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FAILURE DIAGNOSIS USING THE CHI-SQUARE SHEWHART CHART AND THE ART NEURAL NETWORK IN NAVIGATION SYSTEMS

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Abstract – A failure diagnosis scheme using the chi-square Shewhart chart and the ART neural network is presented for navigation systems. After a failure is detected ART neural network is applied to determine when the failure happens, how serious the failure is and where the failure is located. The application of the proposed scheme to the problem of the Global Positioning System (GPS) integrity monitoring is discussed.

Keywords : fault detection, ART neural network, GPS

1. INTRODUCTION

For many safety-critical applications a major problem of the existing navigation systems is lack of integrity monitoring. Integrity monitoring requires that a navigation system detects, isolates faulty measurement sources, and removes them from the navigation solution before they noticeably corrupt the output. In many applications, a navigation system must also be robust enough to operate in more than one dynamical environment. If the dynamics change drastically, or if a sensor failure occurs, the computation algorithm must contain some mechanism to detect and rectify this situation. While catastrophic or hard failures can be easily uncovered, more subtle or soft failures can only be detected and isolated with help of more sophisticated techniques based on error estimation/decision theory [1-3].

This problem can be stated as a particular change detection problem in some convenient modeling framework. The early warning of small - and not necessarily fast - changes is of crucial interest in order to avoid the economic or even catastrophic consequences that can result from an accumulation of such small changes. For example, small faults arising in the sensors of a navigation system can result, through the underlying integration, in serious errors in the estimated position of the user.

The criterion to be used is quick detection, few false alarms and no missed detection. Fast detection is necessary because, between the fault onset time and the detection time, we use abnormal measurements in the navigation equations, which is highly undesirable. On the other hand, false alarms result in lower accuracy of the estimate because some correct information is not used. The optimal solution is again a trade-off between these two contradictory requirements.

A disadvantage of the traditional detection techniques is the existence of the distribution “tail” of the detection delay. Strictly speaking, a small mean delay for detection need not imply that the permitted time to react to a change is negligible. Therefore, we have to minimize the “worst case” probability of missed detection.

We use a statistical hypothesis testing minmax approach applied to fixed size samples of measurements. It is named as χ^2-Shewhart chart (CSSC) because it is based upon a quadratic form of the residuals [4]. CSSC provides good clues for failure diagnosis.

In the recent years, neural networks have been applied to a variety of problems in the areas of pattern recognition, image processing, process identification, etc [5]. One of the major features of the proposed algorithm is the estimation of system parameters using neural network to determine the status of the system. Another important task is to detect new faults or operating conditions. This warrants the use of neural networks with unsupervised learning. The ART (Adaptive Resonance Theory) class of networks [6] is an ideal choice for this application.

When we think of the goal of least squares without restrictions of Gaussianity, one has to wonder why an information theoretic error criterion is not utilized. In this paper we discuss the possibility to extend the proposed scheme with information filtering [7].

The organization of the paper is as follows. Section 2 describes the proposed algorithm and its individual elements. Section 3 presents the algorithm application for failure diagnosis in navigation system. The developed solution is discussed in section 4. The conclusion of the paper is given in section 5.

2. PROPOSED ALGORITHM

The navigation solution is based upon accurate pseudorange measuring from several satellites with known locations to a user. The iterative least squares algorithm provides us with an optimal solution. After a failure is detected, it is highly desirable to know when the failure happened, where the failure is located and how serious the failure is. Input patterns for the ART-2 are organized from the CSSC results at the moment when failure is detected. The ART-2 network either gives a cluster number of an existing failure, or indicates the present condition as a new failure. A failure diagnosis structure for such a purpose for a GPS navigation system is shown in Fig. 1.

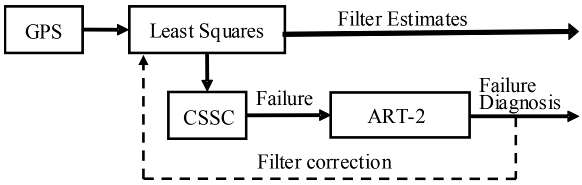

Fig. 1. Proposed fault detection and diagnosis scheme

In the next four sections the each part of the scheme is described.

2.1. The Navigation solution

The pseudorange p_i from the ith satellite to the user can be written as

$$p_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + c\delta_u + \varepsilon_i \quad (1)$$

where x_i, y_i and z_i denote the ith satellite's position in three dimensions, x_u, y_u and z_u is the user's position, c is the speed of light, δ_u is user receiver clock bias, ε_i is an additive measurement error associated with ith satellite, $i = 1,2,\ldots n$ (n is the number of tracked GPS satellites).

