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DIMENSIONLESS SENSITIVITY AS THE BASE FOR ESTIMATION OF ERROR OF METERS OF INTEGRAL CHARACTERISTICS

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Abstract — The new approach for development of determination method of accuracy estimation for meters of integral characteristics is proposed. This method is based on the measurement or determination of dimensionless sensitivity. The dimensionless sensitivity is determined as

$$\alpha(x) = \frac{d(\lg y)}{d(\lg x)} = \frac{x}{y} \cdot \frac{dy}{dx} . \quad (1)$$

For the finite increments of argument and function the expression (1) can be rewritten as:

$$\alpha\left(\frac{x_n + x_{n+1}}{2}\right) = \frac{x_{n+1} + x_n}{x_{n+1} - x_n} \cdot \frac{y_{n+1} - y_n}{y_{n+1} + y_n} . \quad (2)$$

The method can be used for the cases with difficulties for calibration of meters of integral values.

Keywords: Dimensionless sensitivity, step of measurement, integral characteristic.

1. INTRODUCTION

Well known in the literature, the sensitive analysis is based on dimensionless value $\alpha = d(lgy)/d(lgx)$, which is a slope of function $y(x)$ in log-log scale. Nowadays the interest to this approach constantly increases [1-8]. This is mainly due to dimensionless value, which make possible to compare the processes in various ranges of argument and function, to determine parameters and kind of approximations of nonperiodic functions both physical and other nature, to describe adequately the complex and compound functions. Synonyms of this value in the literature are: dimensionless or unitless sensitivity, ratio, coefficient. Dimensionless sensitivity coefficient is usually used, when the parameter values may vary many orders of magnitude.

It is necessary to note that from mathematical point of view the logarithm is always the dimensionless value. In physics the usage of the logarithm of physical value is understood that the dimension value is unit vector scaled. It is necessary to taken into account such difference between mathematical and physical formalities.

Dimensionless sensitivity (DS) was used for comparison of functions [9, 10], for approximation and modelling of functions [11, 12]. Especially wide it was used in semiconductor physics for analysis of harge flow mechanisms and current-voltage characteristics [13-18].

In this article the new possibility for use of dimensionless sensitivity for estimation of error of meters of integral characteristics is represented [19].

2. DEFINITION AND MATHEMATICAL DESCRIPTION OF DIMENSIONLESS SENSITIVITY

The dimensionless sensitivity is determined as

$$\alpha(x) = \frac{d(\lg y)}{d(\lg x)} = \frac{x}{y} \cdot \frac{dy}{dx} , \quad (1)$$

For the finite increments of argument and function the expression (1) can be rewritten as:

$$\alpha\left(\frac{x_n + x_{n+1}}{2}\right) = \frac{x_{n+1} + x_n}{x_{n+1} - x_n} \cdot \frac{y_{n+1} - y_n}{y_{n+1} + y_n} \quad (2)$$

The $\alpha(x)$ value for each x point is the order of the parabola $y = Cx^{\alpha}$, drawn through the point of origin ($x=0$, $y=0$) and two closely located points of a function $y(x)$ (x_n, y_n and x_{n+1}, y_{n+1}), where C is the constant (see Fig.1).

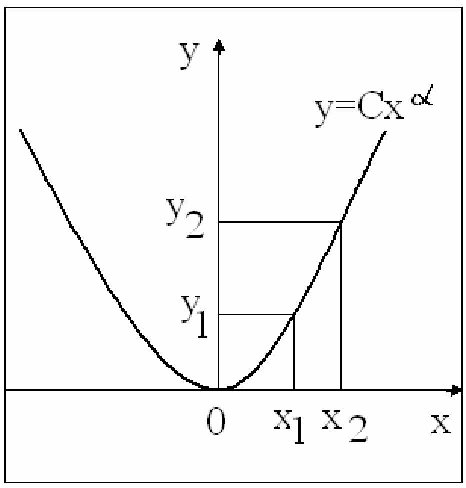

Fig.1. Geometrical definition of dimensionless sensitivity of function $y=f(x)$ in the point x.

It characterises the tangent of the angle of function slope in the vicinity of x point in log-log scale $lg\, y = f\,(lg\, x)$ (see Fig.2.).

