10th IMEKO TC7 International Symposium
June 30–July 2, 2004, Saint-Petersburg, Russia

SYNTHESIS OF TEST MEASURING SIGNALS

Yury K. Rybin

Department of Computer-aided Measurement Systems and Metrology
Electro-Physical Faculty, Tomsk Polytechnic University, Tomsk, Russia

Abstract - Different models of measuring signals are considered. Their merits and demerits are discussed. A generalised model is offered on the basis of pulse string with the given shape, amplitude and duration. This model is equally suitable for describing deterministic, random and pseudorandom signals, both analogue and digital. The results of the synthesis of periodic signals with prescribed spectrum and random signal with given distribution law are presented.

Keywords: synthesis, measurement signal, measurement error

1. INTRODUCTION

Different signals are used in measuring and metrological practice. For example, when verifying ac voltmeters, it is required to estimate how a sine shape influences on measurement error. For this aim, one uses signals with desired harmonic factor, peak factor, averaging factor, spectrum or ratio of spectral components, and probabilistic characteristics are used.

Distinctions of signals used in measuring instruments are as follows [1]:

- repeatability of signals in measuring devices,
- compatibility with measuring devices by type and magnitude of informative parameters,
- awareness of the shape and parameters,
- standardization of uncertainty of principal informative parameters,
- feasibility of an estimation of the reproduction uncertainty in modelling or an experimental way,
- feasibility of revealing measuring information by an optimal way.

These requirements are satisfied by no means all of the signals which are traditionally considered in literature on signal theory. For example, a random signal of the type "white noise" cannot be referred to measuring signals because of impossibility of its reproduction in measuring devices due to its infinite energy. The signals composed of δ-pulses are not measuring ones, as they are incompatible with measurement means by a magnitude.

These and other circumstances allow to put the measuring signals in a special class of signals which are not studied enough from metrological standpoint. Many publications are devoted to the consideration of measuring signal, for instance [1, 2]. They study problems of the estimation of inaccuracy in producing that or other signal, ways of their normalization and etc. Regrettably, not all of the known models of signals are suitable for reproducing by hardware.

The necessity of problem statement for the synthesis of measuring signals with required characteristics is dictated by practice. In the last years of higher demand to metrological characteristics and to functional possibilities of the sources of signals are presented.

The synthesis of signals usually are produced by the presentation of the signal as decomposition into series

$$x(t) = \sum_{n=0}^{\infty} a_n \cdot \psi_n(t), \qquad (1)$$

where: $\psi_n(t)$ are the basic orthogonal functions of time, a_n is a coefficient of decomposition. To solve the problem of the synthesis it's take an ensemble of signals $\{\psi_i(t)\}$, i.e. to choose the function basis. The problem of choosing the basic function of the identically solved problem in [3] is considered as a central problem of the synthesis of signals since the successfully found basis allows to present the class of synthesised signals in the simplest and the most exact way.

The presentation of a signal as a series (1) is based on simultaneous summation of infinite number of function $\psi_n(t)$.

When reproducing the signal, presented in the series (1) by hardware or software programs other series with finite number of members is used

$$\tilde{x}(t) = \sum_{n=0}^{N} c_n \cdot \tilde{\psi}_n(t). \qquad (2)$$

Because of the difference between coefficients c_n and a_n, the restrictions of their numbers, the approximate realization of the function $\tilde{\psi}_n(t)$ and operations of the summation the error in reproducing of a signal appears

$$\Delta x(t) = x(t) - \tilde{x}(t). \qquad (3)$$

Considering the fact that parameters of the measuring signals are expressed through functional $x(t)$, the errors in metrics corresponding to this functional must be minimized.

The values Δ_2 and Δ_1 may be accordingly taken as estimation of the standard of the mean square deviation and maximum absolute error reproduction of a signal. We shall define Δ_2 and Δ_1 for different ways of hardware realization of signals by formula (2).

When reproducing signals by means of digital electronics, the error is defined basically by inaccuracy in presentation of coefficients c_n and inaccuracy in forming basic functions $\{\psi_i(t)\}$. The last component depends on number of binary bits *r*. It is known that the quantization error and has random nature. She does not depend on the value of coefficients c_n and is distributed evenly in the interval $[0,2^{-r}]$.

