Proc. IMEKO-TC7 Symposium (Cracow, Poland, June 25-27, 2002)

Conductivity Measurements of Conducting Materials with the Use of Inductive Transducer in Digital Measurement System

Józef KUSMIERZ
Department of Electrical and Computer Engineering
Kielce University of Technology
Al. 1000-lecia P.P. 7, 25-314 Kielce, Poland
E-mail: kusmierz@eden.tu.kielce.pl

ABSTRACT

This paper presents a processing formula of an inductive transducer for conductivity measurements of conducting materials. On the basis of this formula, an analysis of metrological properties for a flow version of the transducer has been performed. This type of the transducer consists of a coil wound round an insulating pipe of which – coaxially with the coil – a cylindrical sample of investigated material is placed. The aforementioned processing formula of the transducer has the form of a dependence between – on the one hand – the changes of impedance observed at terminals of the coil and – on the other hand – conductivity of the material of the sample, geometrical structure of the transducer and frequency of power supply. This formula allows us to optimize the design of the transducer. The advantages resulting from theoretical analysis have been demonstrated in practical experiment carried out by means of digital measurement system.

Keywords: eddy currents, conductivity measurements, inductive transducer, mathematical model of transducer, digital measurement systems.

1. INTRODUCTION

In metrological practice, as far as measurements of electrical conductivity of conducting materials are concerned, two measurement techniques are applied: a contact technique and a contactless technique. The contact method is characterized by several disadvantages such as, among others, a special preparation of samples of investigated materials, the designing of appropriate terminals connecting the investigated samples with measurement system and a comparability of the terminals resistance and the resistance of samples especially for samples made of well-conducting materials. In contactless technique the aforementioned difficulties do not exist, however, other problems – regarding the interpretation of measurement results – appear. In the literature concerning contacless measurements of conductivity of different materials with the use of inductive (eddy-current-based) principle [2, 3, 6], one cannot find a complete analysis of metrological properties of transducers. One can only find formulas derived under simplified assumptions and not providing opportunities for designing transducers taking into account the influence of all physical quantities upon the processing characteristics. For this reason there is a need to develop mathematical models of this type of transducers and improve numerical methods revealing metrological properties coming out of model solution [6, 7].

Determination of conductivity of a given material in the contactless way, using the inductive transducer, consists in introducing the investigated sample into a variable magnetic field of the transducer and the measurement of the changes of impedance observed at its terminals. These changes are caused by eddy currents induced in the investigated sample [6]. The losses of energy of an electromagnetic field to maintain eddy currents results in increasing the active component of the coil impedance by DR. On the other hand, the influence of a magnetic field of eddy currents upon the external magnetic field manifests in decreasing of the imaginary component of the coil impedance by DX. The knowledge of dependences expressing connections between the changes of impedance components (DR, DX) and the conductivity of the investigated sample, geometrical structure and frequency of power supply allows us to design the transducer in an optimal way and interpret the measutent results.

2. NUMERICAL MODEL OF THE TRANSDUCER

In order to solve the aforementioned problem, the construction model of the inductive transducer in a flow version has been assumed (Fig. 1).

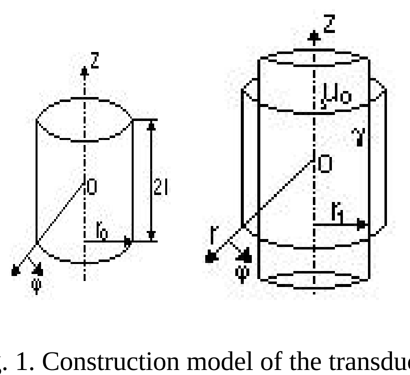


Fig. 1. Construction model of the transducer

In the model, the parameters of particular media are the following:

– the investigated sample

$$0 < r < r_1, \quad g > 0, \quad m = m_0 ,$$

– the inducing coil

$$r = r_0 , \quad i = \begin{cases} \frac{Iw}{2l}, & \text{for } |z| < l, \\ 0, & \text{for } |z| > l, \end{cases}$$

– the air

$$r_0 \geq 0, \quad g = 0, \quad m = m_0 ,$$

where:
w – the number of turns of coil winding,
g – the conductivity of the sample.

