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UNCERTAINTY EVALUATION IN DITHERED A/D CONVERTERS

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Abstract – In this paper formulae for evaluating the uncertainty of measurements (direct and indirect) performed by means of an ADC-based device are presented. The complex technique of dithering is used to reduce the effect of noise and to increase the ADC resolution. The authors discuss a way of writing and interpreting the uncertainty specifications of ADCs.

Keywords uncertainty, measurement error, A/D converters

1. INTRODUCTION

The considerable progress obtained in the field of both electronic technology and digital signal processing is modifying the basic architecture of the measurement devices and the idea itself of measuring. Nowadays it is adequate to conceive an instrument as a data-processing system which acquires the physical variable and yields information necessary to determine the desired figures. Besides, it is very common to build very complex automated measurement systems by interfacing comparatively cheap digital instruments, analog-to-digital conversion boards and personal computers.

A primary problem in this kind of measurement systems is evaluating uncertainty of the obtained measurements and particularly in the interpretation and use of the ADC metrological specifications. In effect, researchers and engineers have been having at their disposal some international Standards about ADC errors (e.g. [1], [2]) and an ISO Guide [3] about uncertainty evaluation. Nonetheless, the authors are not sure that everybody will calculate the measurement uncertainty of a given ADC in the same way. First of all, the cited regulating documents do not address explicitly the issue (ADC standards do not go into uncertainty evaluation; ISO Guide ignores even the most common figures of merit of ADCs). It must be said, too, that manufacturers give ADC specifications in many different (and often ambiguous) forms, so that it is virtually impossible to give a clear path to uncertainty evaluation for all the commercially available digital instruments.

In this paper some very general ADC-related problems of uncertainty evaluation are clearly stated and the relevant solutions are given. Many of the given formulae do not involve striking novelties in uncertainty or ADC characterization theory; nevertheless, they are not obvious and cannot be found in any Guide, Standard or textbook known to the authors. Besides, the formulae for the case of ADC *with dither* are presented [4], [5]; particularly, is shown that a noise intentionally added, reduces the quantization noise via averaging techniques [6], [7], [8]. Generally speaking, the provided formulae (and maybe also the underlying conceptual scheme) could be considered for inclusion in future editions of the mentioned Standards.

2. ADC ERROR MODEL

A real-world ADC differs quite a lot from the ideal ADC, which is described by a perfectly linear quantization characteristic, $quant(x)$. The error model considered in this paper (Fig. 1) is pretty simple:

$$e = y - x = nlquant(x+n) - x \tag{1}$$

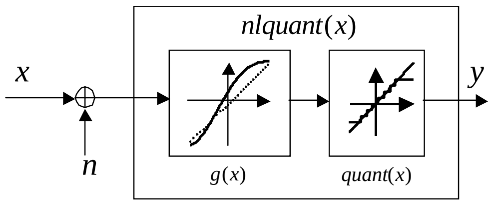

Fig. 1 – ADC error model

where n is a noise voltage added to the analog voltage input x and the function $nlquant(x)$ is the well-known nonlinear quantization staircase, which can be seen as a $quant(x)$ cascaded with a continuous nonlinear function $g(x)$, as illustrated in Fig.2.

The $g(x)$ function can be furtherly decomposed in the sum [9]

$$g(x) = Gx + O + inl(x) = x + \Delta Gx + O + inl(x) \tag{2}$$

being ΔG the *(relative) gain error*, O the *offset error*, $inl(x)$ the *integral nonlinearity error*. In this

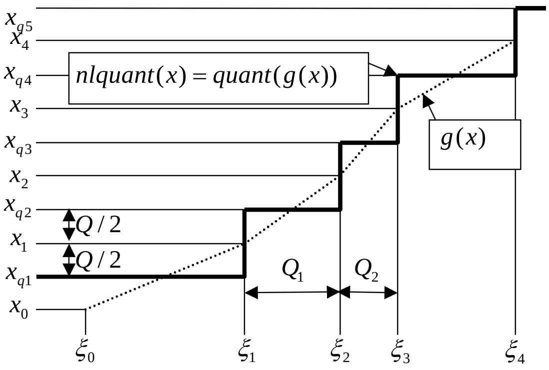

Fig.2. – The nonlinear quantization staircase $nlquant(x)$, as a cascade of the functions $g(x)$ and $quant(x)$.

case the (1) can be written as:

$$e = y - x = \Delta G\left(x+n\right) + O + inl(x+n) + eq(g(x+n)) + n \quad (3)$$

The previous error model is very simple and can be used when the frequency signal of the input is low enough, that's for input signal with *slow temporal variations* with respect to the aperture time and the bandwidth of the ADC.