In forming signals by means of analog technology the error of reproducing the signal is also defined by inaccuracy in reproduction of the basic functions and inaccuracy in summation. If in both cases we take into consideration only one composite part of inaccuracy, i.e. inaccuracy in the set of coefficients c_n, the limit of the bias error can be presented as:

$$\Delta_1 = \Delta \cdot N,\ \Delta_2 = \Delta\sqrt{N},$$

where Δ is inaccuracy in coefficients, *N* is a number of coefficients.

So, maximum absolute error is directly proportional to the number of orthogonal functions used for presentation of the signal and the root mean-square error of reproducing the measuring signal, set in the series (2) and caused by inaccuracy of coefficients, is proportional to the root square of number function. In particular, for the system of trigonometric functions these errors are proportional, accordingly, to the number of harmonicas and to the root square of the number of harmonicas. It is obvious that no matter how small inaccuracy of the amplitudes Δ of the harmonicas is, there is always such a number *N*, under which errors Δ_2 and Δ_1 are bigger than any number given beforehand. Here there is a contradiction: the more exact the way we want to present the measuring signal is, the larger number of the harmonicas *N* must be, on the one hand, but on the other hand, the error of the reproduction increases. In other words, the less methodical errors of reproducing the signal by means of summations of the series we want to get, the bigger random error becomes. The property of incorrectness in summations of Fourier series - functions with inexactly set coefficients - also indicates this. When reproducing periodic signals, the presence of inaccuracy in coefficients of the series (2) brings about appearance of a significant random error and maximum value of the signals.

We shall demonstrate this on some examples.

It is known that any periodic signal, for instance, of triangular form can be presented as a sum

$$x(t) = \sum_{n=1}^{N} \frac{1}{n} \sin\left(n\frac{2\pi}{T}t\right).$$

The increase of the number of members of the series *N* brings about reduction of inaccuracy in presentation of a signal. Moreover inaccuracy can be small everywhere, except for points of the function's break-ups. However, when inaccuracy of coefficients of the series (1/n) appears, the increase of the number of members of the series does not bring about the reduction of inaccuracy. To illustrate this we shall take inaccuracy of the coefficients as a random error portioned evenly in interval from +10% to - 10% with number of members of the series *N* = 100. In this case the signal *x*(*t*) when reproducing is described by other sum.

$$\tilde{x}(t) = \sum_{n=1}^{N} \frac{1}{n}\left[1+0,1(2\text{rnd}(1)-1)\right]\sin\left(n\frac{2\pi}{T}t\right),$$

where *rnd* (1) are the numbers accidentally distributed in interval from 0 to 1.

Fig.1 presents the results of calculations on reproducing the signal of triangular form by means of the summation of the series of trigonometric sine functions with exact (dotted line) and no exact (utter line) coefficients of the series.

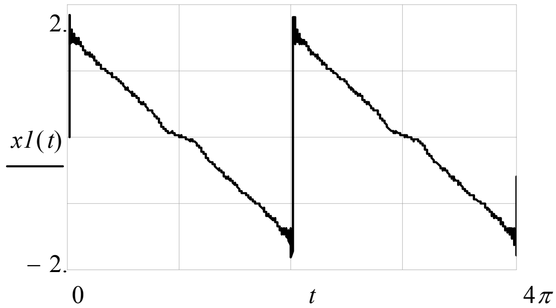

Fig. 1. Reproducing the ramp by means of trigonometric Fourier series with exact and inexact coefficient *N* = 100

So, the presentation of the signal by final sum of members of the trigonometric Fourier series with inexactly given coefficient leads to significant error in reproducing the signal. Moreover, the increase of the number of members of the series does not bring about the reduction of error.

