Transfer Function Identification Based on the Sampled Initial Stage of the Process Step Response

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ABSTRACT

A simple and effective method of parameter identification for a linear model of aperiodic processes with known time delay is presented in this paper. The process time lag order and, subsequently, the remaining model parameters are determined from a few samples of the initial phase of the process step response.

Keywords: discrete measurements, parameter identification, linear modelling.

1. INTRODUCTION

A new identification technique for the order of linear processes utilizing discrete samples of the initial stage of process step responses is introduced here, see [1]. Knowing the process order and utilizing Strejc's model for aperiodic processes one can evaluate the mean time constant of the process, see [2], and subsequently, the spread of its time constants, which may lead to a more complicated multi-time-lag model with different time constants. This is made possible by the use of the identified process order and at least one of the time constants already known (e. g., T_{max} or T_{min}) of the multi-time-lag model.

2. IDENTIFICATION

In [1] a simple method for a deterministic identification of the order n that characterizes a linear multi-time-lag plant model

$$M(s) = \frac{k}{\prod_{i=1}^{n}(1+sT_i)} \tag{1}$$

has been presented.

The time constants in (1) are bounded by the following inequality

$$T_{min} \le T_i \le T_{max} \quad , \quad i = 1, \ldots .. n. \tag{2}$$

evaluated from the sampled initial stage of the plant step response.
It has been shown that it holds for the model order n [1]

$$n = \text{Ent}\ \{1+n_0\} \tag{3}$$

where

$$n_{0_{ij}} = \frac{\ln \frac{h_j}{h_i}}{\ln \frac{t_j}{t_i}} \tag{4}$$

Ordinates h_j, h_i of the step response measured at t_j and t_i time instants respectively lie at the basis of model order n estimation. For a series of i = 1, …, N measurements we get

$$L = \frac{N(N-1)}{2} \tag{5}$$

combinations of unique pairs of ordinates needed to calculate n_0 according to (4).
The deterministic identification of the model order n provides the foundation of identification of the remaining model (1) parameters.
By introducing the notion of the mean time constant $\overline{T}$ [2] based on the mean pole p

$$\overline{p} = \frac{1}{\overline{T}} = \frac{\sum_{i=1}^{n} \operatorname{Re} p_i}{n} \tag{6}$$

of the transfer function (1) we get

$$\overline{T} = \frac{n}{\sum_{i=1}^{n} \frac{1}{T_i}} \tag{7}$$

As was shown in [4] the mean time constant may also be estimated from

$$\overline{T}_{ij} \approx \frac{n}{n+1} \quad \frac{t_j - t_i}{\left(n - n_{0_{ij}}\right) \ln \frac{t_j}{t_i}} \tag{8}$$

after the model had been identified.
By introducing the notion of the spread of time constants (2) characterized by the spread factor λ

$$\lambda = \sqrt[n-1]{\frac{T_{min}}{T_{max}}} \quad , \; 0 \langle \; \lambda \le 1 \tag{9}$$

we may go from the model (1) to the model

$$M_{\lambda}(s) = \frac{k}{\prod_{i=0}^{n-1} \left(1 + s \frac{T_i}{\lambda^i}\right)} \tag{10}$$

where the spread factor λ can be estimated by solving the equation [3]

$$\left.\begin{aligned} &\lambda^{n-1} + \cdots + \lambda + 1 - n \frac{T_{min}}{\overline{T}} = 0 \\ \text{or} \quad &\lambda^{n-1}\left(1 - n \frac{T_{max}}{\overline{T}}\right) + \cdots + \lambda + 1 = 0 \end{aligned}\right\} \tag{11}$$

provided T_{min} or T_{max} is known.

Having known $\overline{T}$ and n we may estimate T_{min} from the inequality

$$\frac{\overline{T}}{n} \langle \; T_{min} \le \overline{T} \tag{12}$$

which follows from the first equation (11).
After λ has been estimated, we get for individual time constants of the model (10)

$$T_i = \frac{T_{min}}{\lambda^{i-1}} = T_{max} \lambda^{i-1} \tag{13}$$

The plant model gain k can be estimated for individual values of the measured samples through the relationship

$$k_{ij} \approx \frac{h\,(t_i)\, n!}{t_i^n \left(1 - \frac{n}{n+1} \frac{t_i}{\overline{T}_{ij}}\right)} \prod_{i=1}^{n} T_i \tag{14}$$

Taking (8) into account, equation (14) can be rewritten in an equivalent form:

$$k_{ij} \approx \frac{h\left(t_i\right) n!}{t_i^n \left[1 - \left(n - n_{0_{ij}}\right) \frac{t_i}{t_j - t_i} \ln \frac{t_j}{t_i}\right]} \prod_{i=1}^{n} T_i \tag{15}$$

Thus, the local simplified linear model (10) of an aperiodic process may be identified on the basis of sampled values of the initial part of the step response.