Nonlinear equations (1) can be solved using iterative technique based on linearization. Let us introduce the following vectors: measured pseudoranges $P = (p_1, \ldots, p_n)^T$ and the state vector $X = (x_u, y_u, z_u, \delta)$. If we know approximately where the user is $X_0 = (x_{u0}, y_{u0}, z_{u0})$, then we can use a first order Taylor series approximation

$$Y = P - P_0 \cong H_0(X - X_0) + \varepsilon \quad (2)$$

where $P_0 = (p_{10}, \ldots, p_{n0})^T$, $H_0 = \left.\frac{\partial P}{\partial X}\right|_{X = X_0}$ is the Jacobian matrix of size n x 4. Then the Least Squares (LS) estimate $\hat{X}$ is

$$\hat{X} = X_0 + A_0(P - P_0) \quad (3)$$

where $A_0 = (H_0^T H_0)^{-1} H_0^T$. When the convergence is reached, the residual vector e is given by the following equation

$$e = P - P_0 \quad (4)$$

2.2. Model of fault

The model for global navigation sets can be represented in the following linear form:

$$Y_k = HX_k + V_k + Z(k, t_0) \quad (5)$$

where $Y_k \in R^r$ is the measurement at time k, $X_k \in R^n$ is the unknown input useful signal, H is a known constant full rank matrix, V_k is nonstationary zero mean Gaussian white noise, $Z(k, t_0)$ is the change vector

$$Z = \begin{cases} 0, k < t_0 \\ Z \neq 0, k \geq t_0 \end{cases} \text{ and } Z_l = (0, \ldots, 0, z_l, 0, \ldots, 0)^T.$$

We assume that there exists measurement redundancy, namely that $r > n$. The covariance matrix R is scalar: $R = \sigma^2 I_r$, where I_r is the identity matrix of size r.

2.3. Chi-Square Shewhart chart

We pursue our discussion on the detection of abrupt changes in the regression model given by (5). The characteristic feature of this problem is the fact that the nuisance parameter X is unknown. Let us start with a hypotheses testing problem that is simple for the informative parameter Z:

$$\begin{aligned} H_0 &= \{Y \approx N(HX, \sigma^2 I_n)\} \\ H_1 &= \{Y \approx N(HX + Z, \sigma^2 I_n)\} \end{aligned} \quad (6)$$

We are interested in detecting a change from 0 to Z. One possible solution is based upon the minmax approach. The main idea of the minmax algorithm for nuisance parameters is to maximize the minimum possible power over the unknown parameters. Therefore, the design of the minmax test consists of finding a pair of least favourable values X^0 and X^1 for which the Kullback-Leibler information $K = K(X^0, X^1)$ is minimum, and in computing the likelihood ratio (LR) L for these values. The Kullback-Leibler information K is given by

$$K = \frac{1}{2\sigma^2}\left\| H(X^1 - X^0) + Z \right\|^2 \quad (7)$$

It is obvious that K is a function of the difference $x = X^0 - X^1$.Therefore, we minimize K with respect to the parameter x. The minimum is obtained for $x^* = (H^T H)^{-1} H^T(Z)$ and is given by

$$K(x^*) = \frac{1}{2\sigma^2} Z^T BZ, \quad (8)$$

where $B = I - H(H^T H)^{-1} H^T$ is the projection matrix, the rank of B is the number $n - 4$ of unit eigenvalues of the matrix B. Finally, we get the following formula of the log LR for hypotheses (6) under the least favourable value x^* of the nuisance parameter

$$S_k^t = \sum_{j=k}^{t} \log L(Y_j; x^*) = \sum_{j=k}^{t}\left(\frac{1}{\sigma^2} Z^T e_j - \frac{1}{2\sigma^2} Z^T BZ\right) \quad (9)$$

where e is residuals of the LS algorithm for navigation solution (4). It is worth nothing that this LR is independent of the unknown values X^0 and X^1 and the LR is a function of the residual vector e.

Therefore, the minmax χ^2-Shewhart chart algorithm is:

$$M = \inf\{t \geq 1 : g_t = \max(0, g_{t-1} + S_k^t) \geq \lambda\} \quad (10)$$

where λ is a given threshold.

We can replace the original problem by the problem of detecting a change in the noncentrality parameter b^2 of a χ^2 distribution with $r - n$ degrees of freedom. CSSC has two tuning parameters: the window size N and the threshold λ. Its properties are given by the mean time between false alarms

$$\bar{T}(N,\lambda)=\frac{N}{1-P\left[\chi^2(r-n)<\lambda\right]}$$

(11)

and the mean delay for detection

$$\bar{\tau}(N,\lambda)=\frac{N}{1-P\left[\chi'^2(r-n,b^2)<\lambda\right]}, \quad (12)$$

where $\chi'^2(r - n, b^2)$ is χ^2 distributed random variable with $r - n$ degrees of freedom and b^2 is noncentrality parameter.

