

## MATEMATICAL METHODS APPLICATION IN METROLOGICAL ASSURANCE OF SPECIAL MEASUREMENT INSTRUMENTS AND INFORMATION MEASUREMENT SYSTEMS

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**Abstract** – The paper addresses the problem of metrological assurance for information measurement systems (IMS) where the conventional approach based on testing, calibration and verification can't be always applied. Metrological diagnostic check (MDC) is regarded as a basic instrument for metrological assurance of such IMS. The paper outlines a general approach for mathematical support of MDC at different stages of it's realization: IMS building, IMS certification and IMS usage

**Keywords** metrological diagnostic check, measurement redundancy, data processing algorithm

### 1. PROBLEM DEFINITION

The conventional approach for metrological assurance of measurement instruments (MI), based on testing and periodical verification of these instruments seems to be not effective for metrological assurance of special measurements instruments and information measurement systems (IMS). Here the IMS is used to denote the instruments for which the special requirement of minimizing the risk of invalid measurement information are expressly stated, when they are used in conditions that differ from normal (typical) conditions at testing, calibration and verification. The conventional methods of metrological assurance seem to be not appropriate because of following reasons:

- IMS is a unique one or there is a small sample of such instruments, hence, conventional testing isn't possible,
- IMS has a complex structure. The elements of this structure are sensors, the channels of transmission and conversion of measurement signals and software. Moreover this system is build directly on object (process), over which it provides the control and management.
- The conventional verification of IMS sensors isn't impossible for some years due to the conditions of their using.

The problem of working out the alternative approaches for the metrological assurance of SMI is of great importance. It's a problem of providing a validity of the measurement results received by IMS as well as the forecast of their residual life. The

analysis of the publications on the problem shows the advantages of implementation of *metrological diagnostic check* (MDC) as a basis for such approach. The essence of MDC consists in comprehensive using of all possible information about the process under the control. Moreover MDC provides special conditions for receiving additional information. The measurement redundancy is provided by using the relations between measurement parameters, that are based on physical or chemical laws, in data processing. Measurement redundancy can be also provided by duplication of sensors or duplication of some elements of the sensors. Here the using of this information means the analysis of the consistency of the information received from different sources, joint processing of the information and decision making on the validity of measurement results just in the process of IMS usage by application of a corresponding software.

Now the development of IMS is a state of art and application of mathematical methods and software differs greatly in different project realisation. It follows from the above said that the development of IMS requires the solution of the variety of mathematical problems of data analysis, data processing and measurement results interpretation at different stages of MDC realisation. Hence the methodological guidelines on the development of IMS are needed greatly. Such guidelines would reduce the expenses for their development and usage as well as would provide the possibility of their interchange. General approach for mathematical support for IMS at different stages of MDC realisation is outlined in the given paper

### 2. THE OBJECTIVES OF MDC

The main task of MDC is to provide the valid estimate of the quality of the results obtained. This task comprises the solution of the following subtasks:

- Control over metrological "good state" of sensors included to IMS. The stable relations between measurement characteristics (MCh) of sensors, that are established at their certification, are used during the control. The information about the interrelating changes in values of different

measurands can be also used in providing the control.

- Control over the conditions of IMS use with the aim to reveal situations that differ from typical ones realizing at the metrological certification when accuracy characteristics of results are determined
- Providing the safety of the information and the life of the IMS in extreme conditions by using the flexible data processing algorithms as well as by information and structure redundancy
- Providing the robust and safe estimates of quality of results in cases when there are some doubts in the validity of the initial information used or this information is restricted
- Correction of sensors MCh in the process of IMS usage
- Short-time forecast of sensors MCh stability and metrological “good state” of IMS.

The aim of the paper is to outline the mathematical tasks of MDC realization and to discuss some approaches for their solution. In this context two points should be taken into account: the IMS tasks (functions) and the kind of information to be processed by IMS. Below the tasks (functions) of IMS are listed with the increasing of their complexity (IMS complexity levels):

- Providing the valid quantitative estimates of the parameters characterizing the state of the object (process) under the investigation
- Providing the control over the object (process). Here the task of parameters estimation is followed by the control procedure (checking that measured parameters are within specified limits) based on these estimates. As parameter estimates are accompanied by associated uncertainties so as decision making should be followed by the estimate of its validity
- Providing the more complex control or diagnostic. In simple case the set of possible parameter values are divided into three subsets that correspond to the following situations: (1) the process is within the specified limits, (2) the crossing of specified limits can be forecasted in nearest future, (3) the process is out the specified limits
- Management over the process that is based on the solution of the above tasks and imply some actions for providing the process to be within the specified limits

Information redundancy, that is internal characteristic of IMS, assume the processing different kind information concerning the object (process) as well as the elements of IMS itself. The following kind of information should be mentioned:

- Measurement results, influence qualities, corrections
- Reference data, typical and “dangerous” situation for the object and IMS state

- Mathematical models: probability distribution laws, differential and algebraic equations systems, functional operators and so on
- Expert knowledge, qualitative estimates

So we meet the problem of joint processing of different kind information. The conventional approach based on theory of probability and math statistics is sometimes inappropriate. So we are forced to apply more flexible approaches such as fuzzy sets and interval calculations. Actually we need the alternative approach for measurement error estimation. The modern approach for uncertainty evaluation can be regarded as an interim step in this direction. It allows to formalise all kind of incomplete, uncertain quantitative information by using corresponding probability distribution functions (pdf). Hence this approach seems to be useful for data processing in IMS of first level of complexity. It allows to extend easily the methods of measurement results error estimation to the methods of accuracy estimation of the procedures of quantitative assessment that use measurement results as well as additional quantitative information. But this approach isn't appropriate for processing the qualitative information as well as for mathematical support of IMS of higher level of complexity.