In the considered model, field equations are Maxwell's equations [1, 3] whose solutions are searched by means of an angle component of vector potential $A_j\ (r,z)$. It is justified due to the axial symmetry of the transducer and the assumed model of the inducing coil in the form of surface currents with angle components flowing on the surface of the cylinder characterized by the radius r_0 and height $2l$. Therefore, the equation describing the vector potential in the cylindrical coordinate system (r, j, z) – after the integral Fourier transform with respect to variable z is applied – assumed the form:

$$\frac{d^2 A_j}{dr^2} + \frac{1}{r}\frac{dA_j}{dr} - (q^2 + \frac{1}{r^2})A_j = 0 , \qquad (1)$$

where:

$$A_j\ (r, l) = \int_0^{\infty} A_j\ (r,z) \cos lz\, dz ,$$

$q^2 = l^2 + jwm_0 g$, w – pulsation of the current.

The solution of the equation (1) with boundary conditions [3] concerning the continuity of A_j on the separation surfaces and the comparison of tangent components of the magnetic field intensity on these surfaces and after applying an inverse transformation [4] has the form:

$$A_j\ (r,z) = \frac{m_0 Iwr_0}{pl} \int_0^{\infty} f(q,r) K_1(l, r_0) \frac{\sin ll}{l} \cos lz\, dl \ ,(2)$$

where

$$f(q,r) = I_1(lr) + \\ + \frac{lI_0(lr_1)I_1(qr_1) - qI_1(lr_1)I_0(qr_1)}{lK_0(lr_1)I_1(qr_1) + qK_1(lr_1)I_0(qr_1)} K_1(l, r_1),$$

I_0, I_1, K_0, K_1 – modified Bessel functions.

In turn, the tangent component of the electric field on the surface of the cylinder modelling the coil is determined:

$$E_j\ (r_0, z) = -jwA_j\ (r_0, z)$$

and the voltage on the coil terminals is calculated

$$U = \frac{2pr_0 w}{2l} \int_{-l}^{l} E_j\ (r_0, z)\, dz$$

as well as the impedance – on the basis of the Ohm's law – $Z = \frac{U}{I}$ is obtained.

3. PROCESSING FORMULA

The processing formula has been defined as follows:

$$D^0 Z = \frac{Z - Z_0}{Z} , \qquad (3)$$

where:
Z_0, Z – the coil impedance without and with the sample, respectively.

$$D^0 Z = \frac{j \int_0^{\infty} F(y, a, d)[K_1(yd) \frac{\sin ydx}{y}]^2 dy}{\int_0^{\infty} I_1(ydK_1(yd)[\frac{\sin ydx}{y}]^2 dy} ,$$

$$F = \frac{yI_0(x_1)I_1(x_2) - \sqrt{y^2 + j}\ I_1(x_1)I_0(x_2)}{yK_0(x_1)I_1(x_2) + \sqrt{y^2 + j}\ K_1(x_1)I_0(x_2)} ,$$

$$x_1 = yad , \quad x_2 = ad\sqrt{y^2 + j} ,$$

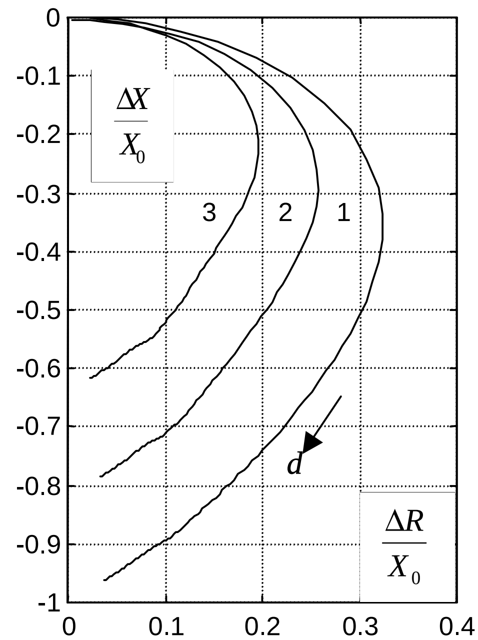
$$y = \frac{l}{\sqrt{wm_0 g}} , \quad d = r_0\sqrt{wm_0 g} , \quad x = \frac{l}{r_0}$$