When the input frequency of signal is above a certain limit value the nonlinear dynamic effects must be considered. In this case the error model must take into account the following important phenomena:

- linear dynamic transformations of the input signal (like those due to the finite bandwidth of the instrument);
- nonlinear dynamic transformations of the input signal (which cause a typical THD increase at higher input frequencies);
- timebase errors (both systematic, like a fixed error in the sampling frequency, and random, like the aperture uncertainty or the jitter in the sample clock signal).

3. MEASUREMENT ERROR AND UNCERTAINTY SPECIFICATIONS

To evaluate errors in a real ADC it is necessary to know the parameters *Q, n,* ΔG, *O*, and $inl(x)$ in (3). Unfortunately, only the quantization step Q is exactly known whereas all the others are effectively unknown. The manufacturer cannot know and provide these values, but can assure something about their *distributions*.

The "distribution of ΔG" is, in the following, the one obtained by considering *the population of all the ADCs with the same metrological specifications* (think, for example, of all the ADCs of the same kind from the same manufacturer, correctly maintained). This definition is a more operational and "engineer-like" way and different to the notion employed by ISO Guide that uses the definition of "subjective probability".

We define in the same way the distribution of *O* and $inl(x)$. As regards the integral nonlinearity, we formulate the quite reasonable additional assumption that the distribution (for all the set of ADCs defined above) does not depend on *x*. Finally, as regards the noise *n*, we can employ the usual notion of distribution of a stationary and ergodic process. As a final needed hypothesis, we will suppose all the distributions to be zero-mean and *independent each other* (this is quite reasonable considering the nature of the considered error parameters).

By considering the *worst-case uncertainties* U_G, U_O, U_{inl}, defined by:

$$U_G = \max\left|\Delta G\right|,\ U_O = \max\left|O\right|,\ U_{inl} = \max\left|inl\left(x\right)\right|, \quad (4)$$

and assuming the distributions of ΔG, *O*, and $inl(x)$ uniform in $\pm U_G$, $\pm U_O$, $\pm U_{inl}$, the *standard uncertainties* given by

$$\sigma_G = \sqrt{E[\Delta G^2]},\ \ \sigma_O = \sqrt{E[O^2]},\ \ \sigma_{inl} = \sqrt{E[inl(x)^2]},$$
$$\sigma_n = \sqrt{E[n^2]}. \quad (5)$$

becomes

$$\sigma_G = U_G/\sqrt{3},\ \sigma_O = U_O/\sqrt{3},\ \sigma_{inl} = U_{inl}/\sqrt{3} \quad (6)$$

The uniform distribution is the only sensible option when no information about the distribution is available even if engineering practice suggest that the distribution of these parameters will be similar to a truncated Gaussian (where the truncation is due to the discarding of clearly out-of-specifications or non-working ADCs). Finally, the worst-case and the standard uncertainty relevant to the ideal quantization are given by the following expressions:

$$U_q = \frac{Q}{2};\quad \sigma_q = \frac{Q}{\sqrt{12}} \quad (7)$$

whereas the worst uncertainty of the *n* is $U_n = \infty$ if the noise is assumed to be Gaussian .

4. UNCERTAINTY EVALUATION

Consider a particular measurement result *y* obtained by an ADC and the distribution of the measurement error $e = y - x$, evaluated for *all the possible measurements yielding the result y*. This situation means to take into account the same information usually available for an ADC user which uses the specifications of the device without having any prior information about the measurand.

