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SCIENCE AND TECHNOLOGY OF MEASUREMENT – A UNIFYING GRAPHIC-BASED APPROACH

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Abstract – Graphic-based representations of theoretical ideas are always useful bases of discussions. This is especially true for measurement sciences where structures are of utmost complexity. It will be shown that these structures are mainly structures of Signal and System Theory.

Keywords: fundamental law of measurement, signal-effect diagram, uncertainty

1. INTRODUCTION

Science and Technology of Measurement

Over the last fifty years innumerable papers have been published on this topic. Nothing shall be added here to the discussion as to what should be meant thereby. Some day, there is bound to be one commonly accepted term for "everything concerning the field of measurement". The term Metrology serves this purpose extremely well, though at present it is generally understood in a narrower sense. But language is a living and dynamic process. We all are allowed to contribute to it. No standard is ever cast in stone. The sub-terms Measurement Science and Measurement Technology, respectively, would subdivide the field of measurement (Metrology) appropriately.

Graphic Representations

But not only the field of measurement as a whole should be well defined. All details without exception must be considered and have to comply with neighbouring sciences, like Signal and System Theory, Stochastics and Statistics, Modelling and Identification and all propaedeutic fields, which may serve as the basis of measurement equipment.

In that sense Measurement Science is a very narrow field. Additions to other sciences lie only in the definition of measurement tasks and thereby in structures of influences, effects and interactions of quantities and signals. They are described best by mathematical and logical theorems and are illustrated clearly by different types of graphical diagrams, supporting a thorough understanding of the field in a convenient manner.

Even the concepts of errors and uncertainties are well-defined within other disciplines.

The main aim of this paper is the presentation of a unifying graphic representation of measurement tasks in the so-called signal-effect diagram, also known as signal flow diagram.

We start with basic assumptions, progressing step by step to more challenging goals, increasing size and complexity of the diagrams accordingly. A solid understanding will be established, one problem following and solving the other. Such a procedure is the most suitable for all levels of measurement education.

2. WHAT DOES MEASUREMENT MEAN?

As L. Finkelstein defined in 1994, "Measurement is the assignment of numbers or other symbols, by an objective, empirical process, to attributes of objects or events of the real world, in such a way as to describe them" [1]. An attempt to represent this statement graphically might yield the diagram shown in Fig. 1.

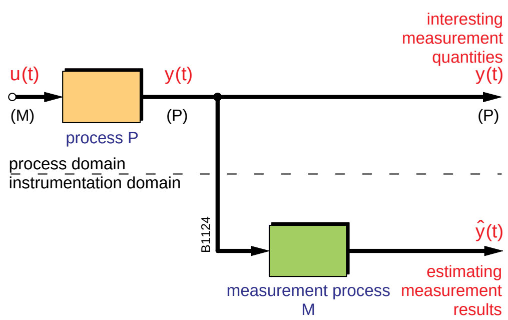

Fig. 1. Basic diagram of measurement

We assume that more than one attribute of objects or events (property) must be measured. This fact is indicated in the diagram by vectors and bold lines, representing these quantities. Furthermore we imply time-dependent properties throughout. We clearly distinguish between the multivariable process P under future observation (i.e. a process without instrumentation) and the multivariable measurement process M (objective, empirical operation M [2]), within which any instrumentation may be applied to the process.

From the point of view of Signal and System Theory there is absolutely no difference between the process P and the measuring process M. Both can be described separately or combined by systems of differential equations in the time domain (state domain representation) or by systems of temporal or spectral transfer functions.

This basic concept of a graphic-based representation of the measuring procedure has to be refined step by step according to the needs of new questions and definitions.

3. FUNDAMENTAL LAW OF MEASUREMENT

What is the primary goal of the measurement procedure? Generally measurement means the acquisition, processing, and presentation of information. Therefore, in a first approach it should be evident that the estimating measuring results $\hat{\mathbf{y}}(t)$ achieved have to equal the unknown measurement quantities $\mathbf{y}(t)$ of interest.

