10th IMEKO TC7 International Symposium
June 30–July 2, 2004, Saint-Petersburg, Russia

USING ORTHOGONAL SEQUENCES IN "BLIND" CALIBRATION OF AN ARRAY OF ANALOG-TO-DIGITAL CONVERTERS

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Abstract – Mismatches in errors between the individual ADCs, more than the errors themselves, have been known to be the dominating cause of signal-distortion in certain systems like a Time-interleaved ADC, implemented using an array of ADCs. A new technique, using orthogonal sequences for effectively eliminating these mismatches, has been proposed in this paper as a scheme suitable for calibrating such an array of ADCs. Ease of implementation compared to currently existing techniques is the main advantage of this new scheme. Online calibration can be performed at the expense of some circuit overhead involving one additional ADC as a reference ADC, switches, DSP and memory, while offline calibration does not require the additional ADC, since any particular ADC in the array may be used as the reference. The reduction in signal-distortion for different resolutions of the reference ADC has been substantiated through simulations performed on a modelled 4-path Time-interleaved ADC. When all the ADCs have been modelled with 12-bit resolution and with up to 1% offset error and 5% gain error, the improvement in SFDR achieved using this calibration scheme is up to 54 dB.

Keywords: Analog-to-Digital Converter (ADC), array, calibration

1. INTRODUCTION AND OVERVIEW

Arrays of (nominally identical) devices are used in certain applications where data sourced from different locations in space or points in time require concurrent processing. A spatially distributed sensor array, associated with its array of ADCs for data-conversion, provides an example scenario for the case where data are sourced from different locations in space, while a Time-interleaved ADC serves as an example of simultaneous processing of the data sourced at different points in time. In general, a combinational space-time distribution of an array of devices may be required in an application.

The fidelity of the data processed using such arrays of devices depends, not surprisingly, on the extent of deviations of the individual devices within a given array from their ideal behaviours. There is an important class of arrays of devices, however, where such fidelity (or a lack of it) depends more directly on the extent of *mismatches* between the deviations from the ideal of the individual device behaviours, than on the extent of these deviations themselves.

In this paper, we propose a new calibration scheme suitable for improving the fidelity of data-conversion in an array of ADCs, which is otherwise degraded largely due to mismatches in the non-ideal behaviours among the individual ADCs in the array. This work will focus on correcting the ill-effects suffered by the array of ADCs due to mismatches in gain and offset errors of the individual ADCs. In order to quantify the improvement achieved using the proposed calibration scheme, a Time-interleaved ADC (N-channel/path case shown in fig. 1) is taken as a case study, with the signal-distortion at its output, expressed as Spurious Free Dynamic Range (SFDR), being the quantity used to measure the improvement. However, the proposed calibration scheme is equally applicable for any array of ADCs where fidelity of data-conversion is limited largely by the mismatches in the gain and offset errors among individual ADCs in the array.

The organization of this paper is as follows. Section 2 provides a window to the currently existing literature on the subject. In Section 3, the calibration scheme is evolved by starting with an observation of the utility of orthogonal sequences in certain spread-spectrum techniques like Code Division Multiple Access (CDMA). This section also explains the advantages of the proposed scheme relative to some cur-