Afterwards, we consider the following kinds of measurement:

A) Measurements with no or negligible noise

1) single or "direct" measurement y
2) combined or "indirect" measurement $y = g(y_1,...,y_n)$

B) Measurements with noise, but without averaging

1) single or "direct" measurement y
2) combined or "indirect" measurement $y = g(y_1,...,y_n)$

C) Measurements with noise and with averaging

1) single or "direct" measurement y
2) combined or "indirect" measurement $y = g(y_1,...,y_n)$

For all the measurements listed above we give formulae for the following quantities:

$$US(y) = \sqrt{E[e^2]} \quad (8)$$

$$U(y) = \max(|e|) \quad \text{(when applicable)} \quad (9)$$

naming the first "standard uncertainty" (according to the familiar terminology of the ISO Guide) and the second "worst case uncertainty" (this is simply an expanded uncertainty with confidence level =1). Of course the latter is applicable only to case with no or negligible noise (*case A*), whereas in the other cases the worst uncertainty is $U(y) = \infty$ (*case B* and *C*).

In order to provide formulae for the evaluation of uncertainty, we adopt an operational viewpoint, in the same spirit of the uncertainty parameters definitions of section 3. We will use for the evaluation the usual "sensitivity coefficients"

$$k_i = \frac{\partial g(y_1,...y_n)}{\partial y_i} \quad (10)$$

Here are the formulae for the cases *A* and *B*.

A1)

$$US(y) \cong \sqrt{\sigma_G^2 y^2 + \sigma_O^2 + \sigma_{inl}^2 + \sigma_q^2} \quad (11)$$

$$U(y) \cong U_G\,|y| + U_O + U_{inl} + U_q \quad (12)$$

A2)

$$US(y) \cong \sqrt{\sigma_G^2\left(\sum_{i=1}^n k_i y_i\right)^2 + \sigma_O^2\left(\sum_{i=1}^n k_i\right)^2 + \left(\sigma_{inl}^2 + \sigma_q^2\right)\sum_{i=1}^n k_i^2} \quad (13)$$

$$U(y) \cong U_G\left|\sum_{i=1}^n k_i y_i\right| + U_O\left|\sum_{i=1}^n k_i\right| + \left(U_{inl} + U_q\right)\sum_{i=1}^n |k_i| \quad (14)$$

B1)

$$US(y) \cong \sqrt{\sigma_G^2 y^2 + \sigma_O^2 + \sigma_{inl}^2 + \sigma_q^2 + \sigma_n^2} \quad (15)$$

B2)

$$US(y) \cong \sqrt{\sigma_G^2\left(\sum_{i=1}^n k_i y_i\right)^2 + \sigma_O^2\left(\sum_{i=1}^n k_i\right)^2 + \left(\sigma_{inl}^2 + \sigma_q^2 + \sigma_n^2\right)\sum_{i=1}^n k_i^2} \quad (16)$$

As regards these equations, it can be highlighted that the correlation between errors (in the case of combined uncertainties) is accounted for in a very simple and elegant way, without explicit evaluation of correlation coefficients.

It is very easy to verify, for example, that a measurement of the kind $y = y_2 - y_1$ is exempt from offset errors, one of the kind $y = y_2 / y_1$ is exempt from gain errors, and one of the kind $y = (y_4 - y_3)/(y_2 - y_1)$ from both.

Case C is a quite different matter because dithering transforms part of the systematic error (quantization) in random error (additional noise) (Figs. 3 and 4).

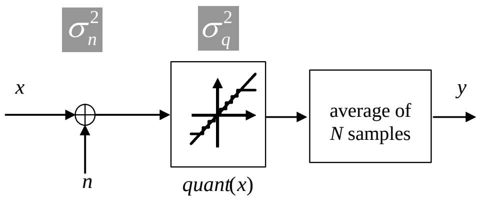

Fig. 3 – Scheme of measurement with averaging in presence of noise (dither)

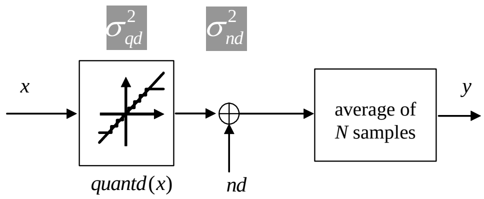

Fig.4 – Equivalent model of the measurement in Fig. 3.

