ideal corrective filter can be unstable, because inversion of a system forms a feedback loop, which can destabilise the system. In the time domain this property will appear as the infinite impulse response, which is not absolutely summable.

2. ANTICIPATIVE DIGITAL FILTERS

Treating the corrective system as anticipative and dividing its structure into two parts leads to obtain the stable but noncausal impulse response. One part is an anticausal system and includes the poles located outside of the unit circle and the other which is a causal system includes the poles located inside the unit circle. The impulse response of such systems is the sum of a left-sided $\{x_n^-\}$ and a right-sided $\{x_n^+\}$ sequence. This two-sided $\{x_n\}$ sequence is absolutely summable.

$$x_n = x_n^- + x_n^+ \tag{2}$$

Eq. (3) and (4) describes the sequences $\{x_n^+\}$ and $\{x_n^-\}$, respectively.

$$x_n^+ = \begin{cases} 0 & \text{for } \; n<0 \\ \dfrac{1}{2\pi}\oint\limits_{\Gamma_1} X(z)\cdot z^{n-1}dz & \text{for } \; n\geq 0 \end{cases} \tag{3}$$

$$x_n^- = \begin{cases} 0 & \text{for } \; n\geq 0 \\ -\dfrac{1}{2\pi}\oint\limits_{\Gamma_2} X(z)\cdot z^{n-1}dz & \text{for } \; n< 0 \end{cases} \tag{4}$$

In such approach to the discrete systems the notation of an asymptotic stability does not exist. The system can be at the most on the limit of stability. It means that it has the poles lying on the unit circle. In this case a filter should be designed, which would be stable in the above sense and the best approximating properties of the ideal corrective filter described by Eq. (1).

3. QUASI-INVERSE FILTERS

The problem of finding the corrective filter should be formulated universally with the help of a generalised algebraic formula, which will be correct in the time domain as well as in the frequency domain. It is achieved by using a dot product, which has its representation in the time and frequency domain

$$(x,y)=\sum_{n=-\infty}^{+\infty} x_n \cdot y_n = \frac{1}{2\pi j}\oint_{|z|=1} X(z)\cdot Y\left(z^{-1}\right) d\ln z \quad (5)$$

where:

x_n n-th sample of the signal $\{x_n\}$,

y_n n-th sample of the signal $\{y_n\}$,

$X(z)$ the z-transform of the signal $\{x_n\}$,

$Y(z)$ the z-transform of the signal $\{y_n\}$,

Therefore, to describe the stability of the filter, the so-called *energy stability* is assumed.

$$\|o\|^2 = (o,o) = \sum_{n=-\infty}^{+\infty} |o_n|^2 \le q \quad (6)$$

where $\{o_n\}$ is an output signal of a filter.

The metric of the dot product of signals can be used to define an *approximation ratio*. This ratio shows how much designed stable corrective filter renders properties of the ideal corrective filter. The smaller is its value, the better a real signal $\{r_n\}$ rendering a model signal $\{w_n\}$.

$$\|r-w\|^2 = (r-w, r-w) = \sum_{n=-\infty}^{+\infty} | r_n - w_n |^2 \quad (7)$$

The stabilisation of the inverse filter being on the limit of stability can be performed in two ways:

1. assuming the stability on the definite level q

$$(x,x)=q \quad (8)$$

and finding the corrective filter x that in the best way approximates the ideal corrective filter, so that

$$(hx-1, hx-1) \to \min \quad (9)$$

where: h and x are the transfer function $H(z)$ of the corrective system and the transfer function $X(z)$ of corrected filter, respectively or their impulse responses h_n and x_n, respectively. 1 describes the identity system;

2. assuming the approximation of the corrective filter x on the definite level q

$$(hx-1, hx-1) = q \quad (10)$$

and searching for the inverse filter x satisfying (11).

$$(x,x) \to \min \quad (11)$$

Below the solution for the first case is presented. To take simultaneously into consideration conditions (8) and (9) the Lagrange's functional is assumed.

$$f(x,\lambda) = (hx-1, hx-1) + \lambda \cdot [(x,x)-q] \quad (12)$$

A factor λ determines an influence of the stabilisation ratio on the final solution. The functional has the minimum in the point x_λ, if for any change of δx the inequality

$$f(x_\lambda + \delta x, \lambda) - f(x_\lambda, \lambda) > 0 \quad (13)$$

is true. The inequality (13) assumes the form (14) by using the properties of the dot product.