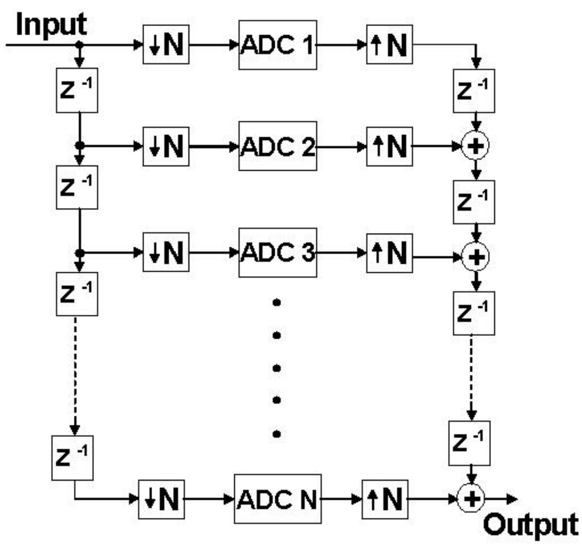


Fig. 1 – N-path Time-interleaved ADC

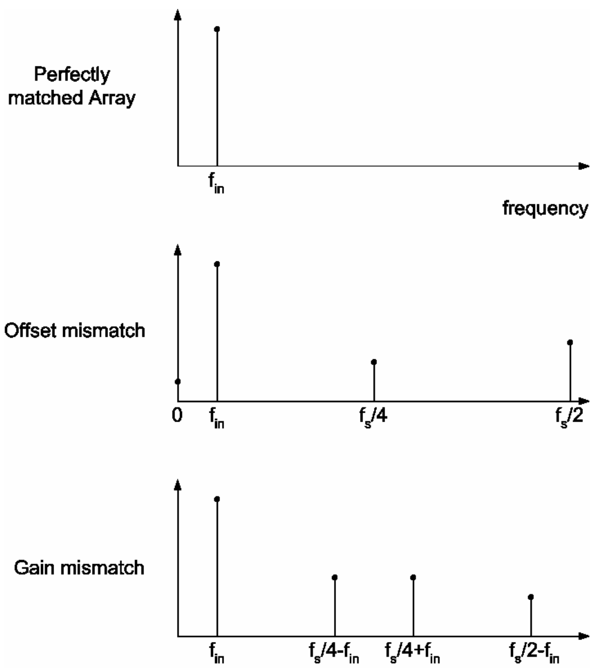


Fig. 2 – Spectrum of reconstructed sinusoid for a 4-path Time-interleaved ADC band-limited to half the sampling frequency, f_s. (courtesy Black et al [1])

rently existing calibration techniques and examines the circuit overhead required for implementing this scheme from the perspectives of both offline and online calibrations. Section 4 provides the results of simulations performed on a modelled 4-path Time-interleaved ADC, taken as a case study. This paper is then concluded in Section 5.

2. COMMENTS ON THE PREVIOUS WORK

Much of the currently existing literature on the errors in arrays of ADCs focuses on Time-interleaved ADCs. In [1], Black *et al* presented their pioneering work in identifying the effects of mismatches among the individual ADCs in a Time-interleaved ADC array, as illustrated in fig. 2 for a 4-path case. This was subsequently extended by Petraglia *et al* in [2].

Methods using analysis/synthesis Quadrature Mirror Filter (QMF) banks for overcoming mismatch errors in ADC arrays were examined in [3-6] by Lookabaugh *et al* and Petraglia *et al*. Though ingenious, these methods pose problems in implementation owing to the requirement for high-quality switched-capacitor filters needed for the analysis bank at the (analog) inputs of the ADCs in the array. These switched-capacitor filters not only introduce their own signal distortion, but they also are an expensive solution since one such filter is needed for each individual ADC in the array. Even though, being digital filters, they do not contribute to additional distortion, the problem of high cost exists for the filters in synthesis bank too as their number also equals the number of ADCs in the array.

Another drawback of the calibration using analysis/synthesis QMF banks is that it is based on the suppression of aliasing tones that are caused by undesirable mismatches among individual ADCs in the array. It does not repair the root cause, which is the mismatch itself. Hence, these methods' utility is restricted only to Time-interleaved ADCs, and cannot be applied to overcome mismatch-induced errors in a more general set of arrays of ADCs.

Thus the need is for a calibration scheme that directly repairs the root cause – the mismatch – for wider applicability.

3. THE NEW CALIBRATION SCHEME USING ORTHOGONAL SEQUENCES

3.1 "Separation" using orthogonal sequences

The utility of orthogonal sequences in data-channel separation has been long established, as in the case of Spread-spectrum schemes like CDMA [7] used in data communication. In this present paper, we use this *separation* property of orthogonal sequences for separating the contributions of offset and gain errors to the observed output in the presence of unknown input signal (as emphasized by our use of the term "blind" in the title of this paper). We show that the separation of signal from error facilitates offset removal and gain calibration with respect to a reference ADC.

