10th IMEKO TC7 International Symposium
June 30–July 2, 2004, Saint-Petersburg, Russia

MEASURING INSTRUMENTS IN ECONOMICS

Marcel Boumans

Department of Economics, University of Amsterdam, Amsterdam, The Netherlands

Abstract – The measurement set-ups for measuring economic phenomena are economic systems that cannot be controlled or shielded. Economists have to infer the desired quantitative facts about the phenomenon under study from the data generated by these systems. Accurate measurement results are therefore not obtained by adjustments and improvements of the measurement system, but must result from building accurate models of these systems. By taking into account the background noise when modelling the phenomenon, the impossibility of control for bias and error is compensated. As a result, accuracy is obtained by first finding accurate representations of the relevant economic system incorporating both phenomenon and its environment. Secondly, these representations are applied in the methods for achieving precision of the measurement results. Because the unobserved facts about the relevant economic phenomena are inferred from data generated by the underlying system, measurement in economics is always associative in which the inferences are based on the models that function as representations of this association. Section two accounts for this kind of measurement. The principles of a broad range of modelling strategies in economics will be discussed in section 3. The general principles of making measurement results precise will be explained in section 4.

Keywords: model, passive observation, ceteris neglectis

1. INTRODUCTION

In empirical economics, models are built to provide facts about phenomena. Though phenomena, like business cycles, GDP and unemployment, are the objects of explanation and prediction of economic theories, these theories usually do not generate quantitative facts about them. For example, theories tell us that capitalist economies give rise to business cycles, but not the duration of recovery. These facts about economic phenomena are not directly observable, but nevertheless have to be converted from data, i.e. observations (see [1] for a elaborate discussion of the distinction between data and facts about phenomena). If data are arranged in a considered way, meeting specific requirements, they provide reliable quantitative information about phenomena. In economics, models function as such devices. In other words, the measuring instruments of economists are models, that are located on the theory-world axis mediating between facts about phenomena and data, see Fig. 1. The dotted line in Fig. 1 represents the indication that theories do not provide (quantitative) facts about phenomena.

Theory
↓
Phenomenon
....
Facts about the phenomenon
↑
Measuring Instrument
↑
Data

Fig 1. Position of measuring instrument on theory-world axis

There is, however, one crucial feature of models that has profound consequences for achieving accurate measurement results. Namely, models are ideal systems, and so all kinds of material interventions to make measuring instruments reliable, such as control, shielding and insulation, are not possible for mathematical objects. To function as an accurate measuring instrument, a model should include a representation of an as far as possible invariant relationship between facts and data. Though invariance is hard to find in an economic world of constant flux, this paper will explore some strategies of finding representations of invariant relationships. These strategies are based on works of economists and econometricians who suggested influential solutions to this problem of invariance.

Before these works will be discussed, we first, in section 2, have to clarify the concept of a model as an ideal measuring instrument. Therefore, we will use Heidelberger's [2] correlative interpretation of the representational theory, which is based on Fechner's correlational theory of measurement. In Heidelberger's work a correlation is linked to a measuring instrument ensuring the invariance of this relationship. Generally for material instruments this can be done by insulation, or in other words by taking care that *ceteris paribus* requirements are fulfilled. In economics, we often cannot create *ceteris paribus* environments and so have to search for invariant relationships in the open air. Do constellations in nature exist that can be used as measuring instruments and how do they look like? The works of Trygve Haavelmo [3], Herbert Simon [4], Milton Friedman [5], and Robert Lucas [6] show that we do not necessarily need *ceteris paribus* environments for accurate measurements.

