Proc. IMEKO-TC7 Symposium (Cracow, Poland, June 25-27, 2002)

# Investigation of the Reference Model Used for Verification of the Conductance Tomography System

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## ABSTRACT

In this article the metrology verification technique of electrical conductivity tomography system is proposed and investigated. The verification is based on the reference model of conductivity distribution that is constructed as a network of the discrete resistors (conductors). Proposed technique permits to verified the data acquisition system, reconstruction algorithm and approximation effects jointly or separately. The reference network parameters and the stages of the verification algorithm are in details described.

**Keywords**: Verification, Tomography System, Electrical Conductivity, Reference Model, Instrumental, Algorithm and Approximation Errors.

## 1. CONDUCTANCE TOMOGRAPHY

Electrical conductance (or resistance) tomography technique can be used to image reconstruction of space distribution of electrical conductivity and other physical quantities depended of them in medicine and industry [1-4]. The data acquisition system (DASY) with the measuring electrodes and the computer are the base components of typical electrical tomography system (Fig.1). Measuring electrodes are placed on periphery of research object and are used for its current or voltage excitation. The object responses on this excitation in the form of electrode currents or interelectrode potential differences (or sometimes interelectrode conductances (resistances) are measured and are collected in DASY. These measurement results are used in reconstruction algorithm that forms the image of a search conductivity distribution $\gamma(x,y)$. A reconstruction algorithm usually is iterative. In general case to solve the tomography inverse problem the continuous conductivity distribution must by approximated by discrete elements, for example, using the finite elements method. Therefore the quality of the conductivity image mainly depends on of the electrode values measurement errors, the reconstruction algorithm errors and the approximation error.

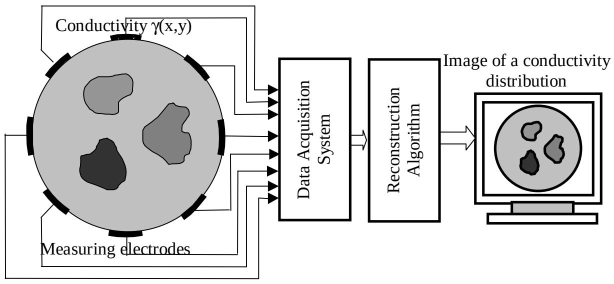


Fig.1. The main components of the electrical tomography system.

## 2. PHYSICAL REFERENCE MODEL OF THE SPACE CONDUCTIVITY DISTRIBUTION

According to tomography principle the local conductivity significance $\gamma_{in}$ inside object cannot be measured directly. Therefore the verification of EIT system required a reference physical model of the conductivity space distribution [6].

### 2.1. Structure of reference model

The external structure of the reference model (quantity of measuring electrodes, current and potential values) must correspond to the structure of the object being investigated. The internal structure of this model should be corresponded to a method of approximation of the electrical field inside research object. The electrical field in conducting environment with non-uniform conductivity $\gamma(x,y)$ at absence of internal current sources can be described by generalised Maxwell equation [1,5]

$$-\nabla[\gamma(x,y)\nabla\varphi(x,y)] = J_e(x,y), \quad (1)$$

where $\varphi(x,y)$ is potential distribution inside object, $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}$ is the differential operator, $J_e(x,y)$ is the current density.

Researches results have shown that the most effective decision of the equation (1) is obtained using finite element method (FEM) [3-5]. The circuit constructed on triangular elements (Fig.2,a) in which is used linear approximation of potential

$$U_i(x,y) = a_{0i} + a_{1i}x + a_{2i}y, \quad (2)$$

and the constant conductivity ($\gamma_i(x,y)=\gamma_{ei}$=const) is the most known [3-5]. The approximated potential distribution (2) of a triangular element is uniquely determined by the nodes potentials and the values of element angles. Each element can be described by a characteristic matrix, received using variational decision of the equation (1) at linear potential (2).

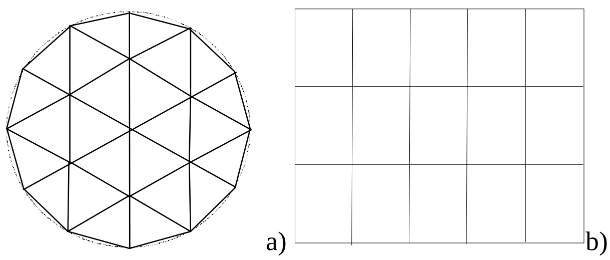


Fig. 2. FEM approximation using triangle (a) and rectangle (b) elements.