$$2\cdot\left(h^* h x_\lambda - h^* + \lambda x_\lambda, \delta x\right) + \|h\delta x\|^2 + \lambda \cdot \|\delta x\|^2 > 0 \quad (14)$$

Thus, the necessary and sufficient condition of the minimum existence is the equation

$$h^* h x_\lambda - h^* + \lambda x_\lambda = 0 \quad (15)$$

The obtained solution can be presented as operative fraction

$$x_\lambda = \left(\lambda 1 + h^* h\right)^{-1} h^* = h^*\left(\lambda 1 + h^* h\right)^{-1} \equiv \frac{h^*}{\lambda + h^* h} \quad (16)$$

which describes a family of filters called *λ–family of quasi-inverse filters*. The sign * means the conjugation operation. The factor λ makes possible the control of the poles location in relation to the unit circle.

On the basis of this solution, the asymptotic stability and the Parceval's theorem the function $F(\lambda)$ is defined and called *the stabilisation function*

$$F(\lambda) = (x_\lambda, x_\lambda) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left|H\left(e^{-j\omega}\right)\right|^2}{\left(\lambda + \left|H\left(e^{-j\omega}\right)\right|^2\right)^2} d\omega \quad (17)$$

Similarly is defined the so-called *approximation function* $\Phi(\lambda)$

$$\Phi(\lambda) = (hx_\lambda - 1, hx_\lambda - 1) = \\ = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\lambda^2}{\left(\lambda + \left|H\left(e^{-j\omega}\right)\right|^2\right)^2} d\omega \quad (18)$$

The graphs of the stabilisation and approximation functions are showed in Fig.1. Full correction of the corrected system is obtained when λ=0. It means, that the stability assumption has not been taken into consideration during seeking the final solution. The corrective system is the exact inversion of the corrected system.

When the ideal inversion of the corrected system is unstable (value of the stabilisation function approach an infinity, which is marked by the thick dashed line on Fig.1), selecting λ>0 allows design the stable corrective filter. In this case a full correction is not possible, because the value of the approximation function is different zero.

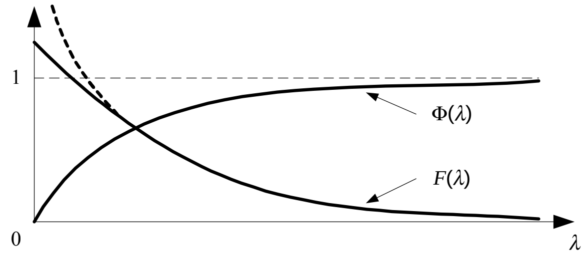

Fig.1. Graphs of stabilisation function F(λ) and approximation function Φ(λ) for the first optimisation problem.

In practice, the stabilisation function should take the prescribed value q, therefore λ should be calculated solving the equation

$$F(\lambda) = q \tag{19}$$

The solution of the Eq. (19) can be carried out by the Newton's method, applying the iterations

$$\lambda_{n+1} = \lambda_n + \frac{q - F(\lambda_n)}{F'(\lambda_n)} \to \lambda_* \tag{20}$$

It can proved, that the iterations are always convergent to the limit λ_* practically independently of the initial value.

In the same way the solution of the second optimum problem is obtained, with the difference, that Lagrange's functional assumes the form

$$f(x,\lambda) = (x,x) + \lambda \cdot [(hx-1, hx-1) - q] \tag{21}$$

The solution has now the form

$$x_\lambda = \frac{\lambda h^*}{1 + \lambda h^* h} \tag{22}$$

Similarly *the stabilisation function* $F(\lambda)$

$$F(\lambda) = (x_\lambda, x_\lambda) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\lambda^2 \left|H\left(e^{-j\omega}\right)\right|^2}{\left(1 + \lambda\left|H\left(e^{-j\omega}\right)\right|^2\right)^2} d\omega \tag{23}$$

and *the approximation function* $\Phi(\lambda)$ are defined.

$$\Phi(\lambda) = (hx_\lambda - 1, hx_\lambda - 1) = = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\left(1 + \lambda\left|H\left(e^{-j\omega}\right)\right|^2\right)^2} d\omega \tag{24}$$

The graphs of the functions are illustrated in Fig.2.