3.2 Modelling an individual ADC

Since, for the moment, we are concerned about the mismatches in gain and offset errors, let us disregard other limits (like quantization error and nonlinearity) and model a sub-ADC (an individual ADC in an array) by only its offset, a, and gain, k, so that its output-sample sequence $\{z\}$ is a transformation of its input-sample sequence $\{x\}$, given by

$$\{z\} = a + k\{x\} \tag{1}$$

3.3 Offset modulation

Let $\{s\}$ be a sequence of length M, composed of elements s_j such that

$$s_j \in \{-1,1\} \;\; \forall j = 1,2,..,M \tag{2}$$

If the input-sample sequence $\{x\}$ and the output-sample sequence $\{z\}$ of a given sub-ADC are both modulated (modulation operation is denoted by "●" in the text and "**X**" in figures) by the same sequence

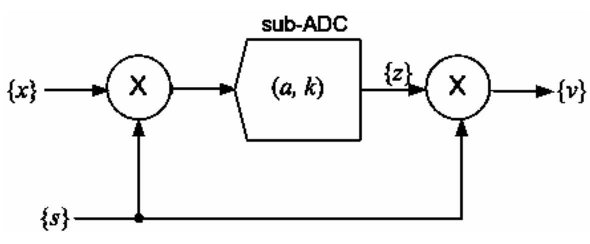


Fig. 3 – Offset modulation

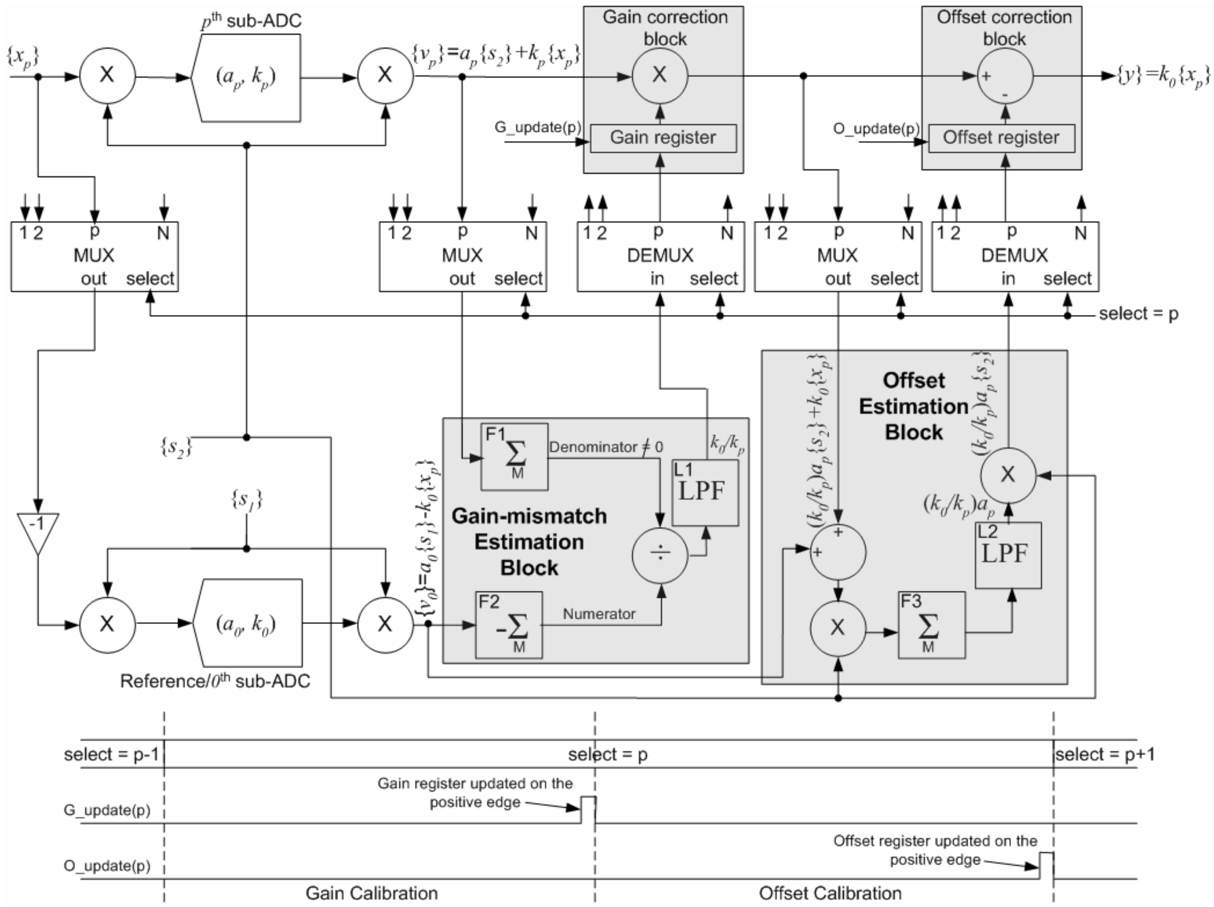


Fig. 4 – The proposed calibration scheme and the timing diagram

$\{s\}$ satisfying (2), as shown in fig. 3 (assuming zero delay from input to output of sub-ADC), then the resultant sequence $\{v\}$ is given by

$$\{v\} = \left[a + k\{x\} \bullet \{s\}\right] \bullet \{s\} = a\{s\} + k\{x\} \qquad (3)$$

$\because \{s\} \bullet \{s\} = \{1\}$, the sequence of all 1's. In other words, modulating both the input and output of a sub-ADC by the same sequence $\{s\}$ satisfying (2), is tantamount to modulating its offset error alone by that sequence $\{s\}$. This principle of offset modulation given by (3) forms the basis for the proposed calibration scheme, as explained below.

3.4 The Calibration Scheme

Let $\{s_1\}$ and $\{s_2\}$ be two orthogonal sequences of length M (M is a positive integer multiple of 4) satisfying (2) and given by

$$\sum_{j=1}^{M} s_{1,j} = 0$$

$$\sum_{j=1}^{M} s_{2,j} = 0$$

$$\sum_{j=1}^{M} s_{1,j} s_{2,j} = 0 \qquad (4)$$

where $s_{i,j}$ stands for the j^{th} element of the sequence s_i.