When synthesising signals with the use of the summation of rows formed with other functions the inaccuracy of reproduction also depends on the inaccuracy of the coefficients. As an example we'll give the results of calculation on reproducing the signal of a sine form presented by the final sum of sedate functions. The signal of a sine form can be written as the row

$$\tilde{x}(t) = \sum_{n=0}^{N} (-1)^n \frac{t^{2n+1}}{(2n+1)!}.$$

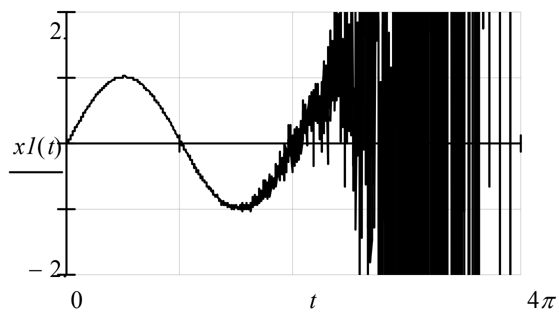

Fig. 2. Reproduction of sine signal by means of power series with inexact coefficients

In Fig. 2 a sine signal is shown, presented by the sum of 10 members of the sedate row. First of all, it is seen that reproducing of the periodic signal is realized on terminated time period only. Outside this period the inaccuracy sharply increases. It is obvious that for reproducing the signal in greater time period it is necessary to enlarge the number of members of the row.

It is clear that sedate rows can be used only for reproducing the signal in a limited time period.

But this is not the only defect.

In presence of inaccuracy in row coefficients the inaccuracy in reproduction of signals increases. The result of calculation with relative inaccuracy of the row coefficients distributed evenly in interval from +1% to -1% is presented on Fig. 2. Here is also seen a sharp growth of inaccuracy reproducing the signal even with such a small inaccuracy of a row coefficients.

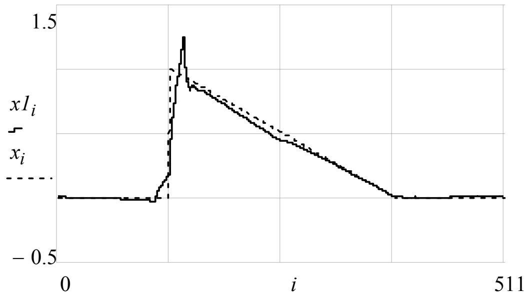

Fig. 3. Reproducing the triangular signal by means of series of Daubeshie's wavelet function with inexact coefficients

And finally, here is an example of presentation of signals as decomposition by Daubechie's wavelet functions. In Fig. 3 one can see the signal of triangular form *x*(*i*) and its presentation by the sum of five wavelets that is a function with inexact coefficients $x_1(i)$. To increase accuracy of the presentation of a signal it is also necessary to enlarge the number of members of decomposition, but satisfactory accuracy here can be reached with fewer members of the row. However, when inaccuracy in coefficients appears (in Fig. 3 inaccuracy is also accepted as distributed in interval from -10% to +10%), a significant inaccuracy is seen on the whole area of the signal. The similar calculations are also possible to execute with approximations of other functions by means of series (2). Consequently, the presentation of numerous realized signals in a row (2), which is suitable for analysis of the signal, is almost unsuitable for hardware or programme syntheses and for the reproduction of any complex measuring signals because of need of forming a large amount of elementary functions: trigonometric, sedate, wavelet-functions and their summations.

2. RESULTS OF THE STUDIES

Another presentation of signals is considered below which is founded on summation of elementary functions occurring at different times.

Let us divide the interval *T* of determination of the signal $x(t)$ into segments $T_1, T_2, .., T_m$. We shall take a system of finite functions, the intervals of which coincide with T_i, as a function $\psi_i(t)$, i. e.

$$\psi_i(t) = \begin{cases} \psi_i(t), \text{ при } & t \in T_i,\ T_i = t_i - t_{i-1}, \\ 0, \text{ при } & t \notin T_i. \end{cases}$$

Let us assume that $T_i \cap T_j = 0$ with $i \neq j$ and $\bigcup_{i=1}^{m} T_i = (0, T)$.