2. MEASURING INSTRUMENT

Heidelberger [2] criticizes the representational theory of measurement of having turned too much into a pure mathematical discipline, leaving out the question of how the mathematical structures gain their empirical significance in actual practical measurement. The representational theory lacks concrete measurement procedures and devices. Heidelberger argues for giving the representational theory a 'correlative interpretation'. This interpretation of the representational theory is based on Fechner's theory of measurement. Fechner had argued that the measurement of any attribute x generally presupposes a second, directly observable attribute y and a measurement apparatus A that can represent variable values of y in correlation to values of x. The correlation is such that when the states of A are arranged in the order of x they are also arranged in the order of y. The different values of y are defined by an intersubjective, determinate, and repeatable calibration of A. They do not have to be measured on their part. The function that describes the correlation between x and y relative to A (underlying the measurement of x by y in A) is precisely what Fechner called the measurement formula. Normally, we try to construct (or find) a measurement apparatus that realizes a 1:1 correlation between the values of x and the values of y so that we can take the values of y as a direct representation of the value of x. The correlative interpretation of measurement implies that the scales of measurement are a specific form of indirect scales, namely associative scales.

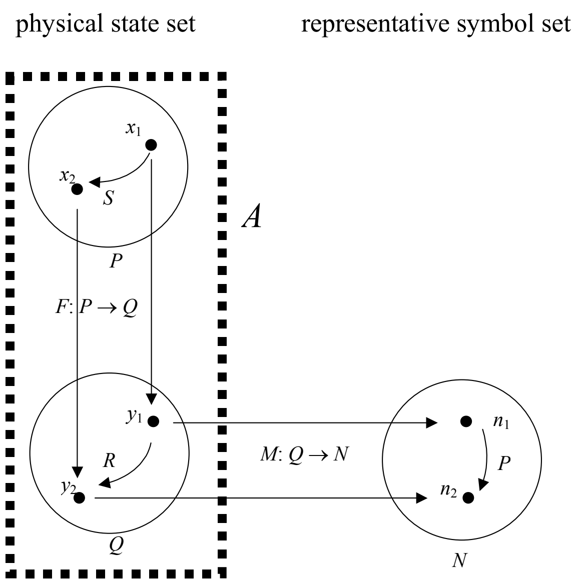

Fig 2. Correlative interpretation of measurement

The association is indicated by F in Fig. 2. An associative scale for the measurement of x is then defined by taking

$$n = M(y) = M(F(x, OC)) \quad (1)$$

where $M(y)$ is the measure of y on some previously defined scale, and F is the correlation between x and y that also involves other influences indicated by OC. OC, an acronym of 'other circumstances', is a collective noun of all the other factors that could have an influence on y.

The central idea of correlative measurement is that in measuring any attribute x we always have to take into account its empirical lawful relation to (at least) another attribute y. To establish this relation we need a measurement apparatus or experimental arrangement, A. The correlative interpretation gives back to measurement the idea that it concerns concrete measurement procedures and devices, taking place in the domain of the physical state sets as a result of an interaction between P and Q. A correlation should not be considered as a numerical law, because that would require independent measurements of both x and y. For each of the variables in a numerical law there must exists a measurement apparatus based on a correlation before a numerical law can be established and tested. The mapping of y into numbers, $n = M(y)$, is not the result of measurement but is obtained by calibration.

As a consequence of this interpretation of measurement, y_i ($i = 1, \ldots, k$) are (indirect) observations of x to be used for the measurement of x. Each observation involves observational errors, ε_i:

$$y_i = F(x, 0) + \varepsilon_i \quad (i = 1, \ldots, k) \quad (2)$$

y is an accurate observation if we are able to reduce the error term considerably. This error term, representing noise, reflects the operation of many different influences. One way of obtaining accuracy is by shielding the apparatus from these other influences, that is by taking care that *ceteris paribus* conditions are imposed. To work out this idea, it is useful to rewrite (2) as an expression of how x and possible other circumstances (OC) influence the observations. Note that the observations y can only provide information about the measurand x when variation of x influences variation of y:

$$\Delta y = \Delta F(x, OC) = \frac{\partial F}{\partial x}\Delta x + \frac{\partial F}{\partial OC}\Delta OC \quad (3)$$

Comparing (3) with (2), one can see that the second term, $\partial F/\partial OC \cdot \Delta OC$, represents the noise term, ε.