Fig. 4 schematically explains how the orthogonal sequences $\{s_1\}$ and $\{s_2\}$ can be used to calibrate p^{th} sub-ADC (belonging to an array of N sub-ADCs) with (offset, gain) = (a_p,k_p) using a reference sub-ADC, (a_0,k_0). Digital correction circuitry includes three recovery filters, F1, F2 and F3, which are Moving-Average filters of length M given by $\sum_{M} \equiv \frac{1}{M}\sum_{l=0}^{M-1} z^{-l}$. These three filters recover the information buried in the output sequences of the sub-ADCs, enabling an estimation of gain and offset errors. The lowpass filters L1 and L2 serve to smoothen out the ripples and noise (due to quantization) in the extracted gain and offset errors, measured with respect to the reference sub-ADC, before being fed to the correction blocks. Equation (4), together with fig. 4, which also shows the expressions for intermediate signals, makes the calibration scheme self-explanatory. Calibration of a sub-ADC is begun by estimating its gain with respect to the reference sub-ADC, and using this information to match their gains. Then, the gain-corrected offset of the sub-ADC being calibrated is estimated and subtracted from the gain-corrected output, to give an offset-free output. One reference sub-ADC is shared among all the sub-ADCs in the array by the use of multiplexers and demultiplexers, which switch the appropriate signals between the sub-ADC being calibrated and the reference sub-ADC. The timing diagram in fig. 4 serves to show the sequence followed in calibrating the array of ADCs.

Matching the offsets, rather than removing them altogether, is a much easier task. It merely involves plotting the output of the sub-ADC being calibrated against the output of the reference sub-ADC when both are subject to the same input signal, and reading the gain ratio and the gain-corrected offset difference as the slope and y-intercept respectively directly from the plot (which is a straight line, if both the sub-ADCs are affected only by gain and offset errors). However, the superiority of the proposed scheme is apparent when its ability to remove offset error altogether is considered.

A calibration cycle involves calibrating all the sub-ADCs in the array against the reference sub-ADC, so that the gain errors after calibration are all matched and identical to that of the reference. Since this scheme altogether eliminates the offsets of all sub-ADCs too, offset-mismatch is no longer a problem.

