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MODELLING OF THE COVERAGE FACTOR IN INDIRECT MEASUREMENTS

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Abstract – The results of examining error evaluation of the coverage factor in indirect measurements have been presented in the paper. To examine coverage factor value, the use of mathematical models, for analysed convolution of component distribution, has been presented. The knowledge of the coverage factor characteristics for the convolution of two Student's distributions and two rectangular distributions was used for the examination.

Keywords expanded uncertainty, coverage factor.

1. INTRODUCTION

Each evaluation of the expanded uncertainty *U* requires the choice of an approximate evaluation method of the coverage factor. In the methods suggested by the international document [1] it is necessary to decide whether the evaluated factor shall approach the factor for a normal distribution or for Student's distribution. Usually the sample size is the decisive factor in the choice. However, how the number of standard component uncertainties influences the choice of the evaluation method is unknown.

The basis for estimating the accuracy of applied approximate method of the estimation of expanded uncertainty is the assumption on the necessity of determining the method, which could be regarded as the exact one. An essentially appropriate concept was adopted, which is taken into consideration, that the method based on the command of the convolution of component distributions may be regarded as an exact method.

Due to complexity and time-consuming character of computing the convolution of many distributions of components, the results of such computing are, in general, hardly ever published. Therefore, approximate methods are generally accepted and recommended.

The present paper continues research in tendencies of changes in error evaluation of the coverage factor $k(\alpha)$ by means of approximate methods with an increasing number of component uncertainties.

As the analysis of precision of approximate methods of evaluation of expanded uncertainty can be regarded in most cases as the analysis of evaluation errors of the coverage factor, in the present paper the analysis of precision of the coverage factor evaluation in case of indirect measurements was conducted, which is characterized by a greater number of standard uncertainties.

Results for a convolution of two Student's distributions and two rectangular distributions are presented in this paper.

2. CHARACTERISTICS OF THE CONVOLUTION OF TWO STUDENT'S DISTRIBUTIONS AND TWO RECTANGULAR DISTRIBUTIONS

A measuring event, which utilizes a convolution of two Student's distributions and two rectangular distributions is an example of indirect measurement carried out by means of two measuring devices, which, in case of repeated measurements, show a scatter of results, and the number of measurements is small (n<30).

Therefore, four standard uncertainties are analyzed: two type-B standard uncertainties, which reflect a standard deviation of rectangular distributions and two type-A standard uncertainties, which reflect a standard deviation of Student distribution.

On the basis of the developed analytical description of coverage factors in case of the analyzed convolutions one is able to identify all parameters, whose function are the factors. One is able to demonstrate that a coverage factor for the convolution S*S*R*R, from now on referred to as factor $k_{SSRR}(\alpha)$ is a function of 6 variables [2]: probability α, number of degrees of freedom $m_1 = n_1 - 1$ and $m_2 = n_2 - 1$ first and second Student's distributions and the ratio of standard uncertainties η_S, η_R and η (1):

$$k_{SSRR}(\alpha) = f(\alpha, m_1, m_2, \eta_S, \eta_R, \eta), \qquad (1)$$

where:

- $\eta_S = \dfrac{u_{A_1}}{u_{A_2}}$ is the ratio of standard uncertainties of type A,
- $\eta_R = \dfrac{u_{B_1}}{u_{B_2}}$ is the ratio of standard uncertainties of type B,

- $\eta = \frac{u_A}{u_B} = \frac{\sqrt{u_{A_1}^2 + u_{A_2}^2}}{\sqrt{u_{B_1}^2 + u_{B_2}^2}}$ is the ratio of combined standard uncertainties of type A to type B.

3. COMPUTATIONAL RESULTS OF THE COVERAGE FACTOR FOR THE CONVOLUTION

Calculations were executed for two selected probability values $\alpha = 0.95$ and $\alpha = 0.99$, for small values m and for the value series η, ranging from 0.1 to 10.

Matlab program was used for the calculations and the following were assumed:

- approximation accuracy of the probability range α over the variable k, ε =1e-4,
- the number of integration ranges in the Simpson's method of integration 300,
- multiple j =20.

Computational results are presented in table 1.

