

Estimating the sampling strategy for the ISO system of limits and fits in coordinate metrology

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Abstract – This paper examines the possibility to set up standardized sampling strategies for circular features, which are tolerated according to the international system of limits and fits. The measuring process is simulated by a mathematical model and the sampling strategy derived for various measurement conditions. One important aim of the proposed model is to control the measurement uncertainty of the measurement process within certain limits of the feature tolerance. To do so the measurement uncertainty is estimated according to international standards. The model of the estimation process considers as uncertainty contributors the coordinate measuring machine, environmental conditions and workpiece deviations. Simulations have shown that it is possible to derive standardized sampling strategies for circular features tolerated according to the ISO system of limits and fits if measurement process parameters are classified.

Keywords: CMM, sampling strategy, ISO system of limits and fits

1. INTRODUCTION

The set up of a measuring strategy for coordinate measuring machines (CMM) is a highly complex task. It includes besides others the selection of a sampling strategy, that is the number and distribution of touch points. Today the operator of a CMM is required to make these decisions. Some guidance, for example in the form of minimum number of points, is given by the standard BS 7172:1989 [1] but is not adjusted to a specific measuring task. As it is shown in several papers [2-5] varying sampling strategies lead to different measuring results. Therefore it is of utmost importance that equal measuring tasks are sampled by the same sampling strategy to make measuring results comparable.

In this paper the *Procedure for Uncertainty Management* (PUMA) is applied to coordinate metrology to estimate what is called ‘task specific measurement uncertainty’. The PUMA method is part of the ‘Guide to the estimation of uncertainty in GPS measurement’ (GUM-GPS) [6]. It is an iterative procedure which can be used for the design and development of measurement processes. In the model proposed in this paper the dependency of the measuring uncertainty from the number of touch-points taken by the CMM is used within the PUMA method to estimate an appropriate sampling strategy. The other parameters of the meas-

urement process which influence the measuring uncertainty are considered for the computation of the uncertainty of measurement, but are constant for the iteration process.

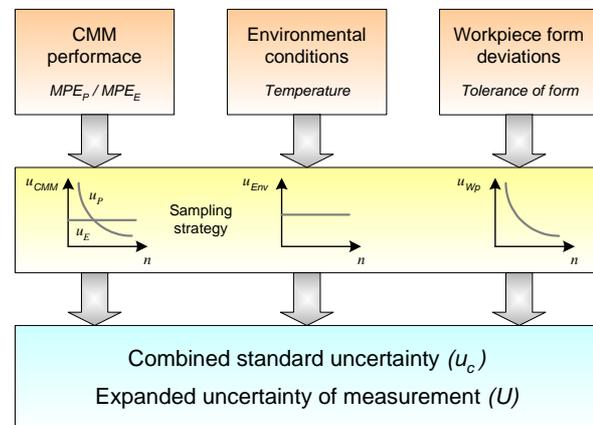


Fig. 1. Depiction of the estimating procedure for the measurement uncertainty

The proposed model works in the following way (Fig. 1). First the measurement process is defined for the following components: CMM performance, environmental conditions and workpiece form deviations. The uncertainties of each individual component, u_{xx} , and its dependency from the number of touch-points, n , are specified in the mathematical model of the CMM measuring process. Based on this information the estimation algorithm computes, for a certain number of touch points, a task specific combined standard uncertainty (u_c) and an expanded uncertainty (U). If the expanded uncertainty is not equal to the target uncertainty (U_T) the next iteration starts. This procedure continues as long as $U \neq U_T$. Finally, if $U = U_T$ the iteration process stops and the expanded uncertainty of measurement and the number of equidistant touch-points are stated. Equidistant sampling is suggested because it has proven to be the most appropriate sampling strategy (e.g. delivered the lowest degree of uncertainty for circular features) and has proven to be most practical for normal users [2]. The grade of the measurement strived for (U_T) can be determined from the feature tolerance. This can be done either according to the ‘Golden rule of measurement’ which

says that the measurement uncertainty should be between 10% and 20% of the feature tolerance zone, or more elaborated according to the *Decision rules for providing conformance or non-conformance with specifications* [7].