Equation (3) shows that accuracy can be obtained by imposing *ceteris paribus* conditions: $\Delta OC \approx 0$, or by imposing even stronger *ceteris absentibus* conditions: $OC \approx 0$. However, both kinds of conditions imply control of circumstances. Generally in measurement, even in a laboratory, there are always circumstances one cannot control. Fortunately, a measuring instrument can also be accurate when it is designed, fabricated or used in such a way that the influences of all these uncontrollable circumstances are negligible. Using the expression of (3), this means that $\partial F/\partial OC \approx 0$. In other words, a measuring device should be constructed and used such that it fulfils the *ceteris neglectis* condition. In economics we generally cannot furnish the environment for measurement purposes, so we have to look for natural measuring instruments satisfying *ceteris neglectis* requirements. Moreover, we are not able to carry out controlled measurements and so will depend on the information obtained by passive observation.

The problem of passive observation is that it is not possible to identify the reason for an (disturbing) influence, say z, being negligible, $\partial F/\partial z \cdot \Delta z \approx 0$. We cannot distinguish whether its potential influence is very small, $\partial F/\partial z \approx 0$, or whether the factual variation of this factor over the period under consideration is too small, $\Delta z \approx 0$. The variation of z is determined by other relationships within the economic system. In some cases a, virtually, dormant factor may become active because of changes in the economic structure elsewhere. However, deciding whether a factor should be accounted for in the measurement relationship between y and x should not depend on such changes. The relationship should be autonomous with respect to structural changes elsewhere.

In practice, the difficulty in economic research does not lie in establishing simple relations, but rather in the fact the empirically found relations, derived from observation over certain time intervals, are still simpler than we expect them to be from theory, so that we are thereby led to throw away elements of a theory that would be sufficient to explain apparent 'breaks in structure' later. This is what Haavelmo [3] called the problem of autonomy of economic relations. Some of these relations have very little autonomy because their existence depends upon the simultaneous fulfilment of a great many other relations. Autonomous relations are those relations that could be expected to have a great degree of invariance with respect to various changes in the economic structure. However, this kind of invariance should not be equated with the observable degree of constancy or persistence of a relation. The degree of autonomy refers to a class of hypothetical variations in structure, for which the relation would be invariant, while its actual persistence depends upon what variations actually occur.

3. ECONOMIC MODELING

When one has to deal with a natural measuring system F which we therefore only can observe passively, the measurement procedure is to infer from the observations y nature's design of this measuring system to determine the value of the measurand x. In other words, we first have to find an adequate representation A of system F before we can estimate the value of x. In economics, both system and its representation are usually assumed to be linear operators. From now on, therefore, capitals like A and F denote matrices and small Latin letters without subscripts, like x and y denote vectors. We assume the following economic system:

$$x_{t+1} = Fx_t + \eta_{t+1} \tag{4}$$

where F is a $N \times N$ transition matrix of state variables and η denotes the rest of the world having only a negligible to none influence on x. The subscript t indicates time. Observations y are proxies of x:

$$y_t = x_t + \varepsilon_t^o \tag{5}$$

where ε_t^o denotes observational error. The economic observations used for the measurement of x_t are time series. So, to measure x_t a model M has to be specified of which the observations y_t function as input and $\hat{x}_t$, the estimation of x_t, as output:

$$\hat{x}_t = M(y_t; A) \tag{6}$$

where $A = (\alpha_{ij})$ are the parameters of the model. $\hat{\varepsilon} = \hat{x} - x$ denotes measurement error. The aim is accuracy by reducing this error term. Because we are dealing with economic systems in which generally many factors are involved, we denote the different system variables by the subscripts i and j. We will now discuss the principles of several modelling strategies to be found in economics.

3.1. System-of-equations approach

Let start with the simplest model, namely take as prediction of next period y_{1t+1} the observation at time t.

$$\hat{y}_{1t+1}^{I} = y_{1t} \tag{7}$$

Then, initially the prediction error is as large as the observational error. The strategy to reduce the prediction error is to add to the model a variable suggested by theory each time the model predictions can be made more accurate. So, we compare (7) with

$$\hat{y}_{1t+1}^{II} = y_{1t} + \alpha y_{2t} \tag{8}$$

to verify whether (8) leads to an improvement of the predictions:

$$\left\| y_{1t+1} - \hat{y}^{II}_{1t+1} \right\| < \left\| y_{1t+1} - \hat{y}^{I}_{1t+1} \right\| \quad (9)$$

where $\|\cdot\|$ denotes a norm. The specification of the norm depends on the problem field at hand, but will be in general the mean squared error discussed below.