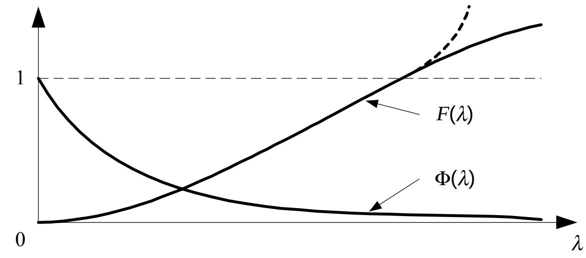

Fig.2. Graphs of stabilisation function F(λ) and approximation function Φ(λ) for the second optimisation problem.

In this case, when the ideal inversion of the corrected system is unstable (value of the stabilisation function approach an infinity, which is marked by the thick dashed line on Fig.2), decreasing λ allows design the stable corrective filter.

Calculation of λ for the given value q, is achieved with the Newton's method, by using the approximation function

$$\lambda_{n+1} = \lambda_n + \frac{q - \Phi(\lambda_n)}{\Phi'(\lambda_n)} \to \lambda_* \tag{25}$$

In each case the stabilisation and approximation functions are mutually inconsistent, i.e. the improvement of the stabilisation leads to the deterioration of the approximation and vice versa.

4. ADDITIVE FACTORISATION OF QUASI-INVERSE FILTER

Let us write the transfer function of the obtained quasi-inverse filter, for the first optimisation problem, in the frequency domain.

$$X_\lambda(z) = \frac{H\left(z^{-1}\right)}{\lambda + H\left(z^{-1}\right) \cdot H(z)} \tag{26}$$

If the corrected system is the FIR system of the order N described by the transfer function

$$H(z) = h_0 + h_1 z^{-1} + \ldots + h_N z^{-N} \tag{27}$$

and the nominator and the denominator of the transfer function $X_\lambda(z)$ is multiplied by z^{-N}, then its denominator can be factorised assuming the form (28), because the complex poles occur foursome. $P(H)$ is the set of the single poles of the denominator of the transfer function $X_\lambda(z)$ lying above the real axis, inside of the unit circle.

$$\prod_{\sigma \in P(H)} \left(1 - \sigma^{-1} \cdot z^{-1}\right) \cdot \left(1 - \left(\sigma^{-1}\right)^* \cdot z^{-1}\right) \cdot \left(1 - \sigma \cdot z^{-1}\right) \cdot \left(1 - \sigma^* \cdot z^{-1}\right) \tag{28}$$

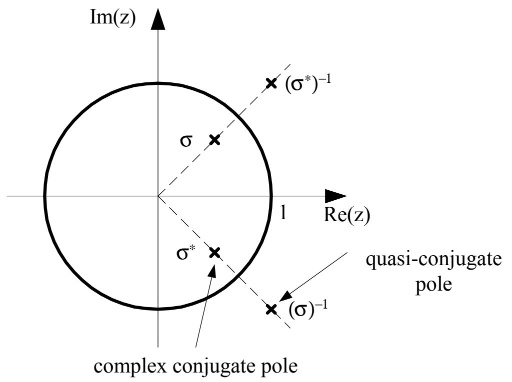

Fig.3. Pole locations for a quasi-inverse filter.

If we divide the factors containing the poles lying outside of the unit circle from the poles lying inside, then the transfer function assumes the form (29).

After the partial-fraction expansion of the transfer function $X_\lambda(z)$, Eq. (29) assumes the factorised form

$$X_\lambda(z) = X_\lambda^-(z) + X_\lambda^+(z) =$$

$$= \sum_{\sigma\in P(H)} \left(\frac{a(\sigma^{-1})}{1-\sigma^{-1}\cdot z^{-1}} + \frac{a^*(\sigma^{-1})}{1-(\sigma^{-1})^*\cdot z^{-1}} \right) +$$

$$+ \sum_{\sigma\in P(H)} \left(\frac{a(\sigma)}{1-\sigma\cdot z^{-1}} + \frac{a^*(\sigma)}{1-\sigma^*\cdot z^{-1}} \right) \tag{30}$$

where

$$a(\sigma) = \left[X_\lambda(z)\cdot\left(1-\sigma\cdot z^{-1}\right)\right]_{z\to\sigma} \tag{31}$$

Because of the partition of the corrective filter into two filters with the transfer functions $X_\lambda^-(z)$ and $X_\lambda^+(z)$, the asymptotic stable anticipative filter can be obtained.