Using the system of such functions, the expression for the ensemble of realized signals can be written as

$$x(t) = \sum_{i=1}^{\infty} a_i \psi_i(t, \tau_i, T_i)[H(t-\tau_i) - H(t-\tau_{i-1})] \qquad (4)$$

where $\psi_i(t, \tau_i, T_i)$ is non-random function of time, describing the form of the impulse with the following parameters: zero hour τ_i and duration T_i; $H\ (t-\tau_i)$ is Hevisayd function.

For signals presented in a row (4), error of the summation with inexactly set coefficients a_i (with inaccuracy ε) is

$$\Delta_2 = \{ \sum_{i=1}^{N} \frac{1}{T} \int_{t_{i-1}}^{t_i} [\varepsilon \quad \psi_i(t)]^2 dt\}^{1/2}.$$

On the basis of the theorem on average number of the integral we find the limitations of inaccuracy where: m_i and M_i are the minimum and maximum of the function $\psi^2{}_i(t)$ accordingly.

Having taken equal intervals of determination of functions to make it simpler and having taken $m_i = 0$, and $M_i = 1$, we get $0 \le \Delta_2 \le \varepsilon$. As a result, inaccuracy in reproducing the signal does not exceed inaccuracy of the coefficient a_i. It is also obvious that maximum absolute inaccuracy makes ε.

Comparing two forms of the presentation of a signal, one can see that in the last case inaccuracy of reproducing a signal does not exceed inaccuracy of either of the coefficients.

Let us consider the models of signals, which can be formed on basis of expression (4). This expression includes as function $\psi_i(t)$, defining the form of the impulse, and its parameters a_i, τ_i, T_i, which can take on different values. The parameters of the signal models can be both known and random, dependent and independent on time, dependent or independent on each other, continuous and discrete and so on.

Representation of a signal in the form of sequence of pulses was earlier used mostly for the description of random pulse streams [8]. Then the scope of such models extended.

The models of periodic signals are get on base of the expression (4) by setting the conditions of periodicity $\psi_i(t + mT) = \psi_i(t)$, where m is an integer number; T is a period of the oscillations. For instance, the model of a sine signal can be set the following way. With

$$x(t) = \sin \frac{\pi}{T_i}(t - \tau_i),\ \tau_i \le t < \tau_{i-1},\ a_i = (-1)^i\ a$$

we have

$$x\ \ (t) = \sum_{i=-\infty}^{\infty} (-1)^i a_i \sin \frac{2\pi}{T}(t - \tau_i) = a \sin \frac{2\pi}{T}(t - \tau_0).$$

Having set the sequence of functions

$$\psi_i(t) = e^{-\alpha_i t} \sin\frac{\pi}{T_i}(t-\tau_i),$$

the model of a signal in mode of establishment of periodic oscillations in RC - generator can be written as

$$x(t) = \sum_{i=1}^{\infty}(-1)^i a_i e^{-\alpha_i t} \sin\frac{2\pi}{T_i}(t-\tau_i) =$$
$$= a \cdot e^{-\alpha t} \sin\frac{2\pi}{T}(t-\tau_i).$$

Having chosen the following functions as $\psi_i(t)$ $\psi_i(t) = \begin{cases} 1, & \text{при } t \in T_i, \\ 0, & \text{при } t \notin T_i. \end{cases}$ it is easily to build the model of pulsed or pseudorandom signal

$$x(t) = \sum_{i=0}^{\infty} a_i \cdot [H(t-\tau_i) - H(t-\tau_{i-1})], \ i = \overline{0,\infty}\}$$

of pulsed generators or model of digital-analogue generators. And finally, taking random quantities as a_i, τ_i, T_i, the expression (4) serves as a model of unceasing or discrete random process of generators of random signals.

So, by controlling the form of the pulse and its parameters it is possible to build the models of different measuring signals, unceasing and discrete-time, periodic and non-periodic, determined and casual. From this viewpoint the expression (4) presents the broad class of practically realized measuring signals.

3. SYNTHESIS OF PERIODIC SIGNAL MODELS

Let's concretise the problem statement of synthesis the periodic signal. It is necessary to define the form of pulses $\psi_i(t)$, their amplitudes a_i and duration T_i so that the constant component and amplitudes and phases of spectral components of the first N harmonics of the signal were equal to the set number of spectral components of the idealised signal $y(t)$, but the rest of them did not exceed the definite values. Set in this way, the problem of syntheses has some peculiarities compared with the classical problem of the syntheses [4].