Def.: **FUNDAMENTAL LAW OF MEASUREMENT**

$$\text{measurement results} \overset{!}{=} \text{measurement quantities},$$

$$\hat{\mathbf{y}}(t) \overset{!}{=} \mathbf{I}\,\mathbf{y}(t)$$

respectively

This result leads to the central condition for an ideal multivariable, time-dependent measurement process.

Def.: **TRANSFER RESPONSE FUNCTION OF THE IDEAL MEASUREMENT PROCESS**

Transfer Response Function = Unit Matrix **I**

This condition is true for all cases. It applies to multivariable, non-linear, and dynamic situations as well.

4. MEASUREMENT ERRORS

A rearrangement of this statement leads to another important definition in the field of measurement, namely to the definition of measurement errors (not to be confounded with measurement uncertainties):

Def.: **MEASUREMENT ERRORS**

$$\text{measurement results - measurement quantities} \overset{!}{=} 0:$$

$$\hat{y}(t) - y(t) \overset{!}{=} 0,$$

or more specifically

measurement errors =

$$\text{measurement results - measurement quantities} \overset{!}{=} 0:$$

$$e_y(t) = \hat{y}(t) - y(t) \overset{!}{=} 0$$

Signal-effect diagram (Fig. 2) is extended.

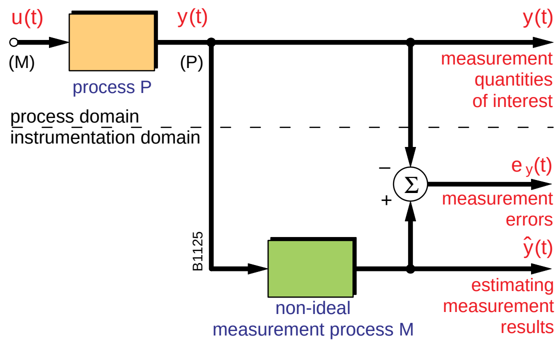

Fig 2. Definition of Measurement Error

This statement is true for all properties of the quantities $\mathbf{y}(t)$ to be measured, whether they are multivariable, constant, time or space dependent, and even random. This means that a measurement process M is ideal as long as it follows any imaginable movement of the measurement quantities $\mathbf{y}(t)$ of interest without any deviations.

If even the slightest deviations occur, they are called measurement errors $\mathbf{e}_y(t)$. Remember: Errors are virtual quantities, since we will never exactly know the quantities desired.

We call such a measurement process M non-ideal. The real transfer response function does not correspond to the ideal, nominal transfer function, which in our case should equal the identity matrix **I**, as stated by the Fundamental Law of Measurement.

Evidently, all preceding statements are variations of the Fundamental Law of Measurement.

5. ACQUISITION AND RECONSTRUCTION OF QUANTITIES

At the front end of a measurement process M there is the sensor process S (sensing sub-process), where purely physical transformations take place on the basis of approved sensing principles according to the objective principle of cause and effect. We obtain new physical quantities $\mathbf{y}_S(t)$, typically electrical or optical ones.

Within a sensor process S we remain completely in the physical domain. But normally we are not interested in those new physical quantities. With very few exceptions they cannot be compared with the measurement quantities $\mathbf{y}(t)$ of interest, as demanded by the Fundamental Law of Measurement. Furthermore, we are looking for symbols as results, normally given as numbers with units as information about all states $\mathbf{y}(t)$ of the process P.

Therefore we need a second sub-process in series with the sensor process S, namely the reconstruction process R [3]. Thereby seemingly we walk (look) back from the known sensor signals $\mathbf{y}_S(t)$ of the sensor process S to the unknown measurement quantities $\mathbf{y}(t)$. This sub-process R fulfils the demands of the Fundamental Law of Measurement as soon as its transfer response function is the mathematical inverse of the transfer response function of the preceding sensor process S (Fig. 3).

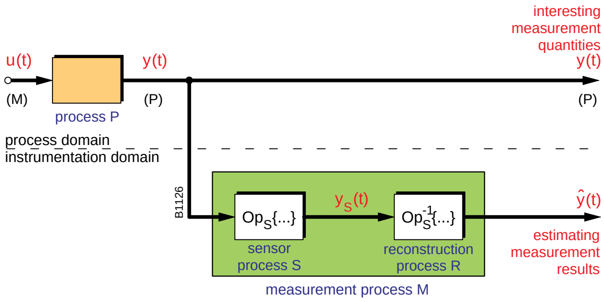

Fig. 3. Sensor and Reconstruction Process

The important consequences that result for technical applications are that a sensor process S does not necessarily have to be ideal! For instance it is allowed to show a non-linear transfer characteristic as long as the following reconstruction process R is able to realize its inverse function. In principle this law is generally true, but for mathematical reasons not all functions and sets of differential equations are invertible.