### 3. MDC REALISATION

Below the problems of MDC realisation are considered at stages of IMS building, of IMS certification and of IMS usage. The IMS of first level of complexity is only considered as a basic one for IMS of higher levels.

#### 3.1 MDC realisation at the stage of IMS building.

At the stage of IMS building it's necessary to provide principle possibility for MDC realisation. It's achieved by unique technical solutions and by corresponding software that allows to safe, to transmit, to convert all possible information in order to get valid estimates of measurement parameters characterising the state of the object. Among technical solutions the following should be mentioned: the possibility to use test signals for self-calibration, the choice of sensors and of their position and others. It should be stress that MDC realisation is closely connected with the particular IMS, which is often build for the control over the concrete object (process).

The software of IMS comprises:

- A. Database that includes the following information (see also tabl.1):
  - Metrological characteristics of sensors, of the channels of transmission and conversion of measurement signals; the structure of IMS; stable relations between sensors MCh, which are used for the internal control of IMS “good-state”.
  - Measurement model, data obtained from sensors, relations between measured parameters, information about possible correlation in

measurement data obtained by different sensors. The choice of the parameters to be measured is beyond our consideration. In order to formalize the problem of parameter choice we should analyze the further usage of the results obtained, answer the question where and how these results will be used. In other words we should consider the IMS of higher level of complexity.

- Conditions of IMS usage: ranges of measurand values and of influence parameters values.

TABLE I. Database content

Element of database	Description
Object	Measurand values (parameters): $\{p_1, \dots, p_M\}$ influence quantities: $\{q_1, \dots, q_K\}$
Measurement conditions	Ranges of measurand values: $F_l^- < f_l(p_1, \dots, p_k) < F_l^+$ , $(p_1, \dots, p_k) \subset (p_1, \dots, p_M)$ , $l = 1, \dots, L$ Ranges of influence quantities values $G_j^- < g_j(q_1, \dots, q_k) < G_j^+$ , $(q_1, \dots, q_k) \subset (q_1, \dots, q_K)$ , $j = 1, \dots, Q$
Measurement model	$Q_i(p_1, \dots, p_M, q_1, \dots, q_K) = 0$ $i = 1, \dots, M$
Relations between measurand parameters	$R_i(p_1, \dots, p_L) = 0$ $i = 1, \dots, r$
Metrological characteristics of sensors	Maximum permissible errors $\Delta_i$ , Calibration curve $\varphi(x)$ Response function $h(t)$ Response time $\tau$

B. Data processing algorithms. Here algorithms of data processing are understood in broad sense, as algorithms for conversion of information that contains measurement data as well as all available information from database. These algorithms include:

- Algorithms for processing of initial data received from sensors. Here the methods of stochastic signal processing, of random noise theory, of correlation and spectral analysis are applied
- Algorithms for checking the consistency of the measurement results of the same measurand, received from different sensors. Here the methods for testing statistical hypotheses are used (Grabs statistics,  $\chi^2$ -statistics,  $F$ - statistic and others)
- Algorithms for joint processing data received from different sources. Here statistical methods for

parameter estimation (weighted estimates, processing of truncated data) are applied

- Algorithms for parameter estimation taking into account the relations between measurands. Here the methods for solution of systems of equations, simulation statistical methods, theory of statistical estimation, variation theory are applied
- Algorithms for database management. Two types algorithms should be indicated: (1) algorithms for data exchanging that safe the measurement results received from sensors and get out these results from data base when it's required and (2) algorithms for correction of the data in the database. The correction can be made on the results of self-calibration, consistence check, additional expert information and so on.

### 3.2 MDC realisation at the stage of IMS certification

As it's mentioned above the conventional methods for measurement instrument testing, calibration and certification are often not appropriate for metrological assurance of IMS. At IMS certification the following experimental and analytical tasks should be solved:

- Determination of metrological characteristics of sensors on the base of the results of measurement experiment designed in appropriate way. For data processing the methods of statistical theory of experimental design, of correlation and regression analysis, of factorial analysis, of time series analysis, of solution of equation systems are used.
- Calculation of IMS metrological characteristics on the base of known metrological characteristic of the IMS elements and IMS structure (algorithms of information conversion)
- And/or determination of IMS metrological characteristics by simulation. It implies the choice of typical model of input signals and the choice of the parameters of simulation experiment to provide the required accuracy of MCh estimation.