Bringing each of the two constituent transfer functions to a common denominator, yield the following result

$$X_\lambda(z) = \sum_{\sigma\in P(H)} \frac{2\,\mathrm{Re}\left[a(\sigma^{-1})\right]\cdot z^2 - 2\,\mathrm{Re}\left[a(\sigma^{-1})\cdot(\sigma^{-1})^*\right]\cdot z}{z^2 - 2\,\mathrm{Re}\left[\sigma^{-1}\right]\cdot z + \left|\sigma^{-1}\right|^2} +$$

$$+ \sum_{\sigma\in P(H)} \frac{-2\,\mathrm{Re}\left[a(\sigma)\cdot\sigma^*\right]\cdot z^{-1} + 2\,\mathrm{Re}\left[a(\sigma)\right]}{\left|\sigma\right|^2\cdot z^{-2} - 2\,\mathrm{Re}\left[\sigma\right]\cdot z^{-1} + 1} \tag{32}$$

As seen from the above formula, the anticipative quasi-inverse filter is a parallel connection of two filters. One of them contains only the poles from outside of the unit circle and an acceleration operations z, i.e. it operates on the samples from the future. The second contains only the poles from inside of the unit circle and a delay operations z^{-1} and operates on the actual and the past samples. Additionally, each filter is composed of a parallel connection of the simple filters, the coefficients of which are calculated for the individual poles.

In this paper an assumption is made, that the quasi-inverse filter contains only the complex poles. In the case of the occurrence of the real poles α (and by the same token, the quasi-conjugate poles α^{-1}), it is necessary to complete the transfer functions $X_\lambda^-(z)$ and $X_\lambda^+(z)$ with the factors that take them into consideration.

5. DESCRIPTION IN TIME DOMAIN

On the basis of Eq. (32) one can obtain the recurrent equations for the individual simple filters

$$o_{n,\sigma}^- = l_{1,\sigma}^-\cdot i_{n+1} + l_{2,\sigma}^-\cdot i_{n+2} - m_{1,\sigma}\cdot o_{n+1,\sigma}^- - m_{2,\sigma}\cdot o_{n+2,\sigma}^- \tag{33}$$

for $n = M-1, M-2, \ldots, 0$ and

$$o_{n,\sigma}^+ = l_{0,\sigma}^+\cdot i_n + l_{1,\sigma}^+\cdot i_{n-1} - m_{1,\sigma}\cdot o_{n-1,\sigma}^+ - m_{2,\sigma}\cdot o_{n-2,\sigma}^+ \tag{34}$$

for $n = 0, 1, \ldots, M-1$, where:

M	the number of the samples of the carrier of the processed signal,
$\{i_n\}$	the input signal,
$\{o^-_{n,\sigma}\}$	the output signal of the anticausal part of the filter for the pole σ,
$\{o^+_{n,\sigma}\}$	the input signal of the causal part of the filter for the pole σ.

The coefficients in Eq. (33) and (34) are described by (35) – (40)

$$l_{0,\sigma}^+ = 2\,\mathrm{Re}\left[a(\sigma)\right] \tag{35}$$

$$l_{1,\sigma}^+ = -2\,\mathrm{Re}\left[a(\sigma)\cdot\sigma^*\right] \tag{36}$$

$$l_{1,\sigma}^- = -\frac{2\,\mathrm{Re}\left[a(\sigma^{-1})\cdot(\sigma^{-1})^*\right]}{\left|\sigma^{-1}\right|^2} = -2\,\mathrm{Re}\left[a(\sigma^{-1})\cdot\sigma\right] \tag{37}$$

$$l_{2,\sigma}^- = \frac{2\,\mathrm{Re}\left[a(\sigma^{-1})\right]}{\left|\sigma^{-1}\right|^2} = \left|\sigma\right|^2 \tag{38}$$

$$m_{1,\sigma} = -2\,\mathrm{Re}\left[\sigma\right] \tag{39}$$

$$m_{2,\sigma} = \left|\sigma\right|^2 \tag{40}$$

$$X_\lambda(z) = \frac{h_N + h_{N-1}z^{-1} + \ldots + h_0 z^{-N}}{\prod_{\sigma\in P(H)}\left(1-\sigma^{-1}\cdot z^{-1}\right)\cdot\left(1-(\sigma^{-1})^*\cdot z^{-1}\right)\cdot\prod_{\sigma\in P(H)}\left(1-\sigma\cdot z^{-1}\right)\cdot\left(1-\sigma^*\cdot z^{-1}\right)} \tag{29}$$

The internal structure of the causal and anticausal filters is illustrated in Fig.4.