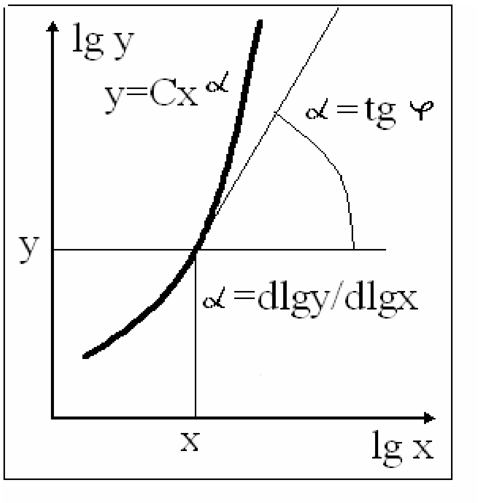

Fig.2. Geometrical meaning of dimensionless sensitivity of function $y=f(x)$ in the point x.

There are the following synonyms for DS: sensitivity coefficient, sensitivity ratio, dimensionless sensitivity coefficient, dimensionless sensitivity gradient, slope, differential slope, slope index, power, power coefficient, power value, exponent, exponent value. All these concepts are the same meaning, i.e. inclination of the curve in log-log scale at some value of argument x.

The error of nonlinearity of the function ($\Delta\alpha$) has statistical and systematic components. The statistical error is due to errors of argument (Δx) and function (Δy). The systematic error is caused mainly by the transition from infinitesimal increment of argument and function (dx and dy) to finite one (Δx and Δy) under differentiation (see (1) and (2)). It is necessary to note that the procedure of minimising of the DS error has been developed to enhance the accuracy of α(x) determination [19]. This procedure implies the x step variation during y measurement and provides the lowest error of α(x) determination. This error minimising is due to the exception of systematic error. The minimised error value (Δ_α) for $\alpha(x)$ is achieved at argument step

$$(\delta x/x)_{opt} = 2[(\Delta_y+\alpha\Delta_x)/\alpha\,/\alpha^2-1\,/]^{1/3} \qquad (3)$$

It is determined by the expression

$$\Delta_\alpha = 1.5\,[\,/\alpha^2-1\,//\alpha\,]^{1/3}(\Delta_y+\alpha\Delta_x) \qquad (4)$$

where $(\delta x/x)_{opt}$ is the optimised step of argument x, $\Delta_{x,y}$ are the relative errors of argument and function determination, correspondingly.

In the case when the δx is less than Δ_x the error of the function y is proportional to the deviation of measurement of this function. The method can be used for estimation of error of meters of integral characteristics.

3. ESTIMATION OF ERROR OF DIMENSIONLESS SENSITIVITY

The value of DS is directly determined by the inaccuracy of the argument x and function $y(x)$. The qualitative view of DS error dispersion is represented on Fig.3.

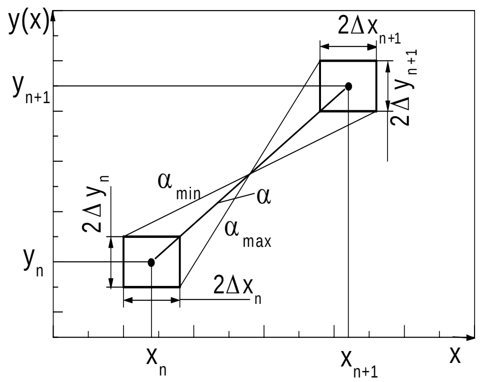

Fig.3. The qualitative view of DS error dispersion in two points (x_n and x_{n+1}) of integral characteristic

We can see from the Fig.3 that the decrease of measurement step (x_{n+1} - x_n) increases the dispertion of DS value and vice versa that the increase of measurement step (x_{n+1} - x_n) decreases the dispertion of DS. The highest possible deviation of DS value from value determined by (2) forms the highest error of α

$$\Delta = (\alpha_{max} - \alpha)\,/\,\alpha = /\alpha_{max}\,/\,\alpha - 1\,/; \qquad (5)$$

which is necessary to determine and minimise.