The main advantage is obvious: Sensor processes are hardware processes and cannot easily be made ideal, whereas reconstruction processes are software processes in many cases, where desired characteristics can be realized without difficulties.

Accordingly, the reconstruction process R simultaneously works as a correction process, which is often less expensive than enforcing an ideal behaviour of sensor process S would be.

Since the model of the reconstruction process R is the inverse of the model of the sensor process S, we do need an appropriate model of the sensor process S. Therefore there is no measurement without a calibration (identification) process!

Here we use the term Model-Based Measurement for the first time, since the inverse model of the sensor process S is directly involved.

6. NON-IDEAL TRANSFER RESPONSE OF A MEASUREMENT PROCESS

Since certain uncompensated non-ideal effects will always remain, we will always have to deal with measurement errors.

Three different reasons can be distinguished for a non-ideal behaviour of the measurement process M.

Def.: **NON-IDEAL MEASUREMENT PROCESS M**

- Disturbance quantities effecting the measurement process M: $\mathbf{v}_M(t) \neq \mathbf{0}$
- Loading quantities of the measurement process M effecting the process P: $\mathbf{z}_M(t) \neq \mathbf{0}$
- Non-ideal transfer response of the measurement process M

- transfer response functions of viable paths $\neq \mathbf{I}$ $(\mathbf{u}_M(t) \rightarrow \mathbf{y}_M(t))$
- transfer response functions of disturbing paths $\neq \mathbf{0}$ $(\mathbf{v}_M(t) \rightarrow \mathbf{y}_M(t))$
- transfer response functions of loading paths $\neq \mathbf{0}$ $(\mathbf{z}_M(t) \rightarrow \mathbf{y}(t))$

Note that we have the choice between two completely different tools for avoiding a measurement error:

- by taking care that an effecting quantity equals zero
- by taking care that a transfer function of an effecting path equals zero

This result is of great importance for the realization of error-robust measurement processes and measurement procedures.

Fig. 4 clearly illustrates these facts and processes accordingly in a signal-effect diagram (Fig. 4). The laws describing the transfer functions and the quantities of interest are always those of Signal and System Theory, supplemented by laws of Stochastics and Statistics insofar as random quantities are involved.

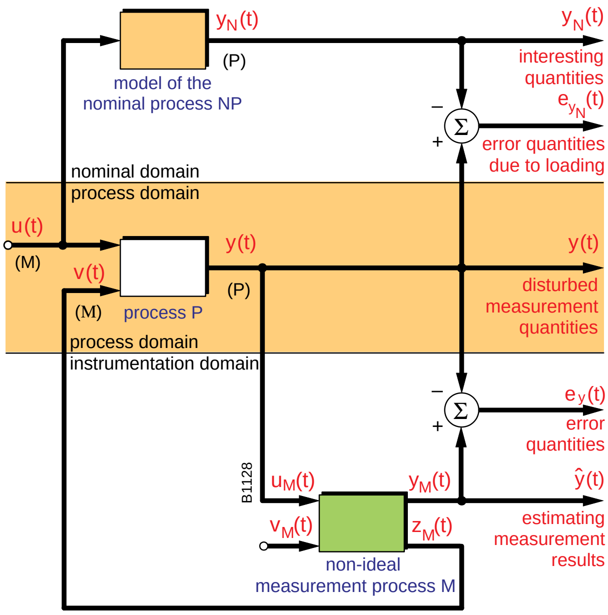

Fig. 4. Non-Ideal Measuring Process

In order to be able to properly discuss loading effects we have to postulate the model of the unloaded measurement process as nominal process NP, which yields reference quantities $\mathbf{y}_N(t)$ for the definition of loading errors $\mathbf{e}_{yN}$. These errors amount to usual measurement errors $\mathbf{e}_y(t)$.