2. THE MATHEMATICAL MODEL

The mathematical model follows the rules for uncertainty budgeting according to the 'Guide to the estimation of uncertainty in GPS measurement' (GUM-GPS) [6]. The aim of this technical specification is to provide secure upper bound uncertainty contributors. As it will be shown some of these uncertainty contributors depend on the number of touch-points.

The first step in estimating the measurement uncertainty for a certain measuring task is to set up a mathematical function of the measuring process (1). In this function the input quantities (X_{xx}) are either intermediate measuring results (e.g. size, temperature) or technical coefficients (e.g. temperature expansion coefficients). The output quantity (Y) is the final measuring result.

$$Y = f(X_1, X_2, \dots, X_{xx}) \quad (1)$$

In a second step the uncertainty budget of each individual uncertainty component (u_{xx}) has to be computed. This can be done according to (2-4). In this equations a_{xx} is the variation limit in micrometers and b_{xx} a dimensionless value which represents the distribution function of the necessary variation limit. c_{xx} is the sensitivity coefficient of the input quantity and is computed by the partial derivative ($\partial Y / \partial X_{xx}$). a_{xx}^* is the variation limit in influencing units of the input quantity (X_{xx}).

$$u_{xx} = a_{xx} * b_{xx} \quad (2)$$

$$a_{xx} = c_{xx} * a_{xx}^* \quad (3)$$

$$c_{xx} = \left| \frac{\partial Y}{\partial X_{xx}} \right| \quad (4)$$

Finally the combined standard uncertainty, u_c , and the expanded uncertainty, U, have to be computed. When quantities are added, of which each contains an uncertainty and the uncertainty contributors are uncorrelated as it is the case here, the law of error propagation can be applied. It says when uncertainty quantities are added their sum contains an uncertainty equal to the square root of the sum of the squares of the individual uncertainty contributors, u_{xx} . Thus the combined uncertainty, u_c , can be stated as following:

$$u_c = \sqrt{\sum_1^i u_{xx}^2} \quad (5)$$

The expanded uncertainty, U, is computed by multiplying the combined uncertainty, u_c , with the coverage factor, k. According GUM-GPS the covering factor $k=2$ in GPS measurements. This represents a confidence level of 95,45 %, which means 95,45 % of the measuring results will lie within this uncertainty range.

$$U = k * u_c \quad (6)$$

In the following this mathematical model will be applied to the components of the measurement process in coordinate metrology. These are the CMM performance, environmental conditions and workpiece form deviations.

2.1. Coordinate Measuring Machine

For the computation of the measurement uncertainty of the CMM the performance standard ISO 10360-1 [8] is applied to the model. According to this two different kinds of errors exist for CMMs. These are the *maximum permissible error of indication of a CMM for size measurement*, MPE_E , and the *maximum permissible probing error*, MPE_p . Both errors are defined in micrometers, μm . The maximum permissible error for size measurement is expressed by the formula: $MPE_E = \pm (A + L/K)$. The maximum permissible probing error is expressed by just a single positive constant: $MPE_p = A$. The 'probing error' A is expressed in micrometers and supplied by the manufacturer. Within the 'size measuring error' K is a dimensionless positive constant supplied by the manufacturer and L is the measured size in millimeters.

