

ON THE “PARADOX OF FOUNDATION” FOR MEASUREMENT

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Abstract – Given the common assumption that measurement plays an important role in the foundation of science, the paper analyzes the possibility that measurement science, and therefore measurement itself, can be properly founded. The conclusion, that such a position unavoidably leads to paradoxical situations, opens the discussion for a new epistemology of measurement, whose characteristics and interpretation are still to be investigated.

Keywords: measurement science; measurement theory, uncertainty in measurement

1. INTRODUCTION

The traditional image of science is related to the metaphor of a *building*, progressively erected with the contribution of the organized collection of the results obtained by researchers. Such a metaphor is not conceptually neutral: rather, its choice reveals the assumption of the hypothesis that the «building of science» is inherently *provided with foundations*.

The term «epistemology» itself, usually deemed as synonymous of philosophy of knowledge, and more specifically of philosophy of science, etymologically originates from the same metaphor of *standing* (the Latin *sistere*, the Greek *histanai*) on (the Greek *epi*).

The epistemological and rhetoric strategy of axiomatization, as originated from the Euclidean geometry, is paradigmatic: the building blocks of a theory, i.e., its theorems, are inferentially obtained from the chosen axioms, which thus play the role of foundational elements for the theory itself.

This foundational function is characterized in terms of not only consistency (foundations must allow to erect the correct building) and completeness (foundations must allow to erect the whole building), but also minimality (foundations must include only what is required to erect the building), therefore recognizing the importance of *clearly distinguishing foundations from development*.

The heritage of formal sciences deeply influenced also *empirical* sciences: on one hand, the belief arose that the axiomatization of the body of knowledge related to a scientific discipline is to be considered the ultimate step for the development of that discipline (as in the case of Newtonian mechanics, axiomatized by Lagrange and Hamilton), thus providing a strong argument to support the conception of physics as the methodological paradigm for all the sciences; on the

other hand, and more generally, the quest for foundations kept to pervade the philosophical instances accompanying the development of empirical sciences. Again, the terminological habits are revealing, as the idiomatic expressions «foundations of *x*», «fundamentals of *x*», «basics of *x*», ... are usually taken as synonyms of discourses on primitive concepts, from which it should be possible to derive all the (theoretical and practical) applications of *x*.

The (usually implicit) hypothesis that such «fundamentals» are definitive truths on which any further scientific development can be cumulated has been traditionally integral part of the physical sciences, in particular after the extraordinary results obtained by the Newtonian mechanics during the XVIII and XIX centuries. As late as 1899 Michelson expressed this position by stating that «the more important fundamental laws and facts of physical science have all been discovered, and these are so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote (...) Our future discoveries must be looked for in the sixth place of decimals» (to consider how firmly such “fundamental laws and facts” were established, remember that in the first years of the XX century the radically new ideas of energy quantization and time relativity were proposed...).

As a consequence of this faith in the cumulative progress of science, it is not amazing that an important part of philosophy of science has been devoted to the search of the «fundamentals» and the discussion on their ability to play the role of «good foundations» for empirical sciences.

In particular, the Hume’s problem on the justification of inductive reasoning can be interpreted as an investigation of the relation between «facts» and «laws» (as expressed by singular and universal assertions respectively) in terms of the ability of the former to *constitute a basis* for the latter.

This standpoint pervaded so many philosophical conceptions, as the traditional distinctions of primary vs. secondary qualities and observational vs. theoretical terms witness, that led the neo-positivistic school to take «the sensation» as the only basis for any scientific construction and even refusing «scientific meaning» to any form of knowledge not «founded» in this way.

2. FOUNDATIONS OF MEASUREMENT?

In its deemed role of objective means for observation and evaluation, measurement received a peculiar function of *protocol of truth* to give foundation to the scientific knowledge (consider the importance traditionally recognized to the so-called «crucial experiments», whose interpretation has been usually considered depending on the precision of the available measurement results, as the previously quoted concept of discoveries to be looked for «in the sixth place of decimals» assumes): to reach truth is the aim of knowledge and measurement is the operative means to get true data.

Given such hypotheses, it is not surprising that measurement science itself adopted this standpoint, as witnessed by the traditionally clear-cut distinction between direct vs. indirect (or derived) measurements, in which a measurement method is considered «direct» precisely because its (assumed) ability to get a direct interface to the physical world and thus acquire «raw sense data». It is known that the very concept of «true value» for a quantity and the derived one of «error», as formalized in the Gauss' theory of errors, have been assumed fundamental and however unavoidable in the interpretation of measurement results, although «the true value of a quantity is an ideal concept and, in general, cannot be known exactly» [1].

As all empirical sciences were asking measurement to play the foundational role of «protocol of truth» and measurement science accepted this function of delegate to deal with «pure data», measurement itself was forced to the paradoxical position of being at the same time the «most empirically objective» (because of its institutional tasks) and the «most metaphysically based» (because of its conceptual foundation on the hypothesis of the existence of true values) operation.

Let us call this twofold position the “*Paradox of Foundation*” (PoF) for measurement.

As a matter of fact, it must be recognized that both the technology of measurement and many areas related to the science of measurement (e.g., signal theory) have not specifically suffered from the existence of the PoF, mainly because they do not require to be explicitly supported by foundational topics for their development (as the terminological frequent, and at the same time operationally immaterial, reference to «true values» witnesses: it is known that the assumption of existence of a true value is not required to justify the adoption of statistical techniques).

A strategy to overcome the conceptual difficulties implied in the PoF was pursued by developing the idea, proposed in the mid of the XIX century by Helmholtz, to characterize measurement exclusively in terms of its *formal* properties: quantities were classified according to the scale transformations under which they are invariant, a work that led to the Stevens' theory of scale types [2]. Once embedded in an algebraic framework formalizing measurements as

(homo)morphic evaluations, the so-called «representational point of view to the theory of measurement» emerged, as systematized in the monumental «Foundations of Measurement», by Krantz, Luce, Suppes, and Tversky [3].