3.5 Quantization error and choice of orthogonal sequences

Choosing the pair of orthogonal sequences involves choosing both their length M and the actual sequences for a given M.

In the presence of quantization error, the perfect cancellation of the contribution of offsets at the outputs of filters F1 and F2 in fig. 4, required for proper calibration, may not happen. However, by choosing M sufficiently large, the resultant residual error due to quantization may be made as small as desired. The assumption here is that quantization error has a uniform probability distribution between its limits.

However, choosing M very large increases the circuit overhead in the scheme, as can be observed from the corresponding increases in the sizes of filters F1, F2 and F3, as well as the size of memory needed to store the sequences $\{s_1\}$ and $\{s_2\}$. The actual choice of M is thus based on the trade-off between the size of circuit overhead and the accuracy after calibration.

Special handling is needed when the output of filter F1 is very close to zero. The corresponding data can be simply ignored from calibration. However, the outputs of filters F1 and F2 in fig. 4 tend to stay close to zero for single-tone input signals at frequencies given by $l\left(\frac{f_s}{M}\right)$, where l is an integer and f_s is the sampling frequency of the sub-ADC. A solution to this problem is in being able to switch between two values of M, say, $M_1 = 4R_1$ and $M_2 = 4R_2$, where R_1 and R_2 positive integers which are co-prime. However, it can be easily seen that no choice of M can give non-zero outputs for the filters F1 and F2, if the input signal is a single tone at frequency $l\left(\frac{f_s}{4}\right)$.

Once M is determined, a standard way of constructing $\{s_1\}$ and $\{s_2\}$ satisfying (2) and (4) is by assigning to it any two of the rows of any order M Hadamard matrix [8] such that $\{s_1\} \neq \{1\} \neq \{s_2\}$ and $\{s_1\} \neq \{s_2\}$. It is not necessary to follow this standard procedure, however. In fact, some practical considerations may impose that the orthogonal sequences chosen be tailored to the requirements in a given situation.

If the estimated (gain-corrected) offset at the output of filter F3 is not exactly equal to its true value, since no calibration can be perfect, it results in a residual offset modulated by the sequence $\{s_2\}$ at the final output. It is hence necessary to keep the spectrum of $\{s_2\}$ as "white" as possible, so that any residual offset does not cause any undesirable spurs after calibration. This imposes the condition that $\{s_2\}$ should be sufficiently random. Once $\{s_2\}$ is chosen satisfying this condition, $\{s_1\}$ can be constructed by toggling any $M/2$ elements of $\{s_2\}$.

3.6 Effect of nonlinearity errors

Inherent nonlinearities in each sub-ADC that cause even and odd order distortions limit the application of this calibration scheme. In the presence of such nonlinearities, (1) needs to be replaced by

$$\{z\} = a + k\{x\} + h_2\{x\}^2 + h_3\{x\}^3 + h_4\{x\}^4 + h_5\{x\}^5 + .. \tag{5}$$

where $\{x\}^i \equiv \{x\} \bullet \{x\} \bullet \{x\} \bullet .....$ i times and h_2, h_3, h_4, h_5, … represent the coefficients corresponding to the higher order harmonics. Substituting $\{y\}$ for $\{x\} \bullet \{s\}$, R.H.S. in (3) modifies to

$$\left[a + k\{y\} + h_2\{y\}^2 + h_3\{y\}^3 + h_4\{y\}^4 + h_5\{y\}^5 + ..\right] \bullet \{s\}$$

$$= \mathrm{E}(\{x\}) \bullet \{s\} + \mathrm{O}(\{x\}) \tag{6}$$

where $\mathrm{E}(\{x\}) \equiv a + h_2\{x\}^2 + h_4\{x\}^4 + ..$ sums the contribution of even order terms to the output while the contribution of odd order terms is given by $\mathrm{O}(\{x\}) \equiv k\{x\} + h_3\{x\}^3 + h_5\{x\}^5 + ..$ This is the consequence of

$$\{s\}^i = \{1\} \text{ if } i \text{ is even}$$

$$= \{s\} \text{ if } i \text{ is odd} \tag{7}$$

Thus it is clear that only the even order contribution is modulated by the modulating sequence.