The matrix of single triangle is shown in Table 1. Here *d* is the thickness of the conducting layer of object and $\gamma_e$ is the average element conductivity. This matrix is symmetric and in only two corners of a triangle are its independent parameters (third corner is connected with two other by trivial dependence $\varphi_k+\varphi_l+\varphi_m=\pi$). Easily to see that the form of this matrix is equivalent to a form of matrix of a nodes potential method for triangle circuit constructed of discrete conductance elements $G_{KL}$, $G_{LM}$, $G_{MK}$ (Fig.3,a).

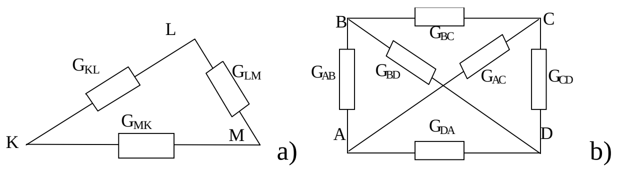


Fig.3. Equivalent electrical circuits of triangle (a) and rectangle (b) finite elements.

Thus, after the comparison these tables, the values of conductance between the triangle vertexes can be calculated by formulas

$$G_{KL}=\gamma_e d\cdot ctg(m);\ G_{LM}=\gamma_e d\cdot ctg(k);\ G_{MK}=\gamma_e d\cdot ctg(l). \quad (3)$$

Table 1. Characteristic matrix of triangular element.

| | | K | L | M |
|---|---|---|---|---|
| K | $\gamma_e d$ | ctg(m)+ctg(l) | -ctg(m) | -ctg(l) |
| L | | -ctg(m) | ctg(k)+ctg(m) | -ctg(k) |
| M | | -ctg(l) | -ctg(k) | ctg(k)+ctg(l) |

Table 2. Matrix of discrete conductance triangle.

| | K | L | M |
|---|---|---|---|
| K | $G_{KL}+G_{MK}$ | $-G_{KL}$ | $-G_{MK}$ |
| L | $-G_{KL}$ | $G_{LM}+G_{KL}$ | $-G_{LM}$ |
| M | $-G_{MK}$ | $-G_{LM}$ | $G_{MK}+G_{LM}$ |

Similar results can be obtained for the rectangular finite elements (Fig.2,b) of the dimensions a×b. Using potential approximation described by formula

$$U_i(x,y) = b_{0i} + b_{1i}x + b_{2i}y + b_{3i}xy, \quad (4)$$

it is possible to calculate the values of the coefficients of the characteristic matrix, which are given in the Table 3.

Table 3. Characteristic matrix of rectangle element.

| | | A | B | C | D |
|---|---|---|---|---|---|
| A | $\gamma_e d/6$ | $2(\nu+1/\nu)$ | $-(2\nu-1/\nu)$ | $-(\nu+1/\nu)$ | $-(2/\nu-\nu)$ |
| B | | $-(2\nu-1/\nu)$ | $2(\nu+1/\nu)$ | $-(2/\nu-\nu)$ | $-(\nu+1/\nu)$ |
| C | | $-(\nu+1/\nu)$ | $-(2/\nu-\nu)$ | $2(\nu+1/\nu)$ | $-(2\nu-1/\nu)$ |
| D | | $-(2/\nu-\nu)$ | $-(\nu+1/\nu)$ | $-(2\nu-1/\nu)$ | $2(\nu+1/\nu)$ |

Where $\nu=a/b$.

This matrix corresponds with the conductance matrix (Table 4) of rectangular circuit constructed on 6 discrete conductance elements (Fig.3,b).

Table 4. Matrix of discrete conductance rectangle.

| | A | B | C | D |
|---|---|---|---|---|
| A | $G_A$ | $-G_{AB}$ | $-G_{AC}$ | $-G_{AD}$ |
| B | $-G_{AB}$ | $G_B$ | $-G_{BC}$ | $-G_{BD}$ |
| C | $-G_{AC}$ | $-G_{BC}$ | $G_C$ | $-G_{CD}$ |
| D | $-G_{AD}$ | $-G_{BD}$ | $-G_{CD}$ | $G_D$ |

Where $G_A=G_{AB}+G_{AC}+G_{AD}$; $G_B=G_{AB}+G_{BC}+G_{BD}$; $G_C=G_{AC}+G_{BC}+G_{CD}$; $G_D=G_{AD}+G_{BD}+G_{CD}$.