The fact should be remembered that in classical variant the problem of syntheses is formulated as a problem of approximations [4] as follows. Supposing the function $y(t)$ is given, which describes measuring signal and due to some reasons is difficult to be realised at miscellaneous *t*. To simplify the problem the function $y(t)$ is changed to approximate $y^*(t)$, represented as the sum of basis (usually orthogonal) functions $\{\psi_i(t)\}$ taken with coefficients a_i. The problem of the syntheses is to define the coefficients of decomposition a_i in accordance with some criterion of vicinity. Unlike classical setting of the problem, in this case the problem lies not in approximation of the function itself, but in approximation of the spectrum of the function $y(t)$ by the spectrum of the function $x(t)$. Moreover, functions with partly coinciding amplitude spectrum can greatly differ from each other in even metrics. This is the first particularity. The second particularity already appears in the process of the syntheses. As it will be shown below, functions $\{\varphi_i(t) \cdot (H(t-\tau_i) - H(t-\tau_{i-1})\}$, forming $x(t)$, linear-independent and orthogonal in the sense of classical setting, become linear-hung in the sense of approximation of amplitude spectrum. This circumstance turns the problem of the syntheses to the class of degenerate problems, having no regular methods of solving. In fact, the problem of the syntheses becomes incorrect.

Later on we'll consider the signal with a given spectrum $y(t)$ as the optimum one.

To approach the spectrum $y(t)$ by spectrum $x(t)$ we'll reflect $x(t)$ from the area of time into the area of frequency (in the space of the spectrum of the signal), using Fourier's transformation.

It is known that spectrum of any function limited in time, including $\varphi_i(t)$, has an interminable number of spectral components, each of which is defined in its turn by number of these functions, that's why Fourier row contains double sums and depends on number of functions $\psi_i(t)$

$$x(t) = \sum_{i=1}^{I} c_i + \sum_{i=1}^{I}\sum_{j=1}^{\infty} c_{ij}\cos(j\omega_0 t) + \sum_{i=1}^{I}\sum_{j=1}^{\infty} d_{ij}\sin(j\omega_0 t).$$

Let's introduce designations $\sum_{i=1}^{I} c_i = c_0$, $\sum_{i=1}^{I} c_{ij} = c_k$, $\sum_{i=1}^{I} d_{ij} = d_k$, then the initial signal has a form of

$$x(t) = \tilde{n}_0 + \sum_{k=1}^{\infty} c_k \cos(k \cdot \omega_0 \cdot t) + \sum_{k=1}^{\infty} d_k \sin(k \cdot \omega_0 \cdot t).$$

The amplitude spectrum of the signal $x(t)$, which is to be reproduced, is

$$q_x = \{c_0,\ c_1,\ c_2,\ c_3, \ldots,\ c_n,\ d_1,\ d_2,\ d_3, \ldots,\ d_n\}.$$

These coefficients are defined by correlation for calculation of coefficients of decomposition in Fourier series.

On the basis of the theorem on average number of the integral let's present c_{ij}, d_{ij} as

$$c_{ij} = \mu_i a_i \int_{\tau_i}^{\tau_{i+1}} \cos(j \cdot \omega_0 \cdot t)dt, \ d_{ij} = \mu_i a_i \int_{\tau_i}^{\tau_{i+1}} \sin(j \cdot \omega_0 \cdot t)dt,$$

where: μ_i - are real numbers, moreover, $m \le \mu_i \le M$, m, M, are, accordingly, the minimum and maximum values of the function , that is 0 and 1.