7. ASSIGNMENT OF NUMBERS AND TRACEABILITY TO STANDARDS

Until now nothing has been said about the assignment of numbers und about the traceability of measurement results to reference quantities (standards). We thus have to refine the structures developed above.

The quantities **y**(t) of interest in a process P are purely physical quantities which are transformed by the sensor processes S into other physical quantities $\mathbf{y}_S(t)$. There are no numbers and units involved.

But in the background we have to refer to international conventions established for seven physical quantities (SI units). When we speak about the magnitude of physical quantities **y** of interest in a process P we always subconsciously refer to these conventions in a virtual rather than in a real process. No real comparison takes place.

However, the electrical quantities $\mathbf{y}_S(t)$ obtained via a sensor process S are compared with electrical reference quantities delivered by an Analog-to-Digital Converter (ADC) located within the front end of the computer, which serves as the reconstruction process R.

These electrical reference quantities are traceable to the international standards by accepted means (Fig. 5). The Analog-to-Digital Converter, as part of the reconstruction process R, assigns numbers and units. For conventional analog devices, the numbers are assigned by conventional graphic means called scales.

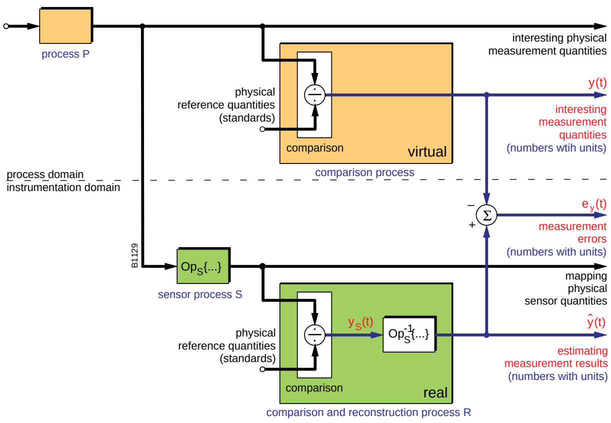

Fig. 5. Traceability to Standards

The definition of measurement errors $\mathbf{e}_y(t)$ as deviations only makes sense if these errors are defined as deviations between numbers that are assigned to physical quantities.

8. CALIBRATION AND IDENTIFICATION

Calibration and identification are identical processes. It is one of the most important goals in the measurement field to identify the transfer response of the sensor process S. Without a thorough identification (calibration) no reconstruction (inversion) is possible. Traceability to international units is only achieved by identification of the measurement process M by applying reference standards (Fig. 6).

For identification (calibration) of the measurement process M as a whole, the sensor process S alone or any other device are connected to known physical quantities (reference quantities, standard quantities) as input signals, which again can be traced back to international units with certain certified uncertainties.

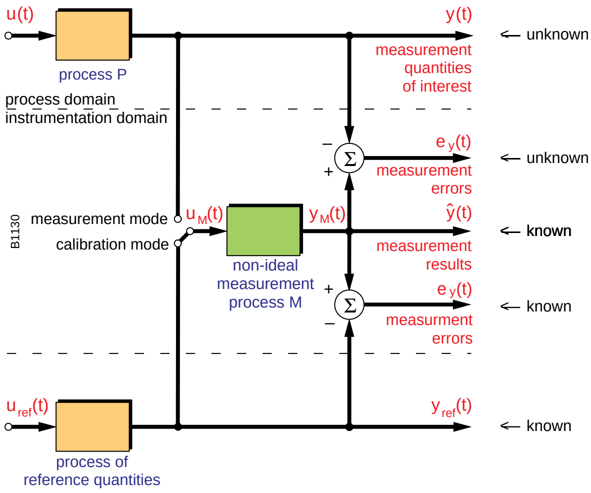

Fig. 6. Calibration Process

The transfer function of interest of process under test PUT, here the measurement process M, is evaluated by given input signals and resulting output signals.

In accordance with to the different possibilities of the non-ideal transfer response of the measurement process M mentioned above there are three paths which have to be identified (calibrated), the viable path, the disturbing path and the loading path. Normally only the viable path is considered.

