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DIGITAL FAULT LOCATION ESTIMATOR FOR POWER LINES BASED ON WALSH FUNCTIONS

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Abstract – Distance relays must estimate accurately and quickly the distance to the fault even in presence of highly distorted input signals. Several digital filtering algorithms for distance relays have been proposed and some of them are used in practical applications. However nonlinear loads are becoming more and more common in electric power systems and that increases the corruption level of signals. For this reason, the study of digital filtering techniques for the extraction of fundamental components from highly corrupt voltage and current waveforms is of vital importance for the development of algorithms applied to the digital protection of transmission lines. The studied technique proposed in this paper is based on Walsh functions.

Keywords: Walsh functions, Digital filtering, Distance protection.

1. INTRODUCTION

Several papers have been published in recent years in digital protection of power systems and in particular in protection of transmission lines because they are primarily exposed to short circuits and consequently present a higher probability of fault occurrence. Transmission lines are normally protected with distance relays; they estimate the impedance between the short circuit and the relay location using voltage and current signals. However during the fault occurrence the input signals to protective relays are contaminated with noise (harmonics and the DC component), that must be rejected while must be retained only signal quantities of interest [1]-[5].

The presence of harmonics and noise determines errors when estimating the fundamental components of voltage and current necessary for the apparent impedance calculation. For this reason it is very important to implement an efficient filtering technique in order to extract the fundamental components of voltage and current and calculate precisely the apparent impedance.

The present paper presents a digital filtering technique based on Walsh functions [6], [7] for extraction of the fundamental components from faulted voltage and current waveforms taken from a typical 345 kV transmission line.

In this paper it has been included a corrective factor that improves the behavior of the measuring system by compensating the error source due to the DC current component consequent to the fault inception.

2. DIGITAL FILTERING ALGORITHM

The Fourier series of a periodic waveform f(t) may be written

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos n\omega t + b_n \sin n\omega t\right] \qquad (1)$$

where a_0 is the mean value and:

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt \qquad (2)$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt \qquad (3)$$

and $T=2\pi/\omega$ is the period of the fundamental frequency 50 Hz.

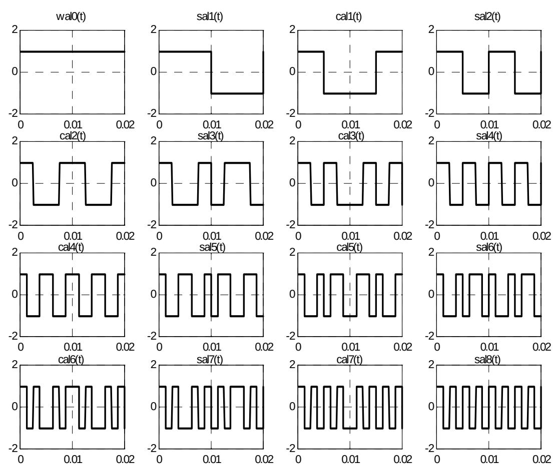

Fig. 1 Walsh functions up to the eighth order

Walsh functions, which are shown in Fig. 1 up to $sal_8(t)$, can be used as an alternative orthogonal family for the series representation of a periodic waveform according to

$$f(t) = A_0 + \sum_{n=1}^{\infty}\left[A_n cal_n(t) + B_n sal_n(t)\right] \qquad (4)$$

where

$$A_n = \frac{1}{T}\int_0^T f(t)\, cal_n(t)\, dt \qquad (5)$$

$$B_n = \frac{1}{T}\int_0^T f(t)\, sal_n(t)\, dt \qquad (6)$$

$$A_0 = \frac{1}{T}\int_0^T f(t)\, wal_0(t)\, dt \qquad (7)$$

The procedure for the Walsh series determination and the corresponding Fourier derivation is given below.

Consider a periodic voltage f(t) whose period is T at the fundamental frequency f =50 Hz. Signal f(t) is sampled with sampling frequency f_s .