The general strategy is to compare two models, I and II, and choose the one that provides the best predictions.

$$\hat{y}^{I}_{it+1} = \sum_{j=1}^{n} \alpha^{I}_{ij} y_{jt} \ \ (i = 1, \ldots, n) \quad (10)$$

$$\hat{y}^{II}_{it+1} = \sum_{j=1}^{n+1} \alpha^{II}_{ij} y_{jt} \ \ (i = 1, \ldots, n+1) \quad (11)$$

If $\left\| y_{it+1} - \hat{y}^{II}_{it+1} \right\| < \left\| y_{it+1} - \hat{y}^{I}_{it+1} \right\|$ for the majority of these error terms (i = 1, …, n), choose model II. Note that for each additional variable the model is enlarged with an extra (independent) equation.

If the prediction errors are reduced by taking into account more and more variables then the measurement errors are reduced. To see this compare two different expression for y_{t+1}:

$$\begin{aligned} y_{t+1} &= \hat{y}_{t+1} + \varepsilon^{A}_{t+1} = Ay_t + \varepsilon^{A}_{t+1} \\ &= Ax_t + A\varepsilon^{o}_{t} + \varepsilon^{A}_{t+1} \end{aligned} \quad (12)$$

$$y_{t+1} = x_{t+1} + \varepsilon^{o}_{t+1} = Fx_t + \eta_{t+1} + \varepsilon^{o}_{t+1} \quad (13)$$

Then the upper limit of the prediction error can be expressed as:

$$\left\| \varepsilon^{A}_{t+1} \right\| \le \left\| Fx_t - Ax_t \right\| + \left\| \varepsilon^{o}_{t+1} - A\varepsilon^{o}_{t} \right\| + \left\| \eta_{t+1} \right\| \quad (14)$$

The aim of the system-of-equations strategy is to reduce the prediction error as much as possible by expanding A till we arrive at an accurate representation of F:

$$\left\| Fx_t - Ax_t \right\| \approx 0 \quad (15)$$

Then,

$$\hat{x}_{t+1} = Ax_t \quad (16)$$

can be used as measurement formula with a rather small measurement error:

$$\left\| \hat{\varepsilon}_{t+1} \right\| = \left\| \hat{x}_{t+1} - x_{t+1} \right\| \le \left\| Ax_t - Fx_t \right\| + \left\| \eta_{t+1} \right\| \approx \left\| \eta_{t+1} \right\| \quad (17)$$

3.2. Decomposition of complexity

The above strategy is based on the idea to build into the model as much background conditions (*OC*) as possible to obtain accuracy; the more comprehensive the models the more accurate they are. In economics, this strategy can result in enormous models containing more than 400 equations. Simon and Ando [4] show how one can simplify the dynamics of such models considerably. One of the main results is that if $\partial F/\partial OC$ is small enough one can legitimately ignore the background conditions for the short run.

Assume that A is nearly decomposable:

$$A = A^* + \varepsilon OC \quad (18)$$

where ε is very small, and OC is an arbitrary matrix of the same dimension as A. A^* can be arranged in the following form after an appropriate permutation of rows and columns:

$$A^* = \begin{pmatrix} A^*_s & 0 \\ 0 & 0 \end{pmatrix} \quad (19)$$

where A^*_s is a ($n \times n$) submatrix and the remaining elements are all zero. Let the roots of the submatrix A^*_s be designated as $\lambda^*_1, \lambda^*_2, \ldots, \lambda^*_n$. We assume that these roots are distinct, and the subscripts are so arranged that $\lambda^*_1 > \lambda^*_2 > \ldots > \lambda^*_n$. If $\Lambda = (\lambda_i)$ and $\Lambda^* = (\lambda^*_i)$ are diagonal matrices whose nonzero elements are the distinct roots of A and A^*, respectively, then there exist non-singular matrices $Z = (z_{ij})$ and $Z^* = (z^*_{ij})$ such that

$$A = Z\Lambda Z^{-1} \quad (20)$$

$$A^* = Z^*\Lambda^* Z^{*-1} \quad (21)$$

We know that

$$x_t = A^t x_0 = Z\Lambda^t Z^{-1} x_0 = Z\Lambda^t w \quad (22)$$

and

$$x^*_t = A^{*t} x^*_0 = Z^*\Lambda^{*t} Z^{*-1} x^*_0 = Z^*\Lambda^{*t} w^* \quad (23)$$