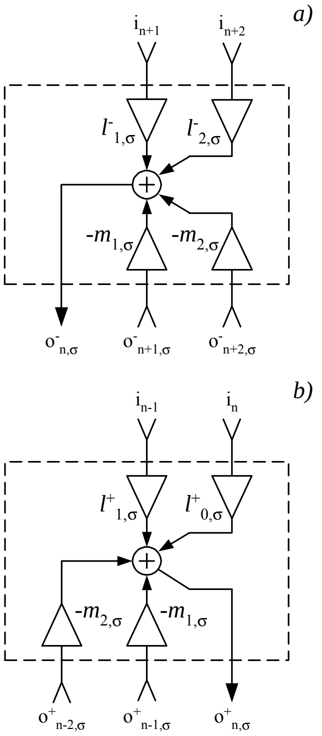

Fig.4. The simple filters of the corrective filter: a) anticausal, b) causal.

The complete output signal is the sum of the corresponding samples of the simple filters responses.

$$o_n = \sum_{\sigma \in P(H)} \left(o^-_{n,\sigma} + o^+_{n,\sigma} \right) \tag{41}$$

If we have an impulse response $\{x_n\}$ of the quasi-inverse filter, then the filtration can be achieved using the convolution

$$o_n = \sum_{m=-\infty}^{+\infty} x_{n-m} \cdot i_m \tag{42}$$

As it was already mentioned, the impulse response of the quasi-inverse filter is the sum of the responses $\{x_n^+\}$ and $\{x_n^-\}$ the causal and anticausal filter, respectively. Thus, Eq. (42) can be written in this form

$$\begin{aligned} o_n &= \sum_{m=-\infty}^{+\infty} \left(x^-_{n-m} + x^+_{n-m} \right) \cdot i_m = \\ &= \sum_{m=-\infty}^{+\infty} x^-_{n-m} \cdot i_m + \sum_{m=-\infty}^{+\infty} x^+_{n-m} \cdot i_m \end{aligned} \tag{43}$$

If the summation will take place in the range $(-\infty, n\rangle$ and $\langle n+1, +\infty)$ then Eq. (43) assumes the form

$$\begin{aligned} o_n = &\sum_{m=-\infty}^{n} x^-_{n-m} \cdot i_m + \sum_{m=n+1}^{+\infty} x^-_{n-m} \cdot i_m + \\ &+ \sum_{m=-\infty}^{n} x^+_{n-m} \cdot i_m + \sum_{m=n+1}^{+\infty} x^+_{n-m} \cdot i_m \end{aligned} \tag{44}$$

But the first and fourth sum in Eq. (44) is equal zero then Eq. (44) one can simplify

$$o_n = \sum_{m=-\infty}^{n} x^+_{n-m} \cdot i_m + \sum_{m=n+1}^{+\infty} x^-_{n-m} \cdot i_m \tag{45}$$

When this is used in a finite set of the non-zero samples indexed from 0 to *M*, then Eq. (45) assumes the final form

$$o_n = \sum_{m=0}^{n} x^+_{n-m} \cdot i_m + \sum_{m=n+1}^{M} x^-_{n-m} \cdot i_m \tag{46}$$

The diagram of the signal processing in the time domain is shown in Fig.5.

6. NUMERICAL EXPERIMENT

A numerical example is given to confirm the theory presented in the previous sections. In this section are presented the simulations in the time and frequency domain. They are calculated for the case where the approximation of the ideal corrective filter is assumed. All simulations are made in Matlab.

The coefficients of the corrected FIR system are presented in Table 1.