The Walsh coefficients A_n and B_n are obtained according to (5), (6), (7). To yield the Fourier spectrum of f(t), A_n and B_n are first multiplied by Walsh-Fourier conversion matrices $\mathbf{F_a}$ and $\mathbf{F_b}$, respectively, to give approximations **a*, b*** to the cosine and sine coefficients of f(t):

$$\mathbf{a^* = F_a\, A}$$

$$\mathbf{b^* = F_b\, B} \qquad (8)$$

where

$$\mathbf{A} = \begin{vmatrix} A_1 \\ A_2 \\ . \\ . \\ A_S \end{vmatrix} \qquad \mathbf{B} = \begin{vmatrix} B_1 \\ B_2 \\ . \\ . \\ B_S \end{vmatrix} \qquad (9)$$

represent the cal spectrum and sal spectrum of f(t), respectively.

The element of the Walsh to Fourier conversion matrices $\mathbf{F_a}$ and $\mathbf{F_b}$ are the Fourier coefficients of the Walsh functions $cal_m(t)$ and $sal_m(t)$, respectively.

Thus

$$a_{n,m} = \frac{2}{T}\int_0^T cal_m(t)\ \cos n\omega t\ dt \qquad (10)$$

$$b_{n,m} = \frac{2}{T}\int_0^T sal_m(t)\ \sin n\omega t\ dt \qquad (11)$$

where n is the index of row (corresponding to the harmonic order) and m is the index of column. Moreover the elements of the conversion matrix $\mathbf{F_a}$ assume the same absolute value of the conversion matrix $\mathbf{F_b}$, but their signs may differ. The conversion matrices are essentially semi-infinite; when applied in (8) however they are truncated to SxS element square matrices. Also an additional matrix multiplication is required which compensated for the Walsh spectral truncation

$$\mathbf{K_a\, a^* = a}$$

$$\mathbf{K_b\, b^* = b} \qquad (12)$$

where **a** and **b** are the desired cosine and sine coefficients, respectively. The simplest form of compensation occurs when $S = 2^k - 1$, i.e., the highest harmonic order in f(t) doesn't exceed the third, or the seventh, or the fifteenth,….

Then compensation takes the form of a diagonal matrix, with fixed elements

$$\left[\frac{\pi n/2^{k+1}}{\sin(\pi n/2^{k+1})}\right]^2 \qquad (13)$$

where n is the harmonic order.

Thus the fundamental waveform $f_1(t)$ of the signal may be written

$$f_1(t) = M_1 \sin(2\pi f t + \varphi_1) \qquad (14)$$

where

$$M_1 = \sqrt{a_1^2 + b_1^2}$$

$$\varphi_1 = \tan^{-1}(a_1/b_1) \qquad (15)$$

3. FILTER PERFORMANCE EVALUATION

In order to test the applicability of the proposed technique, a simulation of the transmission line in a faulted condition was utilized. This paper makes use of a digital simulation of faulted EHV transmission lines, whose model is described in [4] and [5]. A 250 km transmission line model was used for testing the fault distance algorithm. The series parameters of the power system are taken from a typical 345 kV transmission line; the parallel parameters of the line are neglected in this analysis.

As mentioned before, during the fault occurrence, the voltage and current waveforms possess the fundamental components with the addition of harmonics and, in particular the current waveform, a DC decaying offset.

We consider the line in faulted condition, neglecting the effects of the output impedances of power transformers; therefore the system model is linear. Moreover practical considerations such as the effects of quantization, etc. in primary system fault data are also included in the analysis, so that the data obtained are very close to those found in practice.

Fig. 2 shows typical voltage and short-circuit transient current waveforms.

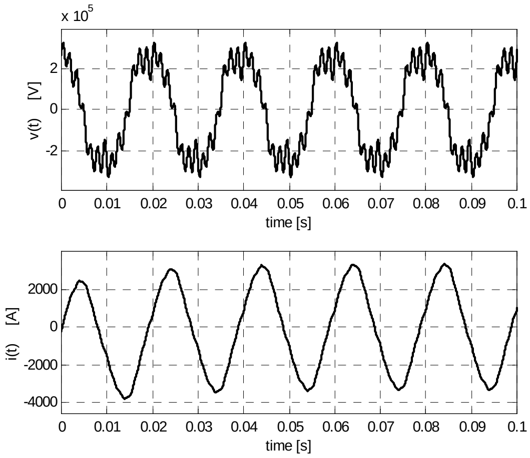

Fig. 2 Typical voltage and short-circuit transient current waveforms

The proposed procedure permits to obtain, besides the DC offset of the current I_0, the fundamental components of current $i_1(t)$ and voltage $v_1(t)$, whose waveforms, compared with the reference ones $i_{1ref}(t)$ and $v_{1ref}(t)$, obtained by the same system model in the known steady state sinusoidal condition, are shown in Fig. 3.