If the least-square (LS) fitting algorithm is used the influence of random errors on the measuring result becomes smaller the more touch-points (n) are taken. This is because the characteristic values of the feature are computed using a mean average value algorithm (sum of the least error squares). Through the application of the law of error propagation the probing variation limit (a_{prb}) for a diameter measurement can be derived:

$$a_{prb \text{ Diameter}} = \frac{2}{\sqrt{n}} * a_{prb}^* = \frac{2}{\sqrt{n}} * \frac{A}{2} \quad (7)$$

The term $2/n^2$ in this equation represents the sensitivity coefficient, c_{prb} , and the term $A/2$ the variation limit in influencing units, a_{prb}^* . The maximum permissible probing error, MPE_p , is by definition without plus or minus sign. However a plus or minus sign is required for the calculation of the variation limit, a_{prb} , and therefore the probing error has to be transformed in a '±'-style. The size of the probing error remains the same by dividing it by two and adding a plus-minus-sign, consequently $MPE_p \approx \pm (A/2)$. For the determination of the distribution factor, b_{prb} , a normal distribution is applied and hence b_{prb} is set to 0,5 [6].

The geometrical errors of a CMM are, unlike the random probing errors, systematic errors regarding a certain measurement [9]. This means the sign and the value keeps the same for repeated measurements. Consequently the law of error propagation can not be applied and the size measuring variation limit (a_{Lme}) of the circle diameter is independent from the number of touch points. The length depending part of the maximum permissible size measurement error, MPE_E , is used as a linear error component in (8). The term $1/k$ represents the sensitivity coefficient, c_{Lme} , and the measuring length is the variation limit in influencing units, a_{Lme}^* . Because geometrical errors may change randomly for different measuring tasks within the working volume of the CMM a normal distribution is applied and the distribution factor, b_{Lme} , is set to 0,5.

$$a_{Lme} = \frac{1}{K} * a_{Lme}^* = \frac{1}{K} * L \quad (8)$$

2.2. Environment

Temperature deviation from the reference temperature 20°C [10] is one of the most important influencing parameters on measurement results and measurement uncertainty. This is because most materials change length with temperature. Corrections for temperatures other than 20°C have to be made for the CMM as well as for the workpiece. This can be done according to the following formula:

$$\Delta L_{Env} = L_0 [\alpha_w * (T_w - 20^\circ) - \alpha_s * (T_s - 20^\circ)] \quad (9)$$

where L_0 = nominal length / measuring result (estimate), α_w = temperature expansion coefficient of the workpiece, α_s = temperature expansion coefficient of the scale, T_w = temperature of the workpiece and T_s = temperature of the scale.

An important matter of fact in temperature compensation is that the thermal expansion coefficient may be very difficult or inconvenient to determine precisely [11]. Also the temperature measurements of the workpiece and the scales may contain errors. Both effects will lead to an overall uncertainty of the nominal temperature expansion. The sensitivity coefficients, c_{xx} , in (10-13) can be computed according (4) by inserting ΔL_{Env} for the output quantity, Y, and in succession T_s , T_w , α_s , α_w for the input quantities, X_{xx} .

$$a_{T_s} = L_0 * \alpha_s * a_{T_s}^* \quad (10)$$

$$a_{T_w} = L_0 * \alpha_w * a_{T_w}^* \quad (11)$$

$$a_{\alpha_s} = L_0 * (T_s - 20) * a_{\alpha_s}^* \quad (12)$$

$$a_{\alpha_w} = L_0 * (T_w - 20) * a_{\alpha_w}^* \quad (13)$$

The scale and workpiece temperature distribution factors (b_{T_s} , b_{T_w}) are set to 0,7. This insinuates an U-distribution which is the most conservative assumption according GUM-GPS. For the thermal expansion coefficients a rectangular distribution is often selected, if there is no additional knowledge of where the actual

value might lie [12]. Therefore the distribution factors (b_{α_s} , b_{α_w}) are set to 0,6.

2.3. Workpiece

The component defined in an engineering specification is an ideal representation of a real workpiece. However, the real workpiece may have numerous deviations because of production spread (for example bores may not be perfectly round). The CMM probes this imperfect workpiece surface point by point. This corresponds to the extraction of a sample from the theoretically infinite number of surface points. Because more touch-points represent the real surface of the workpiece more complete the sample will be the better the more touch points are taken.