Considering the correlation we've got, we can easy write down a system of the equations, linking spectrum of the optimized signal with parameters of pulses

These systems of the equations can be presented in the form of matrix

$$c = Q \cdot E \cdot C \cdot A, \ d = Q \cdot E \cdot D \cdot A, \tag{5}$$

where: $c = \{c_0,\ c_1,\ c_2,\ c_3, ..., \ c_n\}^T$,

$d = \{\ d_1,\ d_2,\ d_3, ..., \ d_n\}^T$,

$A = \{\ a_1,\ a_2,\ a_3, ..., \ a_n\}^T$,

$\mu = \{\ \mu_1,\ \mu_2,\ \mu_3, ..., \ \mu_n\}^T$,

$C = \{c_{ij}\}$, $D = \{d_{ij}\}$,

E - is a single diagonal matrix.

In equations (5) the elements of matrixes C и D satisfy the following restrictions

$$\sum_{i=1}^{I} c_{ij} = [\sin(j \cdot \omega_0 \cdot \tau_{i+1}) - \sin(j \cdot \omega_0 \cdot \tau_{i+I})] = 0,$$

$$\sum_{i=1}^{I} d_{ij} = [\cos(j \cdot \omega_0 \cdot \tau_{i+1}) - \cos(j \cdot \omega_0 \cdot \tau_{i+I})] = 0,$$

because $\tau_{i+I} = \tau_i + T$.

So, the sum of elements in each line is a zero and there is no unambiguous solution of the matrix equations. That's why these equations (5) have a peculiarity - many variants of their solving. Really, the left parts of the equations contain $2N+1$ set coefficients, but the right ones include $3I-$unknown, among which $I-$ unknown functions $\psi_i(t)$, $I-$intervals T_i and $I-$amplitudes a_i . It's clear that to have an unambiguous decision of the system of the equations (5) it is necessary to impose restrictions on the choice $4N$ unknown. An unknown can be either the form of pulses or their duration or their amplitudes. Herewith we can select three main possible variants of setting the problem of syntheses.

These variants correspond to different physical settings of the problem and are not equal in degree of difficulty of their solution. It should be remembered that the purpose of the syntheses of models of a signal is to get such an expression, which could correspond to the simplest type of the dynamic system, which could be easily realized by hardware. In the third variant we can come to a solution, setting in the period of the signal of different forms of pulses, which will considerably complicate the building of the dynamic system and its practical realization without any expected advantages. That's why the most expedient variants for syntheses are the first and the second ones, when the form of the pulse is given, but the unknowns will be amplitudes or duration.

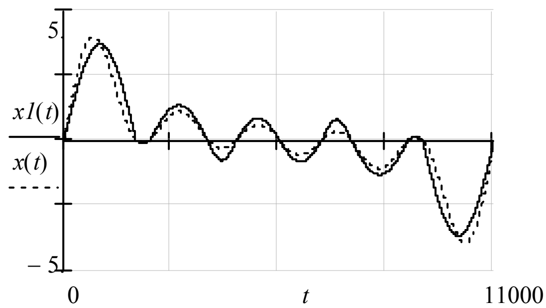

Fig. 4. The ideal (dotted line) and the realized (utter line) signals with given spectrum

In Fig. 4 and 5 we have an example of a graph of a signal with sine-formed pulses and a given time lag, calculated by methods shown above. Fig. 4 shows the idealized signal $y(t)$ (dotted line) composed of the first five harmonicas with amplitude equal to one unit, and $x(t)$ – is the synthesised signal (utter line), made of eleven pulses of sine form.

The first five harmonicas of the signal coincide in amplitude with the first five harmonics of the given signal. However, due to deflection of the form of pulses and non-optimal duration of the pulses the spectrum of the realized signal contains the high harmonicas small and inaccuracy *rms* of the realized signal does not. The power of these components is exceed 2%.

On fig.5 signal with a given THD (total harmonic distortion 0 and 27, 88 percent) is showed

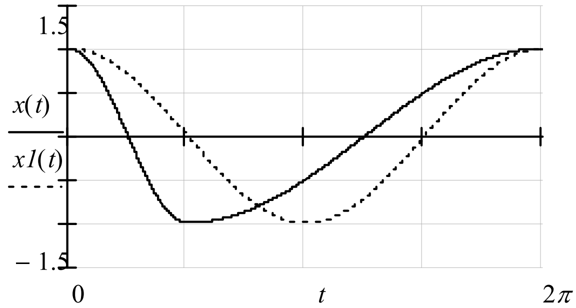

Fig. 5. The signals with a given THD (0 % – dotted line and 27, 88% – utter line)