3. THE ROLE OF THE REPRESENTATIONAL POINT OF VIEW

Peculiar of the representational approach is the hypothesis that foundations of measurement should be looked for in structure, and therefore in informational, instead of empirical, properties, as if «*foundations of measurement*», «*science of measurement*» and «*theory of measurement*» were synonyms. It is not amazing therefore that, as the epigone of this position, an «abstract measurement theory» has been formulated [4], thus removing any empirical connotation to the concept of measurement.

It should be pointed out that the representational point of view does not solve the problems related to the metaphysical stance of the «true value and error» paradigm, simply because it ignores them, as the absence of references to things such as standards, calibration, traceability chains, ... in the approach reveals.

Instead of solving the PoF, the representational point of view has contributed to make it even more critical: in order to comply with the requirement of being the «most empirically objective» operation, measurement has been, and is currently, constrained to decide on two opposite directions: either to maintain the metaphysical foundation implied in the concept of true value, or to accept an exclusively formal characterization, as if measurement were a synonym of (homo)morphic evaluation.

As a consequence, it is usual to find textbooks on measurement science and technology trying «to escape» the PoF with the schizophrenic approach of introducing some basics of measurement (in particular the concept of scale and the classification of scale types) in representational terms and then presenting the applications (e.g., metrological characterization of sensors, calibration and traceability chain, digitalization and digital devices, ...) according to the «true value and error» paradigm.

While social sciences typically adopt the formal condition of (homo)morphism (and the related results, as the concepts of meaningfulness in scale transformations and admissible statistics) as a criterion to validate candidate evaluations, the usefulness of such a condition for empirical sciences is limited because:

* measuring systems typically implement the homomorphism in their physical structure and behavior, so that in their usage, i.e., during measurement, the condition is in principle autonomously, and automatically, fulfilled; the check of homomorphism is fundamental in scale construction, an operation that is rarely part of the work of people involved in measurement in the context of empirical sciences;

* according to the recommendations of the ISO Guide to the expression of uncertainty in measurement [5], any measurement result must be specified as an estimation of both the measurand value and its uncertainty, the evaluation of the latter being a task that must keep into account personal experience, beliefs, and sometimes even ethics, all components that can be hardly formalized in terms of morphic mappings.

4. FUNDAMENTAL PARADOXES OF MEASUREMENT

We believe that several evidences related to the practice of measurement can be taken as suggestions for a *radically different standpoint* to escape the difficulties that the PoF arises, so to assume an epistemologically consistent position with respect to the problem of foundations of measurement. Let us mention a few, possibly the most important, of such difficulties [6, 7].

1. Since measurement results depend on the adopted standard, as formalized by the reference to a measurement unit / scale, critical for measurement is the relation that operatively links measuring instruments to standards through the so-called traceability chain (substantially a sequence of calibrations). This could lead to conclude that standards themselves are the «realizations of the true value» for their quantity and then play the role of actual foundations for measurement. On the other hand, even neglecting the uncertainties the any traceability chain unavoidably introduces, standards must be indeed «realized» by primary laboratories, which are in charge of maintaining high quality national standards and typically accomplish this task by means inter-laboratory comparisons. Therefore this claimed «path towards foundations» cannot but include a component of conventionality.

2. The expression of measurement results generally require the indication of a measurand value and an estimation of its uncertainty, both of them depending for their evaluation on the previous measurement of a given set of recognized influence quantities. Being each of these influence quantities a new measurand, the mentioned dependence should be in principle iteratively applied, with the consequence that a «well founded measurement» would be impossible. The fact that this iterative process is usually stopped at its first step by assuming that the quantities influencing the measurand are not in themselves influenced by other quantities (thus on the hypothesis that they can be «directly measured»), leads to recognize any measurement as an approximation of a more inclusive operation.

3. While the adequacy of empirical models is controlled by means of measurements, the quality of measurement results depends on the validity of the models used for designing measurement instrumenta-

tion and defining measurands. On the other hand, models are valid only within a given scope: the definition of a quantity is always subject to a threshold effect, as the very concept of intrinsic uncertainty makes clear.

The latter issue, the definition of quantities that are object of measurement, is critical.

Given the usual assumptions that measurement is a foundational tool for empirical sciences, and measurands must be (or, at least: usually are) identified before their measurement is performed, the following consequences can be drawn:

A. science is founded on measurement,
and:

B. measurement is founded on the definition of measurands,
and:

C. the definition of a measurand is typically founded on scientific models,
clearly a circular argument.

5. (NON) CONCLUSION

In a paper published a few years ago [8], I analyzed the issues related to the definition of measurands, in particular by highlighting the merits and flaws of the two possible strategies for such a definition: an intensional one and an extensional one, with the related specifications and extensions in operational terms. The quest was explicitly a foundational one.

Since then, I have radically changed my position with respect to the “foundational role of measurement”, and I am now strongly supporting the ideas, proposed for example by Edgar Morin [9], Gregory Bateson [10], and Francisco Varela [11], according to which no absolute foundation is possible for science, because human knowledge is essentially based on a continuously iterative, try-and-revise, adaptive, auto-poietic process, not so different from the way children learn. In this view, knowledge *is a network*, not a “building”, as instead deemed in the classical interpretations. In this view, it is not amazing that at least some measurands cannot be ultimately “defined” and that, strictly speaking, their usage always requires the reference to a whole network of related knowledge.

This standpoint is so new that its consequences are still largely unexplored and, we hope, will be subject of future research in the field of (a foundationless) measurement science.

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