Again after the comparison last two tables we can write down expressions of the values of conductance between the rectangle vertexes

$$G_{AB}=\gamma_e d(2\nu-1/\nu)/6;\ G_{AC}=\gamma_e d(\nu+1/\nu)/6;$$
$$G_{AD}=\gamma_e d(2\nu-1/\nu)/6;\ G_{BC}=\gamma_e d(2\nu-1/\nu)/6; \quad (5)$$
$$G_{BD}=\gamma_e d(\nu+1/\nu)/6;\ G_{CD}=\gamma_e d(2\nu-1/\nu)/6;$$

Thus, to each approximation FEM it is possible to present the appropriate network from the discrete elements (resistors or conductors). The examples illustrating the correspondence of the approximating schematics of finite elements and the grids from the resistance elements are shown in figures 4 and 5. Inside these grids the conductance $G_{ij}$ between $i$ and $j$ nodes (along common line $ij$) are determined by parallel connection of conductance $G_{ij,p}$, $G_{ij\,q}$ of adjacent $p$ and $q$ elements, i.e. it is equal to their sum

$$G_{ij}=G_{ij,p}+G_{ij,q}. \qquad (6)$$

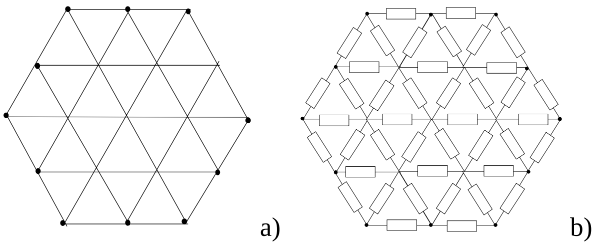


Fig.4. FEM approximation schemes of 24 continuous triangle elements (a) and corresponding to it network constructed from 42 discrete conductance elements (b).

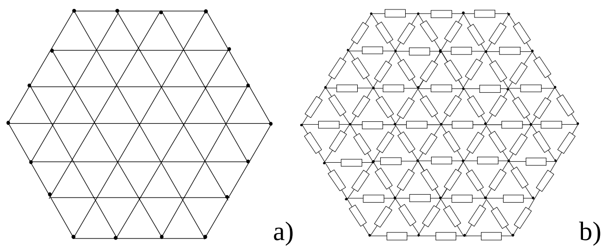


Fig.5. FEM approximation schemes of 54 continuous triangle elements (a) and corresponding to it network constructed from 90 discrete conductance elements (b).

## 2.2. The quantity of discrete elements

The network configuration and the quantity of its discrete elements are depended on the quantity and the form of the elements of the FEM approximation. For example, using a regular approximation of a quasi circular object by triangles in 6 sectors (Fig.4, Fig.5) for a quantity $p$ of approximating circular layers it is possible to calculate:

- the quantity of the output nodes (measurement electrodes)

$$N_{out}=6p^2; \qquad (7)$$

- the total quantity of the network nodes

$$N_n=3p(p+1)+1; \qquad (8)$$

- the quantity of the discrete elements (resistors or conductors)

$$N_G=3p(3p+1). \qquad (9)$$

For example, if a quantity $p$ is equal to $3$ (Fig.5) then a quantity of the discrete resistors is equal $N_G=3\cdot3\cdot(3\cdot3+1)=90$.

## 2.2. The elements accuracy

Electric current badly penetrates the deep layers of the object being investigated. Therefore the sensitivity of measurement results to conductance changes is non-uniform. On the object periphery the sensitivity is maximum and it decreases approximately under exponential law in direction to centre of the object. This in turn specifies a sharp increase of the reconstruction error of the internal elements value. For example, for the two-layered approximation (Fig.4) with 24 triangle elements (42 conductors) the reconstruction error of the internal elements values approximately 10 times is large of the reconstruction error of the outer approximating layer elements (Fig.6). Where $kp_{syst}$, $kp_{rnd}$ are the factors of amplification of systematic and random errors).

The use of one additional approximating layer with 54 triangle elements and 90 conductors (Fig.6) causes the increase approximately 10 times of the reconstruction error of the middle layer elements values and its increase approximately 100 times for the inner layer elements values. Therefore, external elements must have the highest accuracy, and the accuracy of internal elements can be of several times less than of the external elements. Because it does not have sense to require high accuracy to those elements, whose values will be reconstructed with the larger error. In practice the reconstruction error approximately of one percent is acceptable, therefore the accuracy of the reference model elements values must be at the level of the tenth part of the percentage. Therefore the are no problems to choice the conductance of required accuracy.