2.4. ART-2 neural network

The simplest ART-1 architecture is capable of classification only of binary inputs. The more complicated ART-2 architecture can autonomously classify arbitrary sequences of analog input patterns into stable categories after a single learning trial. Fig. 2 illustrates the overall structure of the ART-2 network.

The network consists of three basic layers. F0 layer performs the pre-processing and normalization of the input pattern. F1 layer processes the filtered pattern (output of the F0 layer) and compares it with the learned pattern (stored in the F2 layer). By means of a set of adaptive weights (bottom-up weights), and given a filtered pattern, a F2 node is selected. This node represents a learned pattern (failure category). Once a node is selected, it sends its learned pattern towards the F1 layer by means of a set of adaptive weights (or top-down weights). Then the comparison between the learned pattern and the input pattern is done. If they do not match properly (given a vigilance parameter ρ*), the orientation subsystem activates a reset signal and disables the selected node from now on, and a new search cycle begins at F2 until a new node with a similar pattern or an uncommitted node is found. Otherwise, the adaptive weights of the selected node are updated. Larger value of vigilance parameter ($0<\rho*<1$) encourage the generation of more categories.

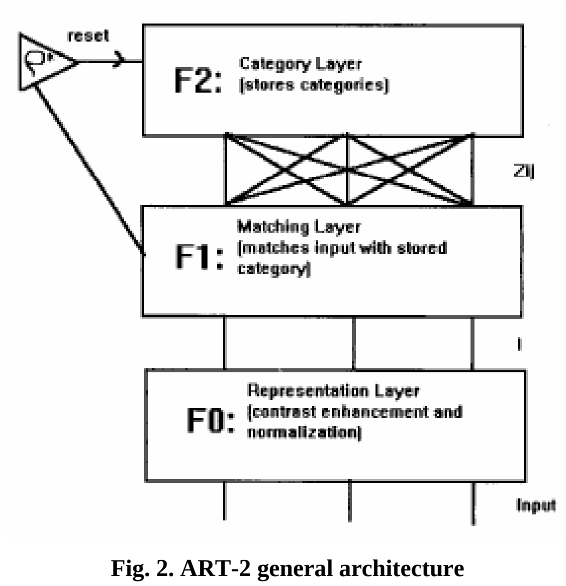

Fig. 2. ART-2 general architecture

TABLE I. χ2-Shewhart chart with nonoptimal sample size (N = 10)

$\bar{T}$	$\bar{\tau}$ Shew	
	r - n = 3	r - n = 4
10^2	11,4	12,5
10^3	17,9	21,2
10^4	37,3	44,3
10^5	86,2	107,4
10^6	230,5	277,3
10^7	621,7	853,6
10^8	1847,0	2493,0

TABLE II. χ2-Shewhart chart with optimal sample size

$\bar{T}$	$\bar{\tau}$ Shew				
	r -n = 1		r - n = 4		r - n = 10
	$\bar{\tau}$ Shew	$\bar{\tau}$ *Shew	$\bar{\tau}$ Shew	N_{opt}	$\bar{\tau}$ Shew
10^3	13,6	20,2	19,4	15	25,3
10^4	20,2	31,4	27,8	22	35,8
10^5	27,5	42,9	35,7	28	45,7
10^6	34,1	53,5	44,5	35	54,9
10^7	42,3	64,3	52,6	41	64,2
10^8	49,8	75,8	60,3	44	73,1

3. FAILURE DIAGNOSIS IN NAVIGATION SYSTEM (EXPERIMENTAL RESULTS)

Labview environment has been used to simulate and test the proposed method [8]. Experimental navigation data was collected using Motorola Oncore 12 channel GPS receiver.

It is difficult to establish the alarm rates (estimate parameters analytically). First, we use a given sample size $N = 10$ as in [9] for navigation systems integrity monitoring. We assume parameter $b^2 = 1$ and consider several values of the number of degrees of freedom $r - n$. These results are in table 1. Second, we fix mean time between false alarms $\bar{T}$, deduce λ and compute $\bar{\tau}$ for χ^2-Shewhart chart. The corresponding results are shown in table 2. In this table, we also add the values of the optimal sample size N_{opt} in the case $r - n = 4$. Furthermore, we also add in the third column of this table the worst mean delay $\bar{\tau}*$ for the χ^2-Shewhart chart for the case $r - n = 1$.