TABLE I. Values of the coverage factor $k_{SSRR}(\alpha)$

$1/\eta$	η	α = 0.95		α = 0.99	
		$m_1 = 3$	$m_1 = 9$	$m_1 = 3$	$m_1 = 9$
		$m_2 = 3$	$m_2 = 9$	$m_2 = 3$	$m_2 = 9$
10		1,9188	1,9047	2,2531	2,2203
5		1,9633	1,9125	2,3875	2,2688
4		1,9950	1,9200	2,4812	2,3031
3		2,0563	1,9350	2,6688	2,3625
2		2,2125	1,9752	3,1141	2,4844
1	1	2,6361	2,0875	4,2031	2,7656
	2	3,0125	2,1875	5,1000	2,9906
	3	3,1313	2,2175	5,3625	3,0578
	4	3,1781	2,2300	5,4500	3,0800
	5	3,2000	2,2375	5,5188	3,0938
	10	3,2350	2,2450	5,5625	3,1125

Fig.1 and Fig. 2 present hypothetical characteristics of coverage factor $k_{SSRR}(0.95)$ $k_{SSRR}(0.99)$ for m_1=m_2=3 and for m_1=m_2=9, η_S=1, η_R=1 in the function of the ratio of standard uncertainties $\eta = u_A / u_B$ and its converse.

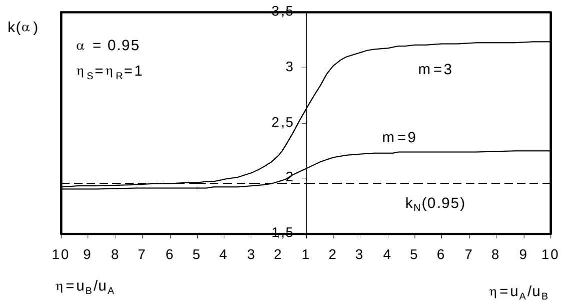

Fig. 1 Characteristics of coverage factor $k_{SSRR}(0.95)$ for m_1=m_2=3 and m_1=m_2=9 in the function of the ratio of standard uncertainties η and its converse.

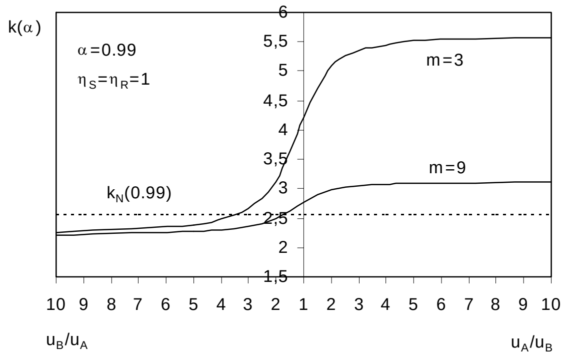

Fig. 2 Characteristics of coverage factor $k_{SSRR}(0.99)$ for m_1=m_2=3 and m_1=m_2=9 in the function of the ratio of standard uncertainties η and its converse

In this situation both samples have the same number of degrees of freedom and none of the component standard uncertainties of type A and of type B is a domineering one. Broken line show characteristics of the coverage factor $k_N(0.95)$ and $k_N(0.99)$ for a normal distribution.

In accordance with the central limit theorem, the characteristics of the coverage factor $k_{SSRR}(\alpha)$ clearly trend to approach the value of the factor $k_N(\alpha)$ as the sample size increases. The phenomenon is observed in the domain where u_A>u_B, further called domain A. Whereas in the domain where u_B>u_A further called domain B, the influence of the sample size is much smaller and fades as the value of the ratio u_B>u_A increases.

In accordance with the assumption that the value of the coverage factor for a tested convolution of distributions of components can be treated as a precise value, the absolute value of evaluation error with the approximate method shall be defined as:

$$\delta = \frac{|k(\alpha) - k_{SSRR}(\alpha)|}{k_{SSRR}(\alpha)}, \qquad (2)$$

where $k(\alpha)$ is the coverage factor evaluated by means of the approximate method, whereas $k_{SSRR}(\alpha)$ is a ratio determined on the basis of the known convolution of distributions of components.