Error and uncertainty analysis in a measurement procedure can be implemented only according to the results of such calibration procedures.

9. IMMEASURABLE QUANTITIES

If there are no reference quantities for quantities to be measured, they cannot be measured. We are familiar with several examples such as comfort, taste, intelligence, poverty, and so on, where we may observe physical reactions indeed, but no objective scales are known: Measurement, weakly defined [2].

We rather look at quantities within a process P, which are not measurable directly under normal conditions. We may ask whether we could find other quantities, which are better accessible for standard sensors instead. We only succeed, if there are causal relations between the desired, but immeasurable quantities **z**(t) and the measurable substitutes **y**(t) in the sense of cause and effect, described by the operator (model) Op_{SP} of the sub-process SP. There are two slightly different possibilities of interconnection, depending on the direction of the effect of the physical relationship.

The first case (Fig. 7.), furnishes an estimate of the desired, but immeasurable quantities **z**(t) by reconstruction (inversion) of the model Op_{SP}.

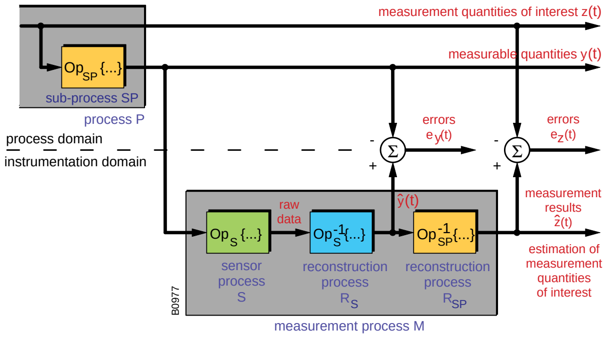

Fig. 7. Immeasurable Quantities

In the second case (Fig. 8) we apply the model Op_{SP} in parallel, which is the simplest form of an observer, the so-called open-loop observer. Many examples use this type of estimation without mentioning the observer solution applied.

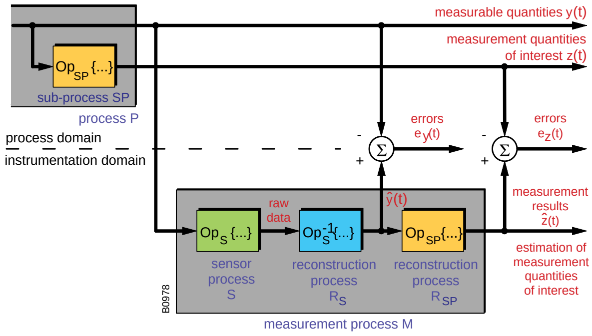

Fig. 8. Immeasurable Quantities

The measurement quantities of interest can be virtual quantities like random values and functions such as mean value, variance, correlation function, spectral density functions, and so on.

Both cases are also called indirect measurement. Anyway, Model-Based Measurement is still the main strategy, now including the model of the sub-process SP.

In both cases we have to handle two types of errors, (1) classical measurement errors $\mathbf{e}_y(t)$ that are solely attached to sensor process S and its reconstruction process RS, (2) extended errors $\mathbf{e}_z(t)$ belonging to the measurement process M as a whole including the model of sub-process SP. Again: Errors are virtual quantities, and we will never exactly know the quantities of interest.

This straightforward strategy is applicable as long as relations between $\mathbf{y}(t)$ and $\mathbf{z}(t)$ within process P remain simple and the inversion of the sub-model SP is granted. Otherwise a further step has to be taken, which leads to the well-known closed-loop observer (Luenberger Observer), which is called Kalman Filter when the quantities of interest are random variables.

10. CLOSED-LOOP OBSERVER

The main idea of a closed-loop observer is quite simple. We use a model of the process P in which the relations between the quantities of interest are described. The process model PM works in parallel to the process P and is fed by the same input information by a (hopefully ideal) measurement process MU (Fig. 9). The observed output $\mathbf{y}_{OB}(t)$ of the model can be compared to the measurable output $\mathbf{y}(t)$, delivered by the (hopefully ideal) measurement process MY. Resulting observation errors $\mathbf{e}_{yOB}(t)$ are used to drive all state variables $\mathbf{x}_{OB}(t)$ within the model PM in order to minimize the errors $\mathbf{e}_{yOB}(t)$ according to suitable optimising strategies.