Under the approximations

$$\mathrm{E}(\{x\}) \cong a \ \& \ \mathrm{O}(\{x\}) \cong k\{x\} \quad (8)$$

which are valid when $|a| >> \left|h_2\{x\}^2 + h_4\{x\}^4 + ..\right|$ and $|k\{x\}| >> \left|h_3\{x\}^3 + h_5\{x\}^5 + ..\right|$, (6) reduces to (3) and calibration by the scheme in fig. 4 is still valid. While these approximations are valid in many practical ADCs, ensuring that the data required for calibration is collected only when $|x|$ is small in relation to the full-scale value, makes their validity even better.

If $\{s_2\}$ is chosen sufficiently random in the manner already explained in sub-section 3.5, it has the effect of spreading the energy in even harmonics (uniformly) throughout the Nyquist interval. Thus, in cases where even-order distortion dominates, this calibration scheme reduces the magnitude of the most dominant harmonic spurs in the output spectrum relative to the fundamental. The energy in odd harmonics, however, remains largely unaffected relative to the fundamental.

3.7 Circuit overhead in Offline Foreground and Background Calibrations

Two scenarios of offline calibration will be considered here, one in which the entire array of ADCs is offline and the other in which a particular sub-ADC in the array is offline while the others are still online.

In the former case, which we will call as Offline Foreground Calibration, one sub-ADC from the array can take the role of the reference sub-ADC. In this case, an additional signal generator may be needed to simulate an incoming signal.

In the latter case, which we will call as Offline Background Calibration, and which is possible if there exists a sub-ADC in the array which experiences idle periods while others are still online, this sub-ADC can take the role of reference sub-ADC and used for calibrating others, during such periods when it can be offline. No additional signal generators are needed in this case, since the sub-ADC being calibrated is still online. In order to calibrate the offset of reference sub-ADC, any of the already calibrated sub-ADCs from the array may be used during their idle durations, with some small modifications to the calibration scheme. In other words, this kind of calibration needs idle durations for at least two sub-ADCs in the array.

In both these cases, however, no additional sub-ADC is needed to act as the reference sub-ADC. Other overhead needed includes switches, DSP and memory, which are the common requirement for all kinds of calibration being discussed here.

3.8 Circuit overhead in Online Background Calibration

In cases like that of a Time-interleaved ADC, no single sub-ADC can be offline while others are online. While this scenario can fall under Offline Foreground Calibration requiring bringing the entire array offline as well as an additional signal generator, depending on the situation, it may be better to subject it what we will call as Online Background Calibration. This requires one additional sub-ADC to act as the reference, but no additional signal generators.

This kind of calibration, when performed on an array of ADCs where all of them need not be simultaneously online or offline, has the advantage that no sub-ADC in the array needs to be interrupted from its normal operation for the sake of calibration. Moreover, this kind of calibration does not depend on the possibly sporadic idle periods of any given sub-ADC from the array. The cost of an extra sub-ADC may be only a fraction of the total cost of the array, especially if N is large. This kind of calibration is also suited for cases where a reference sub-ADC, with a lower resolution (and hence cheaper) than the sub-ADCs in the array, may be sufficient for calibration.