$$a = \frac{r_1}{r_0} .$$

4. NUMERICAL RESULTS

In order to reveal metrological properties resulting from formula (3), it has been subject to numerical analysis. The results of the calculations have been presented in the form of characteristics generated by means of MATLAB files. Fig. 2 presents the characteristics $\frac{DX}{X_0} = f(\frac{DR}{X_0})$ under $a = \text{const.}$ ($a = 1.0,\ 0.9,\ 0.8$) and $d \in (0, \infty)$.

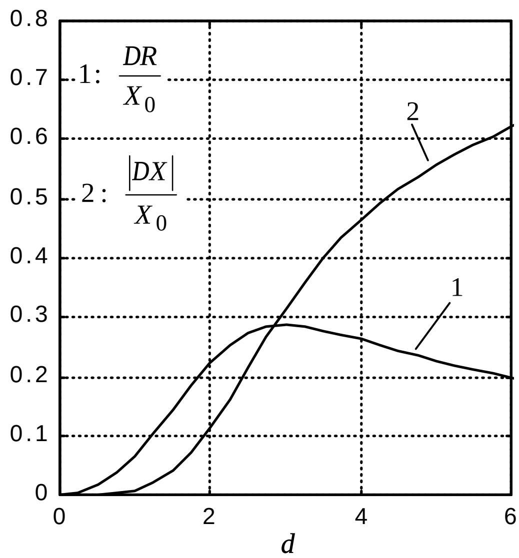
The characteristics of Fig. 2 show that after introducing inside the coil a sample made of material of infinitely large conductivity and characterized by the diameter equal to the coil diameter, one can reach the point at which $\frac{DX}{X_0} = -1$, which means the decay of the magnetic flux and transducer inductance ($X = 0$). In the case of a sample of very small conductivity, the change of the reactance component is equal to zero. On the other hand, the change of the resistance component is equal to zero both for the samples of infinitely large and infinitely small conductivities.



[1: α = 1.0; 2: α = 0.9; 3: α = 0.8]; ξ = 0.5

Fig. 2. Characteristics $\frac{DX}{X_0} = f(\frac{DR}{X_0})$

Figs. 3a and 3b present processing characteristics; the value of variable d under constant pulsation v and constant geometrical dimensions depend on the conductivity of the sample.



$a = 1.0; \ x = 0.5$

Fig. 3a. Processing characteristics in a wide range of changes of variable d

The following conclusions can be drawn from the processing characteristics:

– knowing the approximate range of the changes of the conductivity of investigated sample, geometrical dimensions of the coil and power supply conditions should be designed so as to preserve the monotonicity of the processing characteristics (Fig. 3a),
– in order to obtain possibly large values of the change of impedance, the degree at which the investigated sample fills the coil should be close to 1 ($a \to 1$),

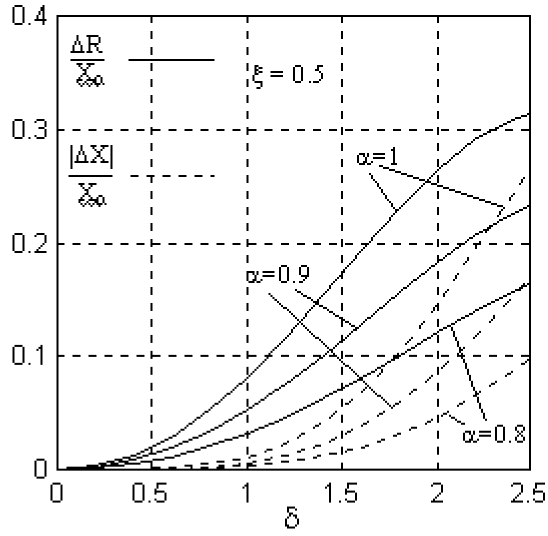


Fig. 3b. Processing characteristics in a range of practical applications

– for $d < 1$, the values of the changes of reactance component in relation to the values of the changes of resistance component are negligibly small (Fig. 3b),
– relative changes of impedance components as functions of variable d are strictly monotonic functions in the interval $d \in [0.4, 2.5]$; this interval should be treated as a working area from which one can determine the measurement range of conductivity and set the power supply conditions.