Comparing the columns corresponding to $r - n = 4$ in tables 1 and 2, we deduce that the sample size plays a key role in the performance of the CSSC algorithm. We also find that the optimal sample size should increase with the mean time between false alarms. Finally, it results from the column $r - n = 1$ of table 2 that the difference between the worst mean delay $\bar{\tau}*$ and the mean delay is significant.

Once the sum of the log LR in the window rise above the threshold, the parameters are sent to the ART-2 network to assess the status of the system. The input to the ART-2 network is the estimated parameters (the state vector X) and the output is the system status number.

Initially, ART-2 network has no knowledge about any failure class. Hence, when it receives the first set of input, it classifies it as a new failure class with number 1. After some time, one more fault is detected. If it is classified as a new fault, then the network creates a new class 2. The advantage of unsupervised learning can be seen in the ability of the system to classify new unencountered situations.

We effectiveness of the proposed scheme is evaluated by 10 simulation runs with jump-type failures. All of the failures were detected by CSSC and classified by ART-2 correctly.

The vigilance parameter ρ^* is adjusted during each run such that the ART-2 network respond to all faults.

4. DISCUSSION

In situations where the converged parameters are known for all possible faults, supervised neural networks such as a back-propagation network can be trained to classify the different faulty situations of the system. However, one of the main purposes of this paper is to introduce an algorithm that can detect new unencountered faults or operating conditions. This can be achieved using neural networks operating under unsupervised learning environment such as the ART based neural networks.

Receiver Autonomous Integrity Monitoring (RAIM) can be implemented using redundant satellite measurements. A minimum of four satellite measurements are required to compute a GPS navigation solution. Most of the time more satellites are visible and it is often possible to make measurements from eight or more GPS satellites simultaneously. If the GPS receiver can track five satellites the RAIM algorithms permit a single satellite failure to be detected. If six satellites are tracked, a satellite failure can be detected, isolated, and the failed satellite rejected from the navigation solution allowing navigation to continue.

The proposed algorithm may indicate a change in operating conditions even if there is none (false alarm), or it may miss a change altogether (missing fault). The false alarm and missing fault rates depend strongly on the parameter estimation scheme and the window length. For example, if the length of the window is big, the algorithm may not respond to faults fast enough due to the strong effect of the past data. On the other hand windows with small length will increase the variance of the identified parameters, therefore, will delay the transfer of the identified parameters to the clustering algorithm.

Someone may want to design GPS system which is able to adjust filter parameters adaptively, in order to correct the effect of soft failures. In that case, the ART-2 may be helpful, since it provides the information about the time when the failure happened and how serious it was.

The mean square error criterion (MSE) has been almost exclusively employed in the training of all adaptive systems including liner filters for many years. There were mainly two reasons behind this choice: analytical simplicity and the assumption that most real-life random phenomena may by expressed accurately by the Gaussian distribution. However, most real-life problems are governed by nonlinear equations and most random phenomena are far from being normally distributed. Therefore, a criterion that considers not only the second order statistics but also higher-order statistical behaviour is a necessity.

The entropy of a given probability distribution function is a scalar quantity that provides a measure of the average information contained in the distribution. In fact, when the entropy of the error is minimized, the expected information contained in the estimation error is minimized. The entropy is invariant to the mean of the error data set. Hence, the adaptive filter is adjusted optimally in the sense that the mutual information between the time series and the model output is maximized.

It is worthwhile to remember that nonparametric entropy training is equivalent to maximum likelihood estimation. Consider a sample of N independent realization $x_1,\dots,x_N$ of a random variable x distributed according to a common density $p^*(x)$. Then the differential entropy $\hat{p}(x)$ is

$$H(p_x(x)) \approx -\frac{1}{N}\sum_i \log \hat{p}(x_i) \to H(p_x(x)) + J(p_x \| \hat{p}) \quad (13)$$

where $J(.||.)$ is the Kullback-Leiber divergence which is symmeterized information of (7). As we can see we do not require a family of distribution when using the entropy minimization approach. This is preferable in many real world problems where we do not know the data distributions.

5. CONCLUSION

A failure diagnosis scheme using the $\chi 2$-Shewhart chart and the ART-2 neural network techniques is presented for dynamic systems. After a failure is detected ART-2 neural network is applied to recognize when the failure happens and how serious the failure is. It is not necessary to have the knowledge of all possible failures since new failures are determined on-line. This is the main advantage of the proposed algorithm. The proposed scheme is successfully applied to diagnose soft failures in GPS navigation system. In the future, we intend to apply sequential change detection algorithm (CUSUM) which takes into account all past and current sets of GPS measurements and FUZZY ARTMAP for failures classification because it automatically determine the vigilance parameter of the ART network. These improvements will help to design an integrated GPS system which is able to adaptively adjust filter parameters.

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