According to the conducted analysis, the application of approximate methods of evaluation of expanded uncertainty in many cases results in exceeding the assumed value of evaluation error coming from equation (2), equal to 20% [3, 4]. It is obvious that convolutions of distributions are usually not known and obtaining them by means of calculations is a time consuming process. Therefore, attempts were made to improve precision of the approximate method by creating mathematical models describing coverage factors for the analyzed convolutions of distributions of components.

There exist several ways of describing the tested sample curves presented in Fig. 1 and Fig. 2. If a particular standard uncertainty, of type A or type B, prevails, the value of the coverage factor converges appropriately with the value of coverage factor $k_N(\alpha)$ for a normal distribution or $k_R(\alpha)$ for a rectangular distribution [2,3].

A measuring situation, where the two uncertainties (type A and B) are comparable, is of particular importance. Therefore, simulation of the coverage factor $k_{SSRR}(\alpha)$ was made concerning changes in the value of the ratio of standard uncertainty $\eta=u_A/u_B$ from 0.5 to 2. The equation describing the model $k_m(\alpha)$ of the coverage factor $k_{SSRR}(\alpha)$ is the relationship:

$$k_m(\alpha) = a + bx \,. \tag{3}$$

Numeric values of parameters a and b were determined taking advantage of the least square method. Simulation was conducted for two selected probability values α=0.95 and α=0.99 and for various numbers of degrees of freedom *m*. On the basis of obtained results, the value of simulation was determined, calculated in accordance with the relationship (2), where coverage factor $k(\alpha)$ assumes values $k_m(\alpha)$.

Fig. 3 and Fig. 4 present hypothetical characteristics of absolute error values δ of factor estimations $k_m(\alpha)$ for probability $\alpha = 0.95$ and $\alpha = 0.99$, η_S=1, η_R=1, m_1=m_2=3 and for m_1=m_2=9, in the function of the ratio of standard uncertainties $\eta = u_A / u_B$ and its converse.

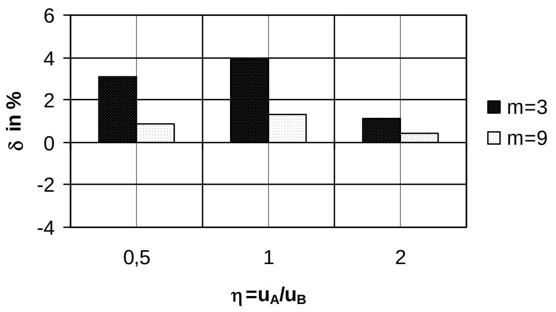

Fig. 3 Absolute error values δ of factor estimation $k_m(0.95)$ in the function of the ratio of standard uncertainties $\eta=u_A/u_B$ and its converse.

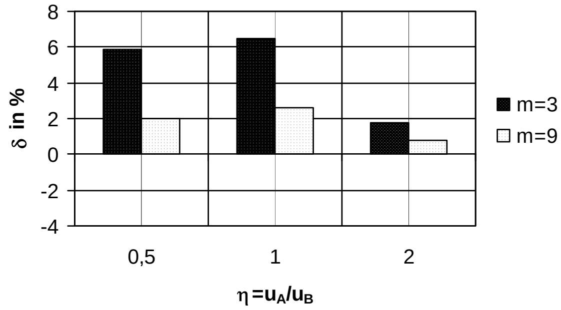

Fig. 4 Absolute error values δ of factor estimation $k_m(0.99)$ in the function of the ratio standard uncertainties $\eta=u_A/u_B$ and its converse

4. CONCLUSIONS

The determined simulation error values rise with an increase in probability α, and with a decrease in the number of degrees of freedom *m*.

The maximum value of an error for the analyzed probabilities α and the number of degrees of freedom *m* does not exceed 6.5%, which constitutesa considerable improvement in precision $k(\alpha)$ evaluation over the values obtained by means of the approximate methods presented in papers [2, 3, 4].

The developed model descriptions have considerably reduced the error of approximate evaluation of expanded uncertainty estimation for a common measure event, which is also considered in the present paper.

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