The processing range can be set by determining the minimal and maximal values of d. The processing characteristics show that the minimal and maximal values of d are equal to 0.4 and 2.0, respectively. In such a range, to each value of d, some value of the conductivity of the sample can be assigned using the following formula:

$$g = \frac{d^2}{r_0^2 w m_0}. \tag{4}$$

Utilizing the characteristics $\frac{DR}{X_0} = f(d)$, $\frac{DX}{X_0} = f(d)$, for each value of conductivity g from the interval $[g_{min}, g_{max}]$, one can determine the corresponding values of the changes of impedance observed at the terminals of the measurement coil. The possibility of such an evaluation is of big significance from the point of view of the selection of appropriate measurement method and measurement devices cooperating with the transducer.

The monotonicity of characteristics provides the basis for performing an inverse operation, that is, the determination of the value of the conductivity of the sample based on measured components DR and DX.

5. MEASUREMENTS

In order to verify the usability of formulas discussed in Section 3 to the designing of inductive transducers, several standard samples of known conductivities have been prepared and, in turn, the measurements have been carried out. A single-coil transducer made as a single-layer coil wound – using a copper wire – directly round the surface of a glass vessel of external diameter equal to 56.0 mm and internal diameter equal to 54.9 mm has been used in the measurements. The length of the coil is equal to 28 mm. The coil was protected against the shifting of the turns and against the moisture by means of two-component glue 'Distal'. The subject of investigations in this experiment were the samples of water solution of sulfuric acid characterized by the following conductivities: $g = 10.8$, 39.2, 54.3, 65.3, 71.7 S/m ($t = 18°C$) which correspond to concentrations: 5%, 10%, 15%, 20% and 25%. Taking into account the considerations presented in Section 4 as well as in the paper [6], a method of the measurement of the transducer magnification has been selected. The measurement principle is presented in Fig. 4.

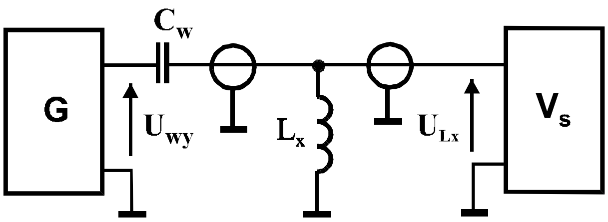


Fig. 4. The system presenting the measurement principle: G – supplying generator, C_w – standard capacitor, L_x – transducer, V_s – peak-value voltmeter.

A generator with controlled amplitude and frequency supplies a typical series resonance system containing a standard capacitor and a measurement coil. The system must fulfill the following conditions:

1. The amplitude of the output voltage must be stable.
2. The output impedance of the generator must be small, however, it is not necessary to fulfill the condition imposed on Q-meters since the system impedance does not change in wide range (a constant value of capacitor C_w).
3. The frequency is selected depending on measurement conditions.
4. The processing formula for the voltmeter has the following form:

$$U_L = QU_{wy}.$$

Therefore, the system enables us to measure the transducer magnification without the sample (Q_0) and with the sample (Q_1), and then the calculation of relative changes of the resistance component $\frac{DR}{wL_0}$ can be made using the following formula [6]:

$$\frac{DR}{wL_0} = \frac{Q_0 - Q_1}{Q_0 Q_1}. \quad (5)$$

The value of $\frac{DR}{wL_0}$ obtained from (5) and the numerically calculated characteristics $\frac{DR}{wL_0} = f(d)$ enable us to determine the value of parameter d and then – using formula (4) – the value of the conductivity of the samples.

In our experiment, the afore-listed tasks are carried out in an automatic manner in the digital measurement system. A scheme of such system which contains typical, series-production-elements is presented in Fig. 5. Non-typical are only the transducer itself and the standard capacitor.