The measurement uncertainties for these two cases are given by the following approximate formulae, that take into account both the noise reduction and the resolution increase:

C1)

$$US(y) \cong \sqrt{\sigma_G^2 y^2 + \sigma_O^2 + \sigma_{inl}^2 + \sigma_{qd}^2 + \frac{\sigma_n^2 + \sigma_q^2 - \sigma_{qd}^2}{m^2}} \quad (17)$$

C2)

$$US(y) \cong \sqrt{\begin{array}{l}\sigma_G^2\left(\sum_{i=1}^{n} k_i y_i\right)^2 + \sigma_O^2\left(\sum_{i=1}^{n} k_i\right)^2 + \\ +\left(\sigma_{inl}^2 + \sigma_{qd}^2\right)\sum_{i=1}^{n} k_i^2 + \left(\sigma_n^2 + \sigma_q^2 - \sigma_{qd}^2\right)\sum_{i=1}^{n}\frac{k_i^2}{m_i^2}\end{array}} \quad (18)$$

where the symbol σ_{qd}^2 denotes the residual *deterministic part of the quantization error*, which is not affected by the averages
It can be shown that this term depends only upon the ratio σ_n / Q with

$$\sigma_{qd}^2 < \sigma_q^2 \text{ , } \sigma_{nd}^2 > \sigma_n^2 \quad (19)$$

Therefore, for any given σ_n it is easy to evaluate numerically the convolution $quantd(x) = quant * f(-x)$ and then the mean squared error σ_{qd}^2 (Fig. 5).

The curve shows that the deterministic quantization error is (obviously) $\sigma_{qd} = 1/\sqrt{12} = 0.2886$ LSB at $\sigma_n = 0$, and that it is (less obviously) practically nullified for $\sigma_n \geq 0.5$ LSB. This is the usual choice of dither level in DAQ boards, and the curve shows that this is indeed an optimal choice, which removes the quantization error by adding only the strictly necessary noise.

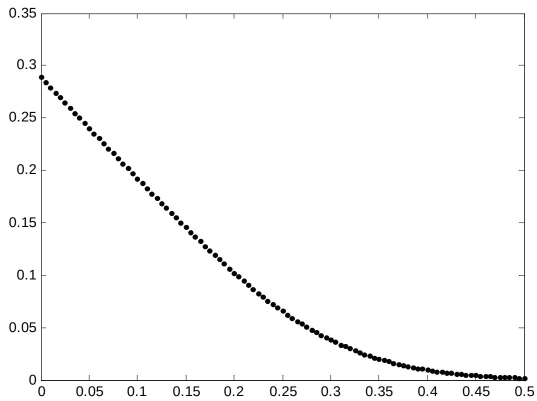

Fig. 5 – Plot of the residual deterministic quantization error σ_{qd} as a function of the input noise σ_n (both are in LSB units).

5. CONCLUSIONS

The paper examines a general way of writing ADC specifications and consequently presents formulae for evaluating the uncertainty of measurements performed by means of ADC affected by gain, offset and nonlinearity error. The suggested method allows to deal very easily with the correlation between measurement due to gain and offset error and with the feared and little-understood topic of dither. Particularly. three cases are distinguished and analyzed: no noise, noise and no averaging, noise and averaging (dither).

For the first two cases the uncertainty derivation is very simple and the result does not consist so much in the formulae themselves, but in the adopted scheme of work. The aim of the authors is to define in a clear and “operational” way the uncertainty specifications, using well-known concepts of the ADC technology avoiding as much as possible the use of subjective figures or assumptions.

The third analyzed case is a bit more difficult. The derived equations, and especially the concept of *residual deterministic part of the quantization error* can be considered, in authors’ view, an actual innovation. Another important result is that the uncertainty evaluation become straightforward, unambiguous and “tigh” since it often happens that the effect of fain and offset, and also of quantization when averaging, is overestimated.

Finally, a way of writing and interpreting the uncertainty specification of ADCs – at least as regards static errors and noise – is suggested.

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