Let us now look at elements of (22) more closely. They have the form

$$x_{it} = \sum_{j=1}^{N} z_{ij} \lambda_j^t w_j \quad (24)$$

From the structure of A^* it follows that

$$z^*_{ij} = 0 \text{ for } i > n, j > n \quad (25)$$

If we define a matrix $U = (u_{ij})$ as

$$Z = Z^* + U \quad (26)$$

we can divide the right hand side of (24) into three sets of terms:

$$x_{it} = z_{i1} \lambda_1^t w_1 + \sum_{j=2}^{n} z_{ij} \lambda_j^t w_j + \sum_{j=n+1}^{N} u_{ij} \lambda_j^t w_j \quad (27)$$

Simon and Ando show that in the short run, that is small t, for a sufficiently small ε, the system characterized by a nearly decomposable matrix A behaves in a manner similar to the behaviour of the smaller subsystem, represented by A^*_s, insulated from the environment. This becomes clear when we express x^*_t as

$$x^*_{it} = z^*_{i1} \lambda^{*t}_1 w^*_1 + \sum_{j=2}^{n} z^*_{ij} \lambda^{*t}_j w^*_j \quad (28)$$

and compare this expression with (27), noting that $z_{ij} \to z^*_{ij}$, $\lambda_j \to \lambda^*_j$ as $\varepsilon \to 0$. In the long run, we may look at our system being dominated by the aggregate variable w_1, ignoring the relations within the subsystem. Thus, if the structure of a dynamic system can be represented by a nearly-decomposable matrix, then such system may be viewed as a composite system, constructed by the superposition of (1) terms representing interaction of the variables within the subsystem; and (2) terms representing interactions with the subsystem and its environment. Simon an Ando showed that, over a relatively short period, the first group of terms dominates the behaviour of the system, and hence the subsystem can be studied (approximately) independent of its environment. Over a relatively long period of time, on the other hand, the second group of terms dominates the behaviour of the system, and the variables within the subsystem move roughly proportionately. Hence, the variables within the subsystem can be aggregated into an index representing the subsystem.

3.3. Keep it simple

The system-of-equations approach is based on the assumption that more comprehensiveness leads to better predictions. However, this assumption is not justified by the empirical results. It appears that very simple univariate naive models (e.g. (7), random walks, low-order autoregressive (AR) models, or simple autoregressive moving average (ARMA) models) perform better in predictions than the larger, more comprehensive models. In interpreting these results, Milton Friedman [5] suggests that the programme of building large-scale models is probably faulty and needs reformulation.

For Friedman, the ability to predict is the quality of a model that should be evaluated, not its realisticness. This methodological standpoint is spelled out in the among economists well-known article “The Methodology of Positive Economics” [5]. The strategy he suggests is to keep the model A^*_s as simple, i.e. n small, as possible by avoiding to model the ‘other circumstances’ OC and instead to search for those objects or systems for which A^*_s is an accurate model (tested by its predictive power). In other words, try to decide by empirical research for which objects or systems the other circumstances OC are negligible: $\varepsilon \approx 0$. Friedman argues that a specification of the domain of objects and systems for which a model applies – the scope of the model – should be attached to the model.

Enlargement of the model is only justified if it is required by the phenomenon to be measured. The relevant question to ask about a model is not whether it is descriptively realistic but whether it is a sufficiently good approximation for the purpose at hand.