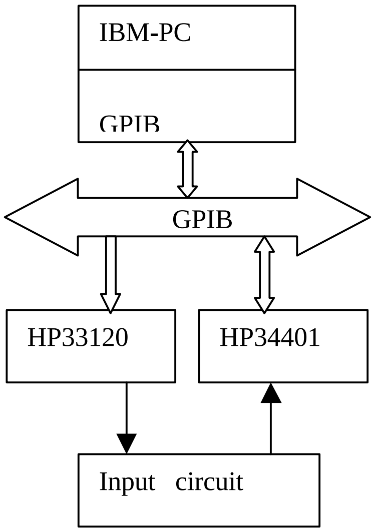


Fig. 5. Digital measurement system

System generator G with digitally controlled amplitude and frequency is run according to the program contained in the memory of the IBM-PC computer. The computer – equipped with the GPIB-standard interface card – plays the role of the system's control unit and successively sets different values of the frequency starting from the initial value equal to $0.9 f_r$ (f_r is an approximate resonance frequency for the system without the sample set by the system before the measurements) until the resonance occurs. The system voltmeter measures the peak value of the voltage after each change of the frequency. Comparing the successive results, the system searches for the resonance point and stores the voltage value which is a function of the transducer magnification and then the system carries out calculations of $\frac{DR}{X_0}$ according to formula (5). The values of $\frac{DR}{X_0}$ obtained in such a way and numerically calculated – based on formula (3) – characteristics $\frac{DR}{X_0} = f(d)$ enable us to determine the values of parameter d and then – from formula (4) – the value of the conductivity of the investigated samples. Fig. 6 presents – in the form of plots – the results obtained in the measurement process (curve 2) and – for the purposes of comparison – theoretical results (curve 1).

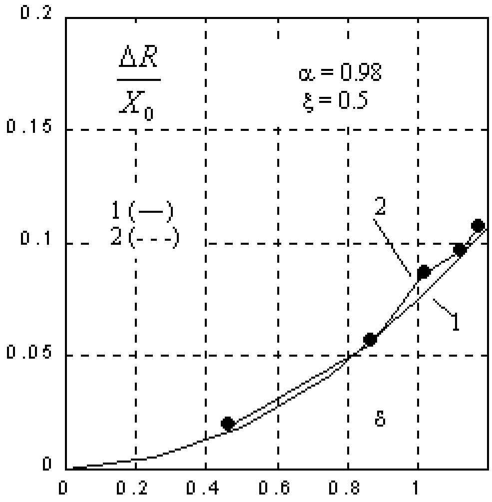


Fig. 6. Experimental (2) and theoretical (1) characteristics

6. CONCLUSIONS

The measurement experiments which have been carried out demonstrate that the measurement system developed in this paper and consisting of input part (the standard capacitor and inductive transducer) and system part (generator, voltmeter, computer) enables us to quickly and efficiently measure the conductivity of conducting materials using a contactless technique.

The derived formulas are useful in designing inductive transducers taking into account the real dimensions of the coil and medium.

9. REFERENCES

[1] H. Hepner, H. Stroppe, *Magnetic and inductive testing of metals*, Wyd. Slask, 1971 (in Polish).

[2] E. Stolarski, “Designing and calibration of inductive indicators”, *Archiwum Elektrotechniki*, Vol. XXIV, No. 91, 1975, pp. 215- 233 (in Polish).

[3] K. Bochenek, *Methods of analysis of electromagnetic fields*, Warsaw, 1969 (in Polish).

[4] G.M. Fichtenholz, *Differential and integral calculus*, Warsaw, 1978 (in Polish).

[5] J. Ebert, “Frequency of self-resonance and self-capacity of shielded solenoid”, *Archiwum Elektrotechniki*, Vol. XVII, No. 2, 1968, pp. 365-375 (in Polish).

[6] J. Kusmierz, “An analysis of processing functions of inductive transducers”, *Metrologia i systemy pomiarowe PAN*, Vol. V, No. 3, 1998, pp. 173-182 (in Polish).

[7] L. Udpa, W. Lord, “Impedance and mesh structure considerations in the finite element analysis of eddy current NDT probe phenomena”, *IEEE Trans. on Magnetics*, Vol. 21, No. 6, 1985, pp. 2269-2272.