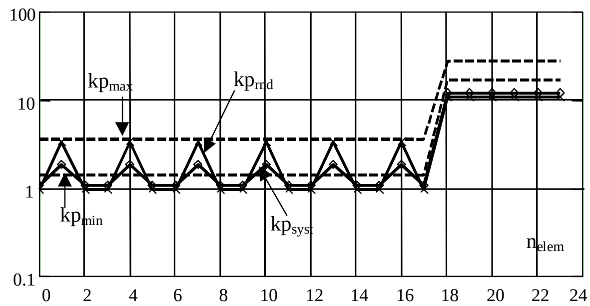


Fig.6. Amplification of instrumental errors for the approximation scheme used 24 triangle continuous or 42 descrete elements.

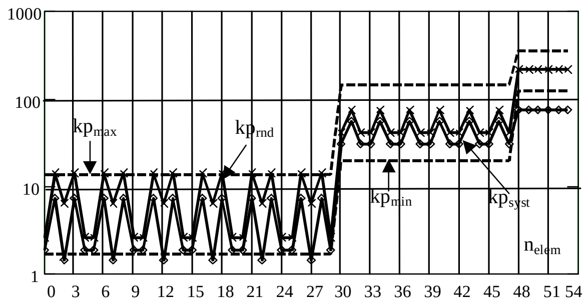


Fig.8. Amplification of instrumental errors for the approximation scheme used 54 triangle continuous or 90 descrete elements.

## 3. METHODOLOGY OF VERIFICATION TOMOGRAPHY SYSTEM

The known conductivity distribution $\gamma^{ref}(x,y)$ is assigned and conductivity $\gamma_e^{ref}$ of triangular elements are calculated. Using equation (3) or (5) and (6) the values $G_{ij}^{ref}$ of the network conductance elements are calculated and are established [6].

**Estimation of the instrumental error.** To estimate the instrumental errors the reference model is connected into DASY inputs instead a real object and the measurements are made. Simultaneously at same conductivity distribution $\gamma^{ref}(x,y)$ the equation (1) is solved numerically and electrode values (vector **E**) are determined. Due to equivalence of the reference model internal structure and the finite elements approximation, the methodical error of the direct problem decision (1) does not change the values of the results. Therefore vector of the instrumental error is calculated by formula

$$\Delta_{instr}=\mathbf{M}-\mathbf{E}. \qquad (10)$$

**Estimation of the reconstruction algorithm error.** To verification reconstruction algorithm the numerically calculated results (vector **E**) are used in reconstruction algorithm (**A**) and appropriate by them elements conductivities values $\gamma_e^{rec}(E)=\mathrm{A}(\mathbf{E})$ are calculated. The reconstruction algorithm error is a difference between reconstruction results on a calculated data and the reference model element values:

$$\Delta_{algr}(\gamma)=\gamma_e^{rec}(E)-\gamma_e^{ref}. \qquad (11)$$

**Estimation of the combine (instrumental + reconstruction algorithm) error.** This error is a difference between of the reconstruction results obtained by the measured data $\gamma_e^{rec}(M)=\mathrm{A}(\mathbf{M})$ and the reference model element values :

$$\Delta_{cmb}(\gamma)=\gamma_e^{rec}(M)-\gamma_e^{ref}. \qquad (12)$$

**Estimation of the approximation error.** The equation (1) is solved numerically using more precision approximation method and electrode values $\mathbf{E}^{(p)}$ are determined. The approximation error can be calculated after the formula

$$\Delta_{apr}(\gamma)=\mathrm{A}(\mathbf{E}^{(p)})-\gamma_e^{ref}. \qquad (13)$$

## 4. CONCLUSIONS

- The verification is based on the reference model of conductivity distribution that is constructed as a network of the discrete resistors (conductors).
- Proposed technique permits to verified the data acquisition system, reconstruction algorithm and approximation effects jointly or separately.
- The external structure of the reference model must correspond to the electrode structure installed into the object being investigated and the internal structure of this model should be corresponded to the used method of the electrical field approximation.
- The network configuration and the quantity of its discrete elements are depended on the quantity and the form of the elements of the FEM approximation.
- The expressions for determining the parameters of the reference model are obtained. The are no problems to choice the conductance of required accuracy
- Methodology of verification tomography system is presented.

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