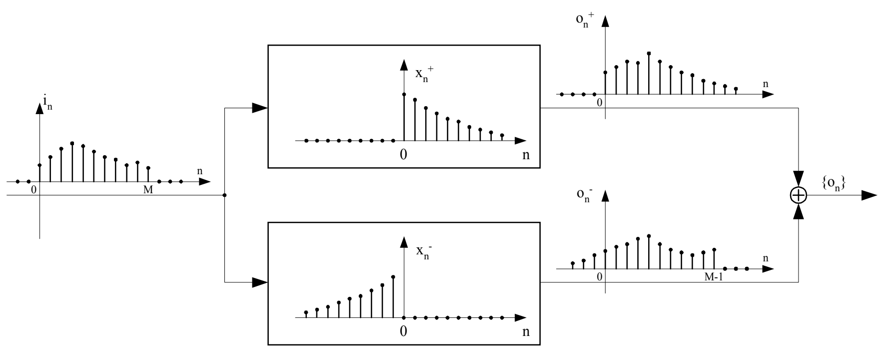

Fig.5. Time domain signal processing of quasi-inverse filter using the causal and anticausal impulse respons.

Table 1. The corrected system coefficients.

No.	Value
0	0,002887873532539
1	0,028012807675437
2	0,088664177134471
3	0,161670176821556
4	0,195312500000000
5	0,161670176821556
6	0,088664177134471
7	0,028012807675437
8	0,002887873532539

Its frequency response is shown in Fig.6. As can be observed this system more or less attenuate all harmonics of an input signal.

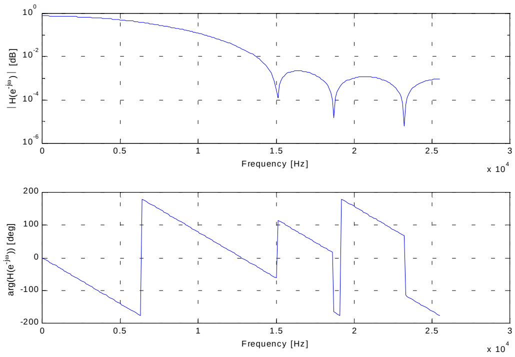

Fig.6. Frequency response of the corrected system.

Assuming that the approximation level of the ideal inverse filter is equal 53%, which is equivalent to q = 0.47, then using Eq. (25) we obtain λ = 2927,6. For the calculated value of λ is drawn a frequency response of the quasi-inverse corrective filter (Fig.7).

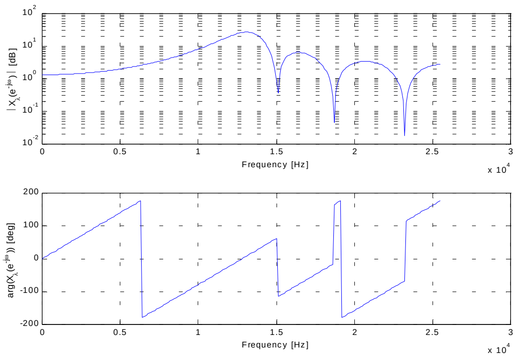

Fig.7. Frequency response of the designed quasi-inverse filter.

Its pole locations are shown in Fig.8. If we limit the set of poles only to the poles lying inside the unit circle and above of a real axis, then each of them has the corresponding complex conjugate pole and all of them have the quasi-conjugate poles.

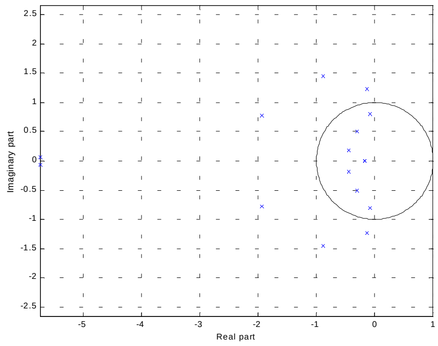

Fig.8. Pole locations for a designed quasi-inverse filter.

Fig.9 presents the resultant frequency response of the corrected system and the quasi-inverse filter. Because the level of the assumed approximation is not equal 100%, so the gain equals unity and the phase shift equals zero only in the limited band of frequency.

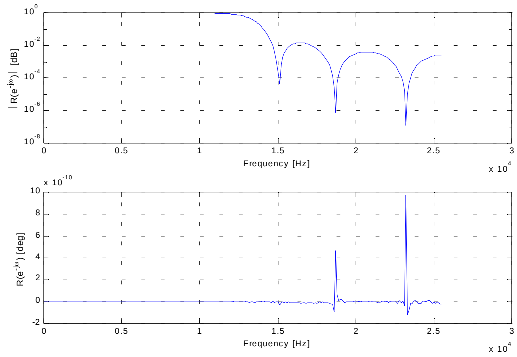

Fig.9. Resultant frequency response of the corrected system and the quasi-inverse filter.