The coordinate values of the real surface (parent population) may have a random or a systematic distribution. In case of random errors the real surface does not have characteristic form deviations. Therefore the computed measurement result is normal distributed if a LS fitting algorithm is applied. Systematic errors may for example occur in turning or centerless grinding, where lobbing effect is typical. Because the LS fitting algorithm delivers a substitute feature which is an arithmetic mean value of all collected measuring points the Central Limit Theorem can be applied. The Central Limit Theorem states that as the sample size increases, the sampling distribution of sample means approaches a normal distribution, whatever may be the probability distribution of the parent population [13]. This means the measuring result will be normal distributed if the sample size (number of touch-points) is large enough. In experiments on 2D circles with superimposed random, systematic and strong systematic errors (lobbing) YAU [5] showed that a sample size of ten proved to be sufficient. In statistical theory however a sample size of $n \geq 30$ is often recommended to approximate reasonably well a normal distribution.

Because the LS fitting algorithm is used and the Central Limit Theorem can be applied, the variation limit caused by form deviations of the workpiece, a_{Wp} , can be computed analogous to the probing variation limit, a_{Prb} .

$$a_{Wp_{Diameter}} = \frac{2}{\sqrt{n}} * a_{Wp}^* \quad (14)$$

The variation limit in influencing units, a_{Wp}^* , is the actual form deviation of the workpiece. If the actual form deviation is not known the form tolerance of the feature may be applied as a worst case scenario. In this case the variation limit, a_{Wp}^* , is 10 microns for a circle with a roundness tolerance of 0,01 mm. If there are good reasons, for example data from statistical process control, one may reduce the variation limit to the 3-sigma or 2-sigma confidence interval.

Because of the Central Limit Theorem the distribution factor, b_{Wp} , is set to 0,5 for a normal distributed variation limit, a_{Wp} .

3. APPLICATION TO THE ISO SYSTEM OF LIMITS AND FITS

Holes and shafts are one of the most frequent and important measuring tasks in industrial production. For the design of mating cylindrical features (holes and shafts) the Standards 286-1 and 286-2 were issued by the International Standards Organization (ISO). In these the size of the tolerance is related to the tolerance grade and the nominal size of a feature. This means the bigger the nominal size of a feature is, the bigger is the related tolerance interval. This is meaningful, because usually deviations or errors arising at processing and inspection are the bigger, the bigger the processed size is. Nominal size intervals are defined between 0mm and 3150mm. In order to secure the same level of accuracy for different nominal sizes additionally the tolerance grade (general designation - IT) is introduced in ISO 286-1 [14]. There are 20 different groups (levels of accuracy), which are characterized by the standard tolerance grades. The tolerance grade is specified by numbers 01, 0, 1, 2, ... 18 in order of decreasing accuracy, and therefore increasing tolerance intervals. The designation of an ISO standard tolerance are for example, depending on the tolerance grade, IT01, IT3, and IT12 etc.

In a first experiment the algorithm is applied to the system of limits and fits for nominal sizes from 0mm to 500mm and tolerance grades from IT1 – IT16. For the computation of the number of touch-points the mean values of the specified nominal size intervals are used (e.g. interval 50mm - 80mm \Rightarrow mean value equals 65mm). The values shown in Fig. 2 are based on the following assumptions about the measuring process: The temperature variation limits, a_{Ts}^* and a_{Tw}^* , are 0,2°C which is typical for conditions in which thermal equilibrium is reached for the workpiece as well as for the CMM [15]. The variation limits of the thermal expansion coefficients are $a_{aw}^* = 2 \mu\text{m}/\text{m}^\circ\text{C}$ for the workpiece (steel) and $a_{as}^* = 1,2 \mu\text{m}/\text{m}^\circ\text{C}$ for the steel scales. The CMM is of the ‘production standard level’ with $A=2 \mu\text{m}$ and $K=250$. Scale and workpiece temperature is 20°C. The workpiece circularity is set to 0,02 mm. The target measuring uncertainty is 10% of the tolerance interval according to the ‘Golden rule of measurement’.