If we represent the nonlinear function $y(x)$ in the power-mode form $y = x^{\alpha_0}$, where α_0 is the constant, we can get from (2) and (5) taken into account the presence of relative errors Δ_x and Δ_y

$$\alpha_{max} = \frac{2-\frac{\delta x}{x}\times\Delta_x}{\frac{\delta x}{x}-2\Delta_x}\times\frac{\left[\left(2+\frac{\delta x}{x}\right)\Big/\left(2-\frac{\delta x}{x}\right)\right]^{\alpha_0}\times(1-\Delta_x)^{\alpha_0}\times(1+\Delta_y)-(1+\Delta_y)^{\alpha_0}\times(1-\Delta_y)}{\left[\left(2+\frac{\delta x}{x}\right)\Big/\left(2-\frac{\delta x}{x}\right)\right]^{\alpha_0}\times(1-\Delta_x)^{\alpha_0}\times(1+\Delta_y)+(1+\Delta_x)^{\alpha_0}\times(1-\Delta_y)} \qquad (6)$$

As $\lim\limits_{\delta x/x\to 0}\Delta_{syst} = 0$ and $\lim\limits_{\delta x/x\to 2}\Delta_{st} = 0$ we can assume their additivity in the region of minimum of Δ. Other words

$$\Delta=\Delta_{st.}+\Delta_{syst.};\ \Delta_{st.}=f(\delta x/x);\ \Delta_{syst}=f(\delta x/x); \qquad (7)$$

The expression for statistical and systematic components of error can be written as [19, 20]

$$\Delta_{st} = \frac{2}{\alpha} \times (\Delta_y + \alpha \Delta_x) \times {}^{x}\!/_{\delta x} \quad (8)$$

$$\Delta_{syst} = \frac{1}{8} \left| \alpha_e^2 - 1 \right| \times \left(\frac{\delta x}{x} \right)^2 \quad (9)$$

under fulfilment of conditions

$$\left(\frac{\delta x}{x} \right)^2 << \frac{2}{\Delta_x + \alpha \Delta_y}; \quad 2\Delta_x << \frac{\delta x}{x} << \frac{2}{\Delta_x + \alpha \Delta_y};$$

$$\alpha << \frac{\left(1 + \frac{2}{\Delta_x^2} \right)^{0,5} + 1}{2} \quad (10)$$

for statistical component and

$$\frac{d\alpha}{x} = 0; \quad \left(\frac{dy}{y} \right)_{opt} = \alpha_e \left(\frac{dx}{x} \right)_{opt} \quad (11)$$

for systematic one.

On the base of (2), (5), (6), (8) and (9) in the region of predominance of statistical component of error ($\delta x/x \to 0$) we get the identity

$$\Delta_y \frac{2}{\alpha} \frac{x}{\delta x} \left[1 - \frac{\alpha^2}{4} (1 + \Delta_\alpha) \left(\frac{\delta x}{x} \right)^2 \right] + \Delta_x \frac{2x}{\delta x} \left[1 + \Delta_\alpha - \left(\frac{\delta x}{2x} \right)^2 \right] = \Delta_\alpha - \Delta_x \Delta_y \left(\alpha - \frac{1}{\alpha} + \alpha \Delta_\alpha \right) \quad (12)$$

From (12) we can get the equation for determination of statistical error Δ_x and Δ_y using two values of nonlinearity

$$\Delta_{xk} = \frac{\alpha_{k+1} \times \Delta_{\alpha(k+1)} \times \left(\frac{\delta x}{x} \right)_{k+1} - \alpha_k \times \Delta_{\alpha k} \times \left(\frac{\delta x}{x} \right)_k}{2\alpha_{k+1} \times \left(1 + \Delta_{\alpha(k+1)} \right) - 2\alpha_k \times \left(1 + \Delta_{\alpha k} \right)} \quad (13a)$$

$$\Delta_{yk} = \alpha_{k+1} \times \alpha_k \frac{\Delta_{\alpha k} \times \left(1 + \Delta_{\alpha(k+1)} \right) \times \left(\frac{\delta x}{x} \right)_k - \Delta_{\alpha(k+1)} \times \left(1 + \Delta_{\alpha k} \right) \times \left(\frac{\delta x}{x} \right)_{k+1}}{2\alpha_{k+1} \times \left(1 + \Delta_{\alpha(k+1)} \right) - 2\alpha_k \times \left(1 + \Delta_{\alpha k} \right)}$$

Under the condition of known error Δ_x it is possible to determine Δ_y with one value of α

$$\Delta_y = \Delta_\alpha \frac{\alpha}{2} \frac{\delta x}{x} - \alpha \Delta_\alpha . \quad (14)$$

4. CONCLUSION

The new application of dimensionless sensitivity for estimation of error of functions is proposed. It can be used for the cases with difficulties for calibration of meters of integral values.

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