### 3.3 MDC realisation at the stage of IMS usage

MDC realisation at the stage of IMS usage is characterised by two important aspects: first, application of flexible algorithms for data processing and, second, checking and correction of MCh of sensors used in the process of IMS usage. To illustrate above said let's consider a simple example.

*Example.* The same measurand value is determined using the measurement results received by three sensors. It's important to stress that these sensors realise different physical laws or at least the possible changes in their calibration coefficients due to input affects will be different.  $X(t)$  denotes the input signal as  $y(t)$  denotes the output signal. The information of calibration coefficient determined at initial calibration and associated uncertainties

$\{(k_1, u(k_1)), (k_2, u(k_2)), (k_3, u(k_3)), k_1 \neq k_2 \neq k_3\}$  is available in database as well as uncertainties of registration of output signal  $\{u(y_1), u(y_2), u(y_3)\}$ . Below the stages of data evaluation and MDC realisation are considered:

- (1) The estimates of input measurand value are calculated using calibration coefficients and output measurement results. The associated uncertainties are also evaluated in relative form:

$$x_i = \frac{y_i}{k_i} \frac{u^2(x_i)}{x_i^2} = \frac{u^2(y_i)}{y_i^2} + \frac{u^2(k_i)}{k_i^2} \quad i = 1, 2, 3 \quad (1)$$

- (2) The consistence check is made. Usually  $\chi^2$ -statistic is used. In the case considered the number of data is small so the analysis of difference between pairs of data is suggested:

$$|x_i - x_j| \leq 2\sqrt{u^2(x_i) + u^2(x_j)} \quad (2)$$

- (3) If the above requirement is satisfied for every  $i, j$ , then the estimate of the measurand value and associated uncertainty are given by

$$\bar{x} = \sum_{i=1}^3 \omega_i x_i \quad \omega_i = \frac{u^{-2}(x_i)}{\sum_{j=1}^3 u^{-2}(x_j)} \quad u^2(\bar{x}) = \frac{1}{\sum_{j=1}^3 u^{-2}(x_j)} \quad (3)$$

- (4) If the requirement (2) isn't satisfied for all  $i, j$  the two situation are possible:

(4.1) Two estimates are consistent with the third one but they aren't consistent with each other. In this case the weighted mean is used:

$$\bar{x} = \frac{1}{4}x_1 + \frac{2}{4}x_2 + \frac{1}{4}x_3$$

$$x_1 < x_2 < x_3 \quad (4)$$

$$u^2(\bar{x}) = \frac{1}{16}(u^2(x_1) + u^2(x_3)) + \frac{1}{4}u^2(x_2)$$

(4.2) If one of estimates isn't consistent with others then data obtained by this sensor are rejected from consideration for some period and the possible reasons of inconsistency are analysed. The measurand value is estimated using only two consistent measurement results

$$\bar{x}' = \omega_1 x_1 + \omega_2 x_2 \quad (5)$$

There could be two reasons for the initial estimates inconsistency: the drift of calibration coefficient  $k_3(t)$  or increasing of uncertainty of output signal registration  $u(y_3)$ . To recognise these situations and to make corresponding corrections in database the following procedure is suggested:

- further current estimates (about 10-15) of measurand are received without using the data of the third sensor from (5) :  $\{\bar{x}'(t_i)\}_1^{15}$  (6)

- the following differences are analysed:

$$\left\{ \bar{x}'(t_i) - \frac{y_3(t_i)}{k_3} \right\}_{i=1}^{15} \quad (7)$$

where  $y_3(t_i)$  are results of registration of output signals for third sensor

- if the changes in sign of the above differences has random nature, then the reason of the above estimates inconsistency is registration uncertainty  $u(y_3)$  underestimation. Hence  $u(y_3)$  should be enlarge to provide the satisfaction with the requirements (2).
- if the changes in sign of the above differences has systematic nature, then the reason is the drift of calibration coefficient  $k_3(t)$ . The improved estimate of  $k_3(t)$  is given by:

$$k_3(t_i) = \frac{y_3(t_i)}{\bar{x}'(t_i)} \quad \tilde{k}_3 = \frac{\sum_{i=1}^{15} k_3(t_i)}{15} \quad (8)$$

$$\frac{u^2(\tilde{k}_3)}{\tilde{k}_3^2} = \frac{u^2(y_3)}{y_3^2} + \frac{u^2(\bar{x}')}{\bar{x}'^2}$$

#### 4. CONCLUSION

The paper outlines a general approach for mathematical support of MDC as a effective instrument of IMS metrological assurance. This approach implies a solution of the sequences of tasks where the solution of the previous one becomes the basis for the consideration of the next one:

- Determining the tasks of MDC in accordance with a kind of information to be processed and functions of IMS.
- Formulation the problems of data modeling and of data processing relating to different stages of MDC and consideration the possible mathematical approaches for their solution
- Choice of mathematical approaches to the typical tasks of IMS metrological assurance
- Working out the data modeling and data processing algorithms for typical tasks of IMS metrological assurance
- Realization of the chosen data processing algorithm for a metrological assurance of a particular IMS.

The paper addresses two of the above problems. The basic ideas of MDC are illustrated in simple example.

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