It should be emphasized that even at the instant

$$t_N \le \; 0{,}7 \; \overline{T} \tag{16}$$

the model structure (i.e. model order n) is known, and approximately its parameters as well. The condition (16), as it was shown in [4], is to be met for convergence of a series, which provides a basis for relationships (4), (8) and (14) or (15). If T_{min} and/or T_{max} are unknown, then, at the t_N instant, the Strejc model

$$S(s) = \frac{\overline{k}}{\left(1 + s\overline{T}\right)^n} \tag{17}$$

is identified with its mean gain

$$\overline{k}_{ij} \approx \frac{h\left(t_i\right) n!}{t_i^n \left(1 - \frac{n}{n+1} \frac{t_i}{\overline{T}_{ij}}\right)} \overline{T}^n \tag{18}$$

The evaluated results of n_0, T, k or $\overline{k}$, the number of which may be L at the maximum according to (5) (extreme values may be dropped), can be averaged.

Employing a series of N measurements enables the effect of noise and disturbances to be substantially reduced.
The described method has been verified using both simulation techniques [5], and physical electrothermal plants.

3. ACCURACY

A high noise-to-signal ratio, which commonly occurs, may become an essential problem in practical applications of this method for model identification of dynamic processes. The influence of the noise increases with the order and can be substantial, since the useful signal is low at the initial stages of the step response. Another problem can be caused by the influence of a static non-linearity on the estimation results.

However, due to the fact that the order is evaluated by (3) and (4), using samples close to each other, the influence of a static non-linearity can be easily minimized or even ignored. To lower the influence of random disturbances added to measured discrete values of the response it is advisable to use a series of N measurements, giving N(N-1)/2 evaluation results instead of a single pair of samples and a single result of evaluation.

The limitation of the method of evaluating the time constants for the model (1) may be that the distribution of time constants T_i is unlike that of eq (13).

An extreme example of this type of process is

$$M(s) = \frac{k}{(1+\alpha\ T)\,(1+T)^{n-1}};$$

$$0 \langle\ \alpha \langle\langle\ 1.$$

It should be emphasized that evaluation of time constants in the model (1) is possible if at least one time constant T_i is known. However, the knowledge of T_{min} (e.g., the time constant of a thermocouple inserted in a furnace) is a more frequent case in practice. The larger the spread of time constants, the smaller n_0 is. In particular, for $T_{min} \to 0$, n decreases by one. Because of the physical meaning only an integer number of n may be accepted. Consequently, according to eq. (3), the order evaluation error becomes negligible (provided the actual time delay τ of the process being identified is known). It should be noted that with the increasing time constants spread factor λ the mean time constant $\overline{T}$ diminishes strongly, which results in a steep decrease (to the power n) in the gain of eq. (18). Therefore, the concept of the mean gain $\overline{k}$ may be of utility, as much as that of the mean time constant. The knowledge of λ allows the evaluation of $k > \overline{k}$ to be made. The accuracy of this method regarding parameters other than the process order n is not high but, nevertheless, it may be sufficient when developing approximate models of actual processes for control purposes. To improve the identification accuracy, instead of a pair samples, a set of many pairs may be used for calculations yielding a set of results to be averaged. Such an approach may provide an efficient means of minimizing the effect of random disturbances superimposed on the sampled step response h(t). The results obtained seem very encouraging, and indicate the validity of the proposed technique. Simulation tests that were carried out also showed its usefulness for purposes applied in controllers.

In [5] 404 results of n_0 calculations have been used to identify the model order from digitally recorded heating-up characteristics of 7 industrial and 2 laboratory electroheated plants rated in the range between 300 W and 90 kW. Of this number only 10 results have been rejected as unreliable, i.e., such ones being higher or lower than the estimated value of n, according to eq. (3). This suggests that model order identification is highly reliable in case of weakly disturbed processes, among which the electroheating processes may be reckoned. The accuracy of the model order identification by the method in hand has been analyzed in [6]. In general, the method is sensitive to disturbances. The following study deals with the so-called worst case of model order evaluation.