3.4. Calibration

A very influential paper in macroeconomics [6] shows that the estimated parameters A are not invariant under changes of policy. The problem is that the model equations in social science are often representations of behavioural relationships. Lucas [6] emphasises that agents form expectations of the future and that these expectations play a crucial role in the economy because they influence the behaviour of the economic actors. People’s expectations depend on many things, including the economic and financial policies being pursued by government and central banks. Thus, estimating the effect of a policy change requires knowing how people’s expectations will respond to the policy change. Lucas argues that the above estimation methods do not take sufficiently into account the influence of changing expectations on the estimated parameter values. Lucas assumes that economic agents have rational expectations, that is expectations based on all information available at time t and they know the model M, which they use to form these expectations.

Parameters invariant for policy changes should be obtained in an alternative way. Either they could be supplied from micro-econometric studies, accounting identities, or institutional facts, or they are chosen to secure a good match between a selected set of the characteristics of the actual time series and those of the simulated model output. Important is that whatever the source is, the facts being used for calibration purposes should be as stable as possible. However, one should note that in social science standards or constants do not exist in the sense as they do exist in natural science: lesser universal, more local and of shorter duration. In general, calibration in economics works as follows: use stable facts about a phenomenon to adjust the model parameters.

Two different calibration methods of obtaining parameter values can be distinguished. The first method is a method of estimation which entails simulating a model with ranges of parameters and selecting elements from these ranges which best match properties of the simulated data with those of the actual time series. An often-used calibration criterion is to measure the difference between some empirical moments computed on the observed variable y_t and its simulated counterpart $\hat{x}_t$. The estimator derived by this method is the so-called Method of Simulated Moments (MSM) estimator [7]. Let $m(y)$ be the vector of various moments of the observed data. $m(y)$ could include the means and variances of a selected set of observable variables. $m(\hat{x})$ is the vector of simulated moments, that is, the moments of the simulations $\hat{x} = M(A)$. Then the estimation of the parameters is based on:

$$A_{MSM} = \arg\min_{A} \left\| m(y) - m(\hat{x}(A)) \right\| \quad (29)$$

The second calibration method works as follows. When for specific circumstance stable facts about the phenomenon are known, we could use this information to adjust the model parameters. This method can be clarified by first rewriting (6) and assuming that M is a linear operator:

$$\hat{x}_t = M(y_t; A) = M(x_t; A) + M(\varepsilon_t^o; A) \quad (30)$$

Express (30) subsequently as a differential equation:

$$\Delta\hat{x}_t = \frac{\partial M(A)}{\partial x}\Delta x_t + \frac{\partial M(A)}{\partial \varepsilon}\Delta\varepsilon_t^o \quad (31)$$

When there are specific circumstances (sc) for which we expect that quantity x is stable ($\Delta x^{sc} \approx 0$), we can adjust the parameters of A such that

$$\Delta\hat{x}_t^{sc} \approx \frac{\partial M(A)}{\partial \varepsilon}\Delta\varepsilon_t^o \approx 0 \quad (32)$$

Although accuracy attained for these specific circumstances does not guarantee accurate results for other circumstances, it does argue for the validity of the measuring results.

4. PRECISION

The above section showed various strategies of obtaining an accurate representation A of a natural measuring system F. When such a model is built, it can be used for measuring x accurately. A useful, though perhaps crude, measure of accuracy is what is called the mean-squared error of the estimator:

$$E[\hat{\varepsilon}^2] = E[(\hat{x} - x)^2] \quad (33)$$

Then one can easily see that

$$E[\hat{\varepsilon}^2] = Var\hat{\varepsilon} + (x - E\hat{x})^2 \quad (34)$$

The first term of the right-hand side of (34) is a measure of precision and the second term is called the bias of the estimator.

If the model is an accurate representation of the economic system, then the estimator is unbiased:

$$\begin{aligned} \left\| x_t - E\hat{x}_t \right\| &= \left\| Fx_{t-1} + \eta_t - EAx_{t-1} \right\| \\ &= \left\| Fx_{t-1} - Ax_{t-1} \right\| + \left\| \eta_t \right\| \approx 0 \end{aligned} \quad (35)$$

Accuracy of the measurement results is then obtained by aiming at precision, i.e. reduction of $Var\hat{\varepsilon}$.