$$I_0 = A_0 \tag{16}$$

$$i_1(t) = I_1 \sin(2\pi ft + \varphi_{i1}) \tag{17}$$

$$v_1(t) = V_1 \sin(2\pi ft + \varphi_{v1}) \tag{18}$$

with

$$I_1 = \sqrt{a_{i1}^2 + b_{i1}^2} \qquad \varphi_{i1} = \tan^{-1}(a_{i1}/b_{i1}) \tag{19}$$

$$V_1 = \sqrt{a_{v1}^2 + b_{v1}^2} \qquad \varphi_{v1} = \tan^{-1}(a_{v1}/b_{v1}) \tag{20}$$

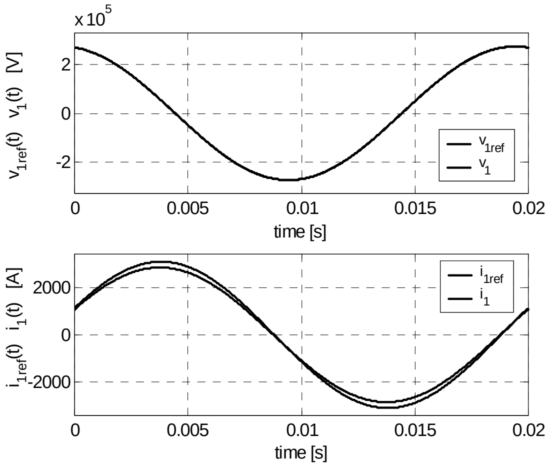

Fig. 3 Fundamental components of the voltage and current waveforms and reference waveforms.

As we can see, the waveforms of $v_1(t)$ and $v_{1ref}(t)$ are practically overlapped, while the waveforms of $i_1(t)$ and $i_{1ref}(t)$ are slightly different, due to the presence of the DC offset.

To obtain a more adequate match between $i_1(t)$ and $i_{1ref}(t)$, two corrective factors, k_A and k_{ph}, are introduced in order to modify amplitude and phase angle, respectively, of the fundamental component of the current.

Corrective factors values, for given parameters of the transmission line, are function of ratios I_0/I_1 and τ/T, where τ is the line time constant; however if τ/T is in the range from 0.65 to 0.85, k_A and k_{ph} are practically functions only of ratio I_0/I_1.

Fig. 4 shows corrective factors k_A and k_{ph} as a function of ratio I_0/I_1 and for τ/T in the above mentioned range.

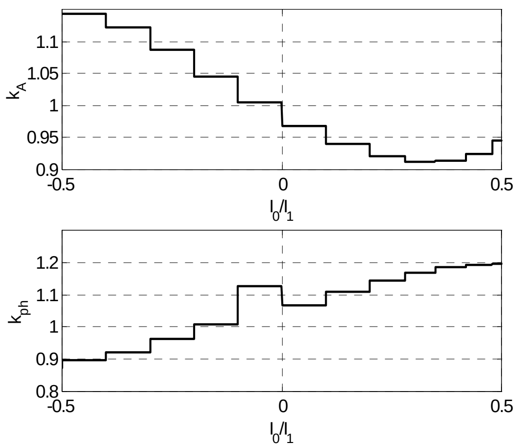

Fig. 4 Corrective factors k_A and k_{ph} as a function of ratio I_0/I_1 and for τ/T in the range from 0.65 to 0.85.

After rearrangement, the corrected fundamental component of the current $i_{1c}(t)$ may be written

$$i_{1c}(t) = I_{1c}\sin(2\pi f t + \varphi_{i1c}) \quad (21)$$

with

$$I_{1c} = k_A I_1$$
$$\varphi_{i1c} = k_{ph}\varphi_{i1} \quad (22)$$

Fig. 5 shows the waveform of the fundamental component of the current $i_{1c}(t)$ after correction, compared with the reference one $i_{1ref}(t)$.