Let the plant step response h(t) be superimposed by random disturbances d(t). Hence, the measured response will be

$$h_m(t) = h(t) + d(t) \tag{19}$$

Assuming the disturbance mean value is equal to zero

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T d(t)\,dt = 0 \tag{20}$$

and the disturbance magnitude meets the condition

$$\left|d(j\omega)\right| \le d\ \ ,\ \omega \in [0,\infty) \tag{21}$$

it may be written for the worst case

$$n_{0d+} = \frac{\ln\dfrac{h_j+d}{h_i-d}}{\ln\dfrac{t_j}{t_i}} \tag{22}$$

$$n_{0d-} = \frac{\ln\dfrac{h_j-d}{h_i+d}}{\ln\dfrac{t_j}{t_i}} \tag{23}$$

Thus, the maximum difference in n_0 evaluation caused by the presence of disturbances d will be

$$\Delta n_{0d} = n_{0d+} - n_{0d-} = \frac{\ln\dfrac{(h_j+d)(h_i+d)}{(h_j-d)(h_i-d)}}{\ln\dfrac{t_j}{t_i}} \tag{24}$$

$$\Delta n_{0d} = \frac{\ln\dfrac{h_j h_i + d(h_j+h_i) + d^2}{h_j h_i - d(h_j+h_i) + d^2}}{\ln\dfrac{t_j}{t_i}} \tag{25}$$

It should be kept in mind that as t_j is increased, the disturbance-to-signal ratio d/h decreases, however even this t_j value adheres to eq. (16) [4]. At the same time, the higher is the order n to be identified, the smaller is the greatest sample h_N related to the t_N instant, which is to be regarded as a disadvantageous circumstance. In [4] it was shown that employing a median filter enables an effective order evaluation to be made even with strongly disturbed processes. To reduce the influence of disturbances d(t) on model order identification an analog filter with known parameters to be taken into account when identifying can be employed.

Let assume the measured values of h are afflicted by a relative error. Then from eq. (24) we get

$$\Delta n_{0d} = \frac{\ln\frac{(1+m)(1+l)}{(1-m)(1-l)}}{\ln\frac{t_j}{t_i}} \tag{26}$$

where m is the relative error of h_j, and l is that of h_i. For example, for m=l=d=0.1, $t_j/t_i=2$ eq. (26) yields $\Delta n_{0d}=0.579$. For m=l=d=0.05, $t_j/t_i=2$ we get $\Delta n_{0d}=0.289$. For d<<h it may be derived from (25)

$$\Delta n_{0d} \approx \frac{2\frac{d}{h_i}\left(\frac{h_j}{h_i}+1\right)}{\left[\frac{h_j}{h_i}+\left(\frac{d}{h_i}\right)^2\right]\ln\frac{t_j}{t_i}} = \frac{2\frac{d}{h_j}\left(\frac{h_i}{h_j}+1\right)}{\left[\frac{h_i}{h_j}+\left(\frac{d}{h_j}\right)^2\right]\ln\frac{t_j}{t_i}} \tag{27}$$

which for d=0.1, $h_j \approx h_i=1$, $t_j/t_i=2$ yields $\Delta n_{0d}\approx 0.571$, and for d=0.05, $h_j \approx h_i=1$, $t_j/t_i=2$ yields $\Delta n_{0d}\approx 0.288$.

4. REFERENCES

[1] S. Skoczowski, „A simple method for identification of the Strejc model based on first discrete samples of step responses (in Polish)", *Kwartalnik Elektroniki i Telekomunikacji*, 1992, Vol. 38, pp.193-209.

[2] N. S. Xu., " Aproximate evaluation of the sampled model parameters for a linear continuons-time with relatively small sampling period", *Int. J. Systems Sci.* Vol. 20, 1989, pp. 811-838.

[3] S. Skoczowski, "Estimation of the order and the spread of time constants for multi-time lag processes using step response", Proc. *First International Symposium on Mathematical Models in Automation and Robotics*, 1994 Międzyzdroje, Poland, pp. 351-357.

[4] S. Skoczowski, "Evaluation of order and the spread of time constants for aperiodic processes using step response", *Control Eng. Practice*, 1997, Vol. 5, No. 8, pp. 1077-1089.

[5] S. Skoczowski, „*Deterministic identification and its use in PID-based robust temperature control"(in Polish),* Wydawnictwo Politechniki Szczecińskiej, Szczecin, 2001

[6] S. Skoczowski, „Accuracy of determining the plant model order from the plant step response", (in Polish), *X Jubileuszowe Sympozjum Modelowanie i Symulacja Systemów Pomiarowych*, Krynica, 2000, pp. 29-34.