Using the Eq. (33) and Eq. (34) are calculated the anticausal x_n^- and causal x_n^+ components of the impulse response of the quasi-inverse filter (Fig.10).

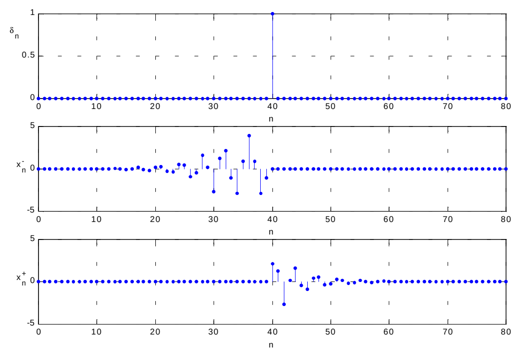

Fig.10. The anticausal x_n^- and causal x_n^+ impulse response of the quasi-inverse filter.

Fig.12 presents the simulations in the time domain, which are performed on the basis of the same equations, where the input signal parameters are presented in Table 2.

Table 2. The parameters of the signal $\{a_n\}$.

Harmonic	Amplitude	Frequency [Hz]
1	4	1600
2	2	3200
5	5	8000

$\{a_n\}$ is the input signal to the corrected system described by the coefficients from Table 1. On the output of the corrected system is obtained the signal $\{i_n\}$, which is the converted signal $\{a_n\}$ according to the frequency response of the corrected system. Their spectrums are shown in Fig.13. Processing this signal by the quasi-inverse filter leads to its full correction.

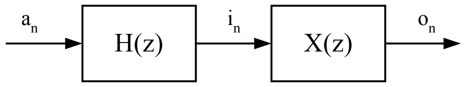

Fig.11. Connection of corrected and corrective systems.

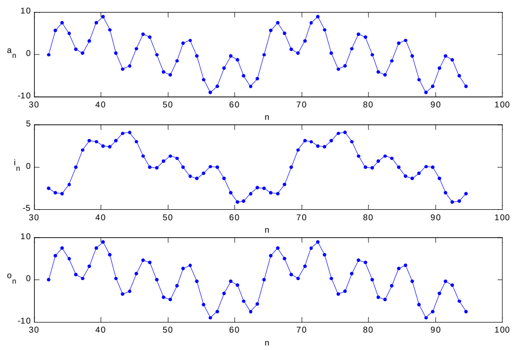

Fig.12. Processed signal in the time domain (2 periods, f_s *= 51200[Hz]).*

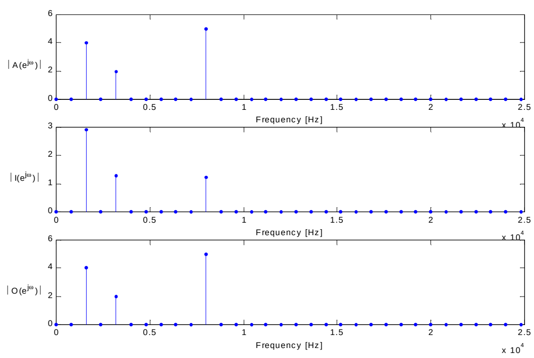

Fig.13. The amplitude spectrum of the processed signals from Fig.12.

7. CONCLUSIONS

In this paper the formulas have been shown, according to which it is possible to design the stable quasi-inverse filter based on existing mathematical model in the form of the transfer function of the corrected system or its impulse response. The advantage of this filter is the possibility of selecting the level of approximation in a ratio to the ideal inverse filter or the level of stability.

The obtained solution is shown in a general form, which can be transformed into the time or frequency domain. On the basis of the obtained solutions the structure of the quasi-inverse filter is presented and also the processing in the time domain. Their certain disadvantage is the necessity of memorising the series of samples in respect to the filter operation on the samples from the future.

In spite of this, that the proposed solution does not belong to the class of the causal but the anticausal filters, it seems that their use in the applications not requiring the online processing can be an alternative for the existing solutions.

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