Fig. 2 shows that the number of touch points rises exponentially with decreasing tolerance grade numbers (IT) and decreasing diameter sizes. This is because the tolerance interval specified in ISO 286-1 decreases with decreasing tolerance grade numbers and decreasing nominal diameter sizes. Hence the smallest tolerance interval (0,8 μm) is assigned to the tolerance grade IT1 and the nominal size interval 0-3mm. The biggest tolerance interval (4000 μm) is assigned to the tolerance grade IT16 and the nominal size interval 400-500mm.

For a CMM equipped with a touch-trigger device the measuring time increases proportional with the

number of touch-points taken. Usually touch-point numbers bigger than 90 are not time efficient to probe. Graphs like the one in Fig. 2 therefore can show the performance of a CMM for a certain measuring task and thus make CMM performance more comparable.

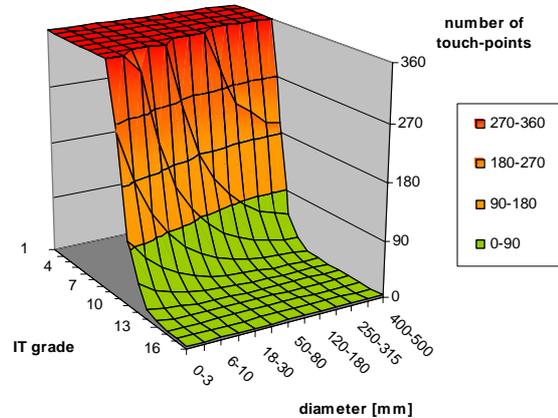


Fig. 2. Number of touch-points depending on IT grade and circle diameter (means of nominal size intervals)

In a second experiment the algorithm is applied to analyze the number of touch-points in greater detail for a smaller IT and diameter range. This is shown in Fig. 3. The tolerance grades are chosen from IT7 to IT16 and the diameter from 18,000 mm to 180 mm. Special emphasis is placed on the transition from one nominal size interval to another by increasing the diameter by just 1 micron and thereby overstepping the threshold for the next higher nominal size interval.

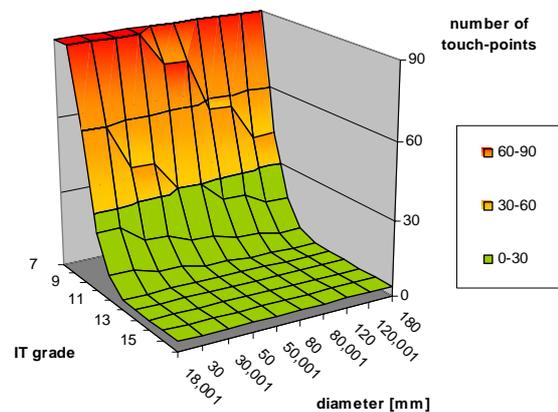


Fig. 3. Number of touch-points depending on IT grade and circle diameter with a focus on transition of nominal size intervals

Analyzing for example the graph for the tolerance grade IT10 shows the following values for the number of touch-points: 23 (18,001-30mm), 16 (30,001-50mm), 12 (50,001-80mm), 9 (80,001-120mm) and 7 (120,001-180mm). In this case (IT10) the number of touch-points changes if the diameter interval is changed. Within a certain interval the number of touch-points remains constant for the whole interval. This is also the case for other tolerance grades and

nominal sizes. For high tolerance grades and large nominal sizes the number of touch points may change in a subordinate manner. This exception arises as this area yields high touch-point numbers and therefore a greater spread can be achieved. Nevertheless the changes are within the 5% range. The effect of (quasi) constant touch-point numbers within certain nominal size intervals can be used to set up standard sampling plans for certain tolerance grades (IT). This means for a certain IT and specified measurement process conditions a fixed number of touch-points can be proposed for the individual size intervals.