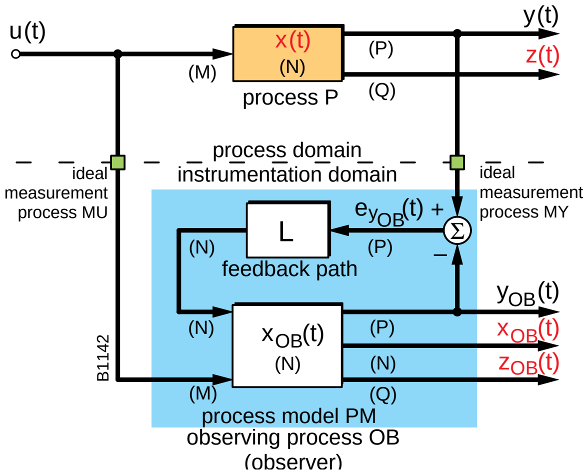

Fig. 9. Closed-Loop Observer

Since the process model PM is running within an instrument or a processor as a simulation process, the internal state variables $\mathbf{x}_{OB}(t)$ and the otherwise immeasurable output variables $\mathbf{z}_{OB}(t)$ can be accessed as estimates of the real quantities of interest $\mathbf{x}(t)$ and $\mathbf{z}(t)$, respectively.

One of the obvious results is a typical control loop whose properties are well defined by System Theory, such that a design of the feedback path (controller) can follow well-known rules. Of course there are some prerequisites, represented by the conditions of controllability and observability, terms that are well defined in System Theory, too.

At this point it is absolutely clear that sophisticated measurement is only realisable by appropriate knowledge, sometimes referred to as a priori knowledge about processes represented by suitable models.

11. CONTROLLED PROCESS, OBSERVING PROCESS – DUAL STRUCTURES

It is most interesting that a measuring process, looked at in a wide sense, includes a feedback path, normally attributed to controlled processes only. If we look at the structures of the controlled process and the observing process concerning their very essential kernels in state space representation, we notice a strange similarity, where K and L represent the feedback laws (Fig. 10; Fig. 11).

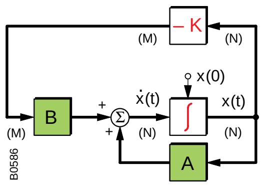

Fig. 10. Controlled System

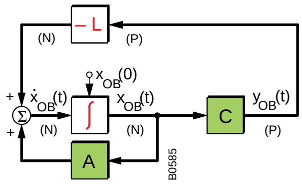

Fig. 11. Observing System

These structures can still be reduced (Fig. 12; Fig. 13):

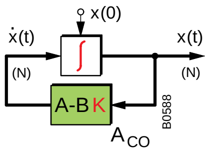

Fig. 12. Controlled System

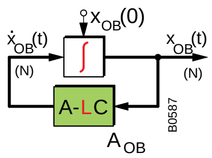

Fig. 13. Observing System

The only difference between a controlled and an observing system is the position of the feedback laws K and L in the feedback path. That means there is no identity. However, laws of Linear Algebra state that these two structures are dual structures with a very close relationship [4].

This result tells the measurement community that Signal, System and Control Theory developed by control people are able to deliver a vast amount of ideas, definitions, methods and tools, which can and should be applied directly within every measurement task. The same is true for the field of Stochastics and Statistics.

12. PROCESS UNDER TEST

Normally measurement is done in close contact with an active, autonomous process P at certain levels of working conditions. We need all sorts of information for different purposes. The input variables $\mathbf{u}(t)$ of such a process P do not depend on measurement needs.

However, there are certain types of processes, passive in behaviour, which are often called objects instead, where either all input signals or some additional input (driving) signals are applied according to measurement needs. To the signal-effect diagram we apply an additional driving process D and a function-generating process FG delivering one or more driving (test) signals $\mathbf{w}(t)$ in order to stimulate the process under test PUT (Fig. 14). These signals may be electrical, optical, acoustical, mechanical signals and so on. Parameters of those driving signals $\mathbf{w}(t)$ have to be chosen according to the temporal and spatial behaviour of the process under test and according to the needs of the measurement (test) procedure. Normally the function-generating process FG and the observing process OB are synchronised. A most prominent example of this measurement task is tomography; there the human body is the process under test.