3.9 Advantages over schemes using analysis/synthesis QMF banks

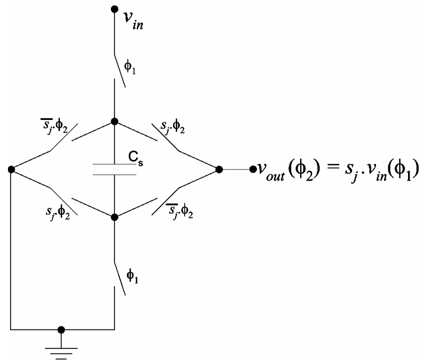


Fig. 5 – Multiplication by ±1 in the analog domain

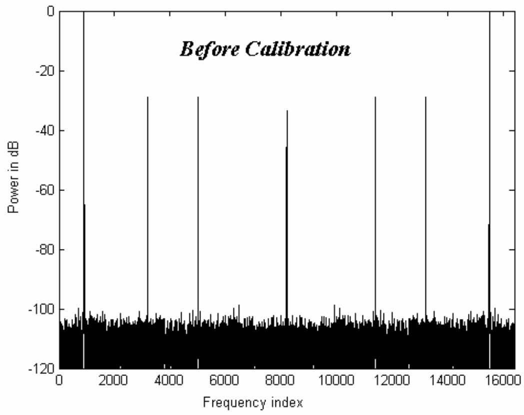
The proposed scheme depends heavily on signal processing in digital domain. The only signal processing required in analog domain is multiplication by ±1. This is demonstrated using switched-capacitors as shown in fig. 5, where ϕ_1 and ϕ_2 are two non-overlapping clock phases and $s_j = \pm 1$. Clearly, this implementation compares very favourably against the much more complex analysis QMF banks, implemented using switched-capacitors and amplifiers [3-6]. Moreover, while the QMF bank technique, based on suppressing the aliasing created by mismatches among the sub-ADCs in an array, works only with Time-interleaved ADCs, the proposed technique has a much wider applicability for arrays of ADCs, since it repairs the root cause – the mismatch itself.

4. SIMULATION RESULTS

MATLAB-SIMULINK simulations were performed on a 4-path Time-interleaved ADC composed of mismatched 12-bit sub-ADCs having up to 1% of full scale offset error and 5% gain error. The length of the orthogonal sequences chosen is $M = 2^{12}$. Calibration is performed using an input signal that has uniform distribution over the full-scale input dynamic

Table 1 – SFDR improvement for different resolutions of reference sub-ADC

Resolution	SFDR after Calibration	SFDR improvement
8-bits	72 dB	37 dB
12-bits	89 dB	54 dB



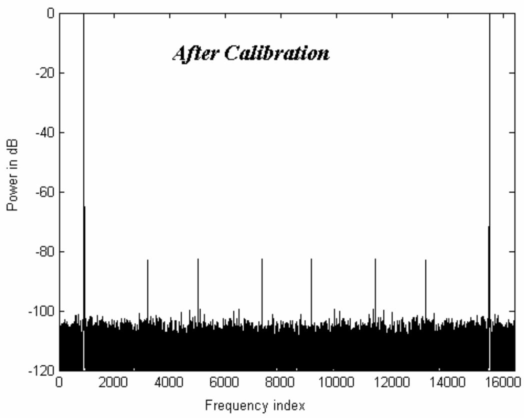


Fig. 6 – Magnitude in dB of 16384-point DFT showing the improvement after calibration in the output spectrum for a single-tone input and using a 12-bit reference sub-ADC

range. Once calibrated, the input excitation was changed to a single-tone sinusoid. The resolution of the reference sub-ADC is first set at 8 bits and then at 12 bits. For the worst-case scenario, with an uncalibrated Spurious-free dynamic range (SFDR) about 35 dB, the improvement is shown in Table 1. Fig. 6 shows the plot of the magnitude of the 2^{14}-point Blackman-windowed DFT of the output spectrum before and after calibration, when the reference sub-ADC has a resolution of 12-bits.

Better improvement can be achieved by choosing longer orthogonal sequences, but at the cost of increasing circuit overhead.

5. CONCLUSION

The "blind" calibration using orthogonal sequences has been shown to be effective in eliminating the undesirable effects of mismatch in gain and offset errors in an array of ADCs, even in the presence of unknown input. The proposed scheme depends heavily on signal processing in digital domain. Signal processing in the analog domain involves only multiplications by ±1, and hence this scheme can be more readily implemented than some others in the literature. Since this scheme repairs the root cause of mismatch, its applicability is wider than just Time-interleaved ADCs alone, which have been the target of much of recent research in the calibration of arrays of ADCs.

The only significant circuit overhead in analog domain is an additional sub-ADC needed for the online calibration of the entire array. This requirement may be swapped for an additional signal-generator in the case of offline calibration. This analog circuit overhead becomes less significant if the number of sub-ADCs in the array increases. Further, any analog circuit overhead may be insignificant in future systems-on-chip where analog circuitry occupies a small fraction. The digital circuit overhead required in this scheme is simple enough to be incorporated into an on-board Digital Signal Processor (DSP) that anyway performs other complex functions.

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