We will now briefly discuss a few methods that assume knowledge of the underlying system and therefore for accuracy purposes aim at precision. These methods can be and in economics are often labelled as 'filtering'.

4.1. Moving weighted average

The moving weighted average (MWA) filter [8] estimates the true value by taking a weighted average of a certain number of consecutive observed values:

$$\hat{x}_t = \sum_{s=-n}^{n} \alpha_s y_{t+s} \quad (36)$$

In most cases the MWA formulas are symmetric: $\alpha_s = \alpha_{-s}$. Substituting (5) into (36), we obtain:

$$\hat{x}_t = \sum_{s=-n}^{n} \alpha_s x_{t+s} + \sum_{s=-n}^{n} \alpha_s \varepsilon_{t+s}^o \quad (37)$$

It is assumed that the system is known, that is we have an accurate representation of it. So we know a set of conditions the α's should satisfy such that the following condition is fulfilled:

$$x_t = \sum_{s=-n}^{n} \alpha_s x_{t+s} \tag{38}$$

Then the measurement error is

$$\hat{\varepsilon}_t = \sum_{s=-n}^{n} \alpha_s \varepsilon_{t+s}^o \tag{39}$$

Because the measurement formula is unbiased, the measurement error is reduced by minimizing its variance. If we assume that each observation error has the same variance, one can derive

$$Var(\hat{\varepsilon}_t) = Var(y_t) \sum_{-n}^{n} \alpha_s^2 \tag{40}$$

So, the α's should be chosen such that $\sum_{-n}^{n} \alpha_s^2$ is minimized.

4.2. Kalman filter

A more efficient filter is the Kalman filter [9], because this method is based on constantly updating the estimator with each new observation available. Let $\hat{x}_{t|t-1}$ denote an estimate of x_t based on an estimate of x_{t-1}, where the subscript $t|t$-1 denotes the estimate made at time t using the information available up to time t-1, and assume for a moment that this estimate is available. A new observation y_t now becomes available; this in itself constitutes an estimate of x_t but it is, of course, subject to noise. To obtain the best unbiased linear estimate of x_t, a weighted linear sum of the two available estimates is formed to yield

$$\hat{x}_{t|t} = L_t \hat{x}_{t|t-1} + K_t y_t \tag{41}$$

where L_t and K_t are time-varying matrices to be specified by imposing on the filter the conditions that the estimator at each time point should be unbiased and of minimal variance.

We first impose the condition of unbiasedness. If the measurement errors are defined as $\hat{\varepsilon}_{t|t} = \hat{x}_{t|t} - x_t$ and $\hat{\varepsilon}_{t|t-1} = \hat{x}_{t|t-1} - x_t$ then we have

$$\hat{\varepsilon}_{t|t} = [L_t + K_t - I]x_t + L_t \hat{\varepsilon}_{t|t-1} + K_t \varepsilon_t^o \tag{42}$$

In this method it is assumed that that the observations are unbiased, $E\varepsilon_t^o = 0$, and if $E\hat{\varepsilon}_{t|t-1} = 0$, the updated estimator is unbiased if $L_t = I - K_t$. When this result is substituted in (41) we have

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t[y_t - \hat{x}_{t|t-1}] \tag{43}$$

K_t is referred to as the Kalman gain and plays a crucial role in the operation of the filter, and so an optimum choice of K_t is desirable to ensure a precise estimate of x_t. To describe the degree of precision of the measurement of the system variables, the variance-covariance matrices of ε_t^o, $\hat{\varepsilon}_{t|t}$ and $\hat{\varepsilon}_{t|t-1}$ are required; these are defined as $R = Var(\varepsilon_t^o)$, $P_{t|t} = Var(\hat{\varepsilon}_{t|t})$ and $P_{t|t-1} = Var(\hat{\varepsilon}_{t|t-1})$. By minimizing the variances of the measurement errors, the optimum choice of the Kalman gain is obtained:

$$K_t = P_{t|t-1}[P_{t|t-1} + R]^{-1} \tag{44}$$

Then,

$$P_{t|t} = [I - K_t]P_{t|t-1} \tag{45}$$

which is the variance-covariance matrix of the measurement errors corresponding to the optimum value of K_t given by (44). This equation provides a means of updating the variance-covariance matrix of the system variables by taking account of the observation made at time t.