The simulations clearly show how the sampling strategy for a circular feature depends on the diameter size and tolerance grade (IT). The computations were based on specific process parameters for the CMM performance, environmental conditions and workpiece deviations. These process parameters have a considerable influence on the measurement uncertainty and thereby on the sampling strategy. For the development of standardized sampling plans for circular features it is therefore necessary to specify these process parameters in meaningful groups.

CMM performance standards could be for example divided up in the following three groups: 'production standard CMMs' ($A=2 / K=250$), 'high precision CMMs' ($A=1,5 / K=350$) and 'ultra high precision CMMs' ($A=1 / K=500$). The environmental conditions may be specified for standard rooms ($T=18-22^{\circ}\text{C}$), modern production sites ($T=18-28^{\circ}\text{C}$) and conventional shop floors ($T=18-35^{\circ}\text{C}$). For the workpiece form deviations intervals could be introduced according to the ratio of the form error to the tolerance size. Roundness error levels could be for example defined for form errors of up to 20%, 50% and 80% of the tolerance zone (form error levels 1 – 3).

In Fig. 4 a sampling plan is shown for the following measuring process parameters: production standard CMM, modern production site and form error level 1. For convenient use the sampling plan could be also represented as a numerical table. In contrast to the two previous examples the number of touch points starts in general on a higher level (distance from ground).

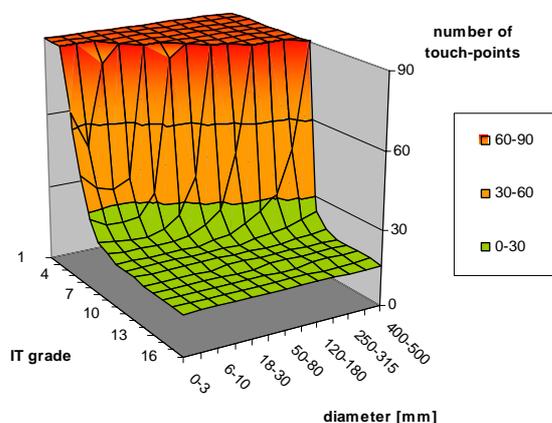


Fig. 4. Number of touch-points depending on IT grade and circle diameter for a specific measurement process

The reason for this is, that the measurement process defined in this example has greater uncertainties as the ones applied in the previous examples and therefore more touch-points are needed to compensate for these. The target measurement uncertainty for all examples was set to 10% of the tolerance interval. A further characteristic of this example is, that the number of touch-points increases with increasing circle diameters. This is because the form error of the feature was set to 20% of the tolerance zone. As the tolerance zone rises with rising diameters also the form error increases with increasing diameters. To map a surface with a higher form error (variation limit) more touch-points are necessary to meet a certain measurement uncertainty (14).

4. CONCLUSION

Based on the findings of this research it can be shown that it is possible to establish standardized sampling plans for the ISO system of limits and fits. A prerequisite doing this is to establish specification groups for the important process parameters. The computation of the number of touch-points and hence the sampling plans can be for example done with the simulation algorithm proposed in this paper.

To get realistic values for the number of touch points it is necessary to map the measuring process in sufficient detail. The components of the measurement process proposed in this paper may need to be extended, e.g. for multiple-stylus probing systems or articulating probing systems. Furthermore the set up of the individual model parameters requires extensive metrological knowledge and experience in working with CMMs.

The advantage of the presented method, compared to a manual approach, is that the estimation of the number of touch points follows sound logical rules. Through this the measuring strategy and result becomes objective and comparable. The CMM operator gets an 'easy to use' suggestion for the sampling strategy which should be preferable applied to a specific measuring task.

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