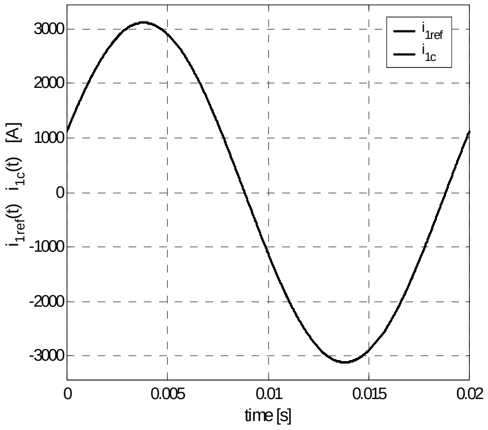

Fig. 5 Fundamental component of the current waveform after correction and reference waveform

The line reactance is given by

$$X_1 = \frac{V_1}{I_{1c}}\sin(\varphi_{v1} - \varphi_{i1c}) \quad (23)$$

Neglecting the fault reactance with respect to the line reactance, the fault distance is determined by

$$d = \frac{X_1}{x} \quad (24)$$

where x is the specific line reactance expressed in Ω/km.

Fig. 6 shows the estimated fault distance (true distance 100 km) as a function of the voltage phase angle φ_{v1} at the instant of the fault inception.

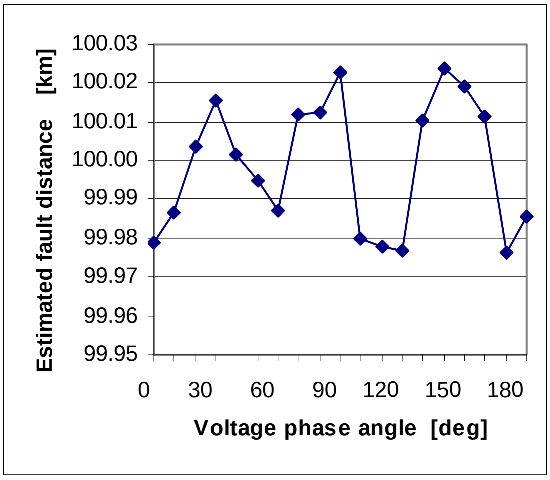

Fig. 6 Estimated fault distance as a function of the voltage phase angle φ_{v1} (true distance 100 km)

Fig. 7 shows the error in the estimated fault distance as a function of the voltage phase angle φ_{v1}.

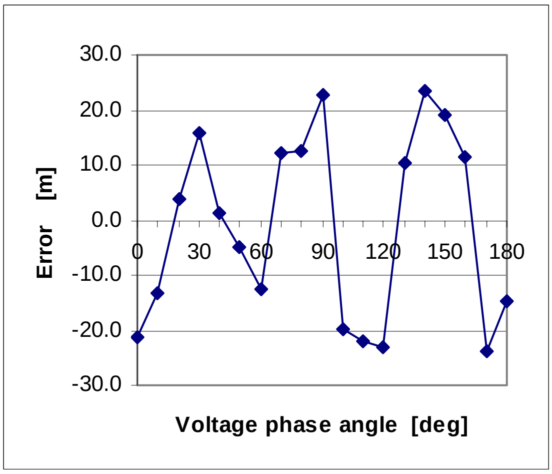

Fig. 7 Percent error in the estimated fault distance as a function of the voltage phase angle φ_{v1}

As observed from Fig. 6 and Fig. 7 the accuracy of the estimated fault distance is within ± 0.025 % for any value of the voltage phase angle φ_{v1}. The uncertainty interval ± Δd is ± 25 m and doesn't depend, in our analysis in which we neglected both the influence of the line parallel parameters both the current transformers, on the fault distance itself.

4. CONCLUSION

The paper has described an algorithm for accurately locating three phase faults on high voltage transmission lines.

The digital filtering technique, based on Walsh functions, extracts the phasors of the fundamental components of voltage and current and calculates precisely the faulted line reactance and then the location of the fault.

This measurement method is fast and accurate and doesn't require expensive instrumentation; moreover it is particularly well-suited to power-frequency waveform measurements, because the synchronization of the Walsh waves with the input voltage signal, suitably conditioned, is not difficult.

The sources of errors due to voltage and current transformers have not been included in the analysis; as well, it has not been taken into account the effect of the output impedances parameters of power transformers and the parallel parameters of the line in the current calculation.

The method can be used for any type of fault on a three phase transmission line by appropriately altering the sequence networks' interconnection.

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