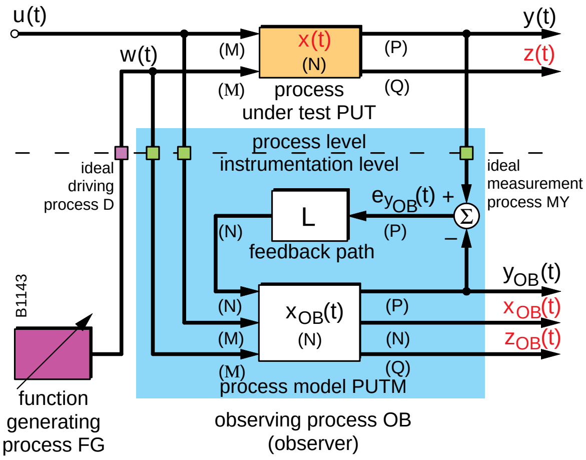

Fig. 14. Process Under Test

Obviously we are not confronted with new structures in this case; just two additional processes D and FG have been added. Besides, this case is almost identical with the calibration process, where the signals of the driving process D and function-generating process FG are reference signals or standards and the process under test PUT is a measurement process M. Especially the important case of dynamic calibration or identification of any given process is represented by this important structure.

In order to keep the diagram simple, disturbing quantities $\mathbf{v}_i(t)$ have been omitted everywhere. Also estimation errors $\mathbf{e}_x(t)$ and $\mathbf{e}_z(t)$ are not shown although they are important when discussing the quality and uncertainty of the measurement or estimation process, respectively.

13. QUALIFICATION PROCESS – MEASUREMENT UNCERTAINTY

One of the most disputed tasks in recent times has still to be included in the discussion of measurement structures, namely measurement uncertainty $\mathbf{u}_y(t)$ assigned to measurement results $\hat{\mathbf{y}}(t)$. It is obvious that this term concerns a process of its own, the quality assurance process Q following the measurement process M (Fig. 15). Without going into detail there are three main inputs to this process determining measurement uncertainty:

- Measurement results
- Errors and uncertainties of all preceding processes
- Demands concerning statistical certainties desired

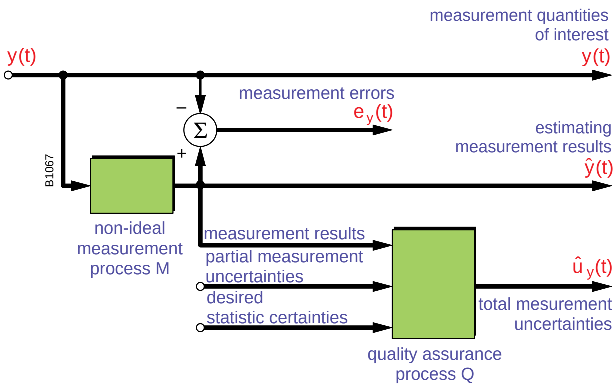

Fig. 15. Quality Assurance Process

Within the quality assurance process Q the models of all processes concerned have to be included in order to track the paths of all errors and uncertainties according to the laws of error propagation. Again, these laws themselves are laws of Signal and System Theory.

It is a known fact that the determination of the uncertainties is a troublesome and laborious task, which is seldom executed satisfactorily. Three main reasons for this:

- models of the processes concerned are often missing
- calibration (identification) procedures of the processes concerned are seldom complete
- separation of random deviations of the process P and the measurement process M within the measurement results can only be done by ambitious means such as Kalman filtering.

14. CONCLUSION

Structures in Measurement Science and Technology

- must be applicable to all tasks in measurement technology without exception
- must be clearly defined as a whole before terms in Measurement Science and Technology are fixed
- must comply with the structures of the subordinate sciences
- must comply with mathematical and logical operations
- should be illustratable by graphical means
- should be as simple as possible concerning details and as a whole
- should give a coherent and understandable impression to everybody in the field of measurement

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