When the filter is updated, the next step is to make new forecasts, using the model describing the system under study:

$$x_{t+1} = \mathrm{A}x_t + \eta_{t+1} \tag{46}$$

where the residual η_t is described by the variance-covariance matrix Q. In accord with (46), the forecast at time t of the state at time t+1 is taken to be

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t} \tag{47}$$

From (46) and (47) the error of prediction is obtained as

$$\hat{\varepsilon}_{t+1|t} = A\hat{\varepsilon}_{t|t} + \eta_{t+1} \tag{48}$$

from which the variance-covariance matrix of the prediction error can be derived as

$$P_{t+1|t} = AP_{t|t}A^T + Q \tag{49}$$

Subsequently these results are updated by using a new observation y_{t+1}. Etcetera.

5. CONCLUSION

Economists have developed all kinds of strategies to compensate for their inability to control the conditions for obtaining accurate measurement results. Instead of making demands on the measuring systems, they require the models to fulfil certain criteria such that they can function as reliable measuring instruments. To reduce the measurement errors they require models to account for the noisy environment. In economics, one can distinguish four general strategies. The first strategy, system-of-equations approach, is to build into the model as much as possible this environment. However, if one uses as a test for accuracy the predictive power of these models, it turns out that even when these models are gigantic their performance can be worse than very simple predictors. The second strategy, decomposition of complexity, shows that there is a trade-off between the time interval over which the accuracy of the prediction will be maintained and the degree of nearness of the system to a really isolated system. The third strategy that can be identified is to keep the models as simple as possible and to look for systems for which the model is an accurate representation. The fourth strategy is a calibration strategy. It doubts whether the models found by the other strategies are representations of invariant systems. Economic systems incorporate expectations about future developments and therefore change when expectations change. The estimation of the model parameters should be based on facts about the economic system that remain invariant under changes of expectations.

Accuracy of the measurement results consists of two aspects: unbiasedness and precision. The first aspect is dealt with by finding accurate representations. Precision is obtained by filtering the data. In economics, two filter methods are broadly applied: the moving-weighted-average and Kalman filter techniques. However, one should note that for both filter methods accurate representations are requisite.

REFERENCES

[1] J. Woodward, "Data and phenomena", *Synthese*, **79,** 393-472.

[2] M. Heidelberger, "Alternative Interpretationen der Repräsentationstheorie der Messung". In G. Meggle and U. Wessels (eds.) *Proceedings of the 1st Conference "Perspectives in Analytical Philosophy"*, Walter de Gruyter, Berlin, New York, 1994.

[3] T. Haavelmo, "The Probability Approach in Econometrics". Supplement to *Econometrica* **12**, 1944.

[4] H.A. Simon and A. Ando, "Aggregation of variables in dynamic systems", *Econometrica*, **29**(2), 111-138, 1961.

[5] M. Friedman, "The Methodology of Positive Economics". In: *Essays in Positive Economics*, University of Chicago Press, Chicago, London, 1953, 3-43.

[6] R.E. Lucas, "Econometric Policy Evaluation: A Critique". In K. Brunner and A.H. Meltzer (eds.) *The Phillips Curve and Labor Markets*, North-Holland, Amsterdam, 1976, 19-46.

[7] C. Gouriéroux, A. Monfort, *Simulation-based econometric methods*, Oxford University Press, Oxford, 1996.

[8] D. London, *Graduation: The revision of estimates*, ACTEX, Winsted, 1985.

[9] P. O'Connell, "Kalman filtering". In: W. Ledermann and E. Lloyd (eds.) *Handbook of Applicable Mathematics, Vol. VI Statistics*, Wiley, Chichester, 1984, 897-938.

Author: Dr. Marcel Boumans, Department of Economics, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands, (tel) +31 20 525 4297, (fax) +31 20 525 4254, m.j.boumans@uva.nl