4. SYNTHESES OF RANDOM SIGNALS

Supposing it is necessary to synthesise the model of a random signal with density of the distribution of probability $P_x(z)$. The syntheses of models of casual (random) signal, as well as models of the deterministic periodic signal will be conducted within the limits of a generalised model (4), taking a_i, τ_i, T_i values as casual, then [5]

$$P_x(z) = \int_0^1 P_\psi(u) \cdot P_a(\tfrac{z}{u}) \cdot P_x(T_j) \cdot \frac{du}{u}, \qquad (6)$$

but

$$P\{\psi(t) \le u\} = P\{0 \le \eta(t) \le \psi^{-1}(u)\} +$$
$$+P\{[1-\psi^{-1}(u)] \le \eta(t) \le 1\} = 2\psi^{-1}(u),$$

then correlation (6) comes

$$P_x(z) = 2\int_0^{-1} \psi^{-1}(u) \cdot P_a(\frac{z}{u}) \cdot \frac{du}{u}. \qquad (7)$$

The equation (7) is functional transformation, which puts the density of distributing probability $P_x(z)$ in accordance with the process $x(t)$ with a function $\psi(t)$ and sharing of the amplitudes $P_a(z)$ in those cases, when there is an inverse function $\psi^{-1}(u)$. It is seen from equation (7) that it is possible to control the density of probability, by means of changing ψ or sharing the amplitudes $P_a(z)$.

Herewith if ψ or $P_a(z)$ are set, the equation (7) moves over to integral equation concerning the second unknown function. So, for instance, with an even distribution of amplitudes

$$P_a(x) = \begin{cases} 1/2, |x| \le 1, \\ 0, |x| > 1. \end{cases}$$

the solution (7) will be an equation

$$\psi^{-1}(y) = -\int_0^y z \cdot \dot{P}_x(z) dz. \qquad (8)$$

But if we choose the certain function $\psi(t)$, after changing of the variable $y = z/u$ the equation is reduced to an integral Volterra equation of the first kind relative to $P_a(x)$

$$P_{x(t)}(y) = 2\int_v^\infty \frac{1}{v} \cdot \psi^{-1}\left(\frac{v}{z}\right) \cdot P_a(z) dz, v \ge 0. \qquad (9)$$

Let's take into consideration the distance between density, which is set by the density of distribution $P_{y(t)}(z)$, and the density, which is got as a result of the syntheses $P_{x(t)}(z)$. As we know [5], the distance between densities can be introduced in even and square metrics by means of different criteria. Let's use the most "sensitive" module criterion

$$\rho = \int_{-\infty}^{\infty} \left| P_{y(t)}(z) - P_{x(t)}(z) \right| dz.$$

Supposing $\rho = 0$, then $P_{y(t)}(z) = P_{x(t)}(z)$. Then on the left side of the equation (9) there is a density of the distribution of probability given to the syntheses; but on the right side there is a functional correlation, connecting it to the function, describing the form of the impulse, and density of the distribution of amplitudes of pulse. The last formula gives us a possibility, having assigned one of unknown, to define the second one and, consequently, synthesize the model of the casual signal with required density of the distribution.

The situation of sine-formed impulses $\psi(t) = \sin \pi \ \ t, \ 0 \le t < 1$ is the most interesting one for the practice of distributing of signals, as this form can be easily reproduced by means of *RC*- or *LC*-generator. Then (9) is transformed into Abel equation relative to $P_a(z)$

$$P_{y(t)}(v) = \frac{2}{\pi} \int_v^\infty \frac{P_a(z)}{\sqrt{v^2 - z^2}} \cdot dv, v \ge 0, \qquad (10)$$

for which there is a formula of transformation [5]

$$P_a(z) = -z \int_v^\infty \frac{\dot{P}_{y(t)}(v)}{\sqrt{v^2 - z^2}} dv, z > 0. \qquad (11)$$

The expression (11) complies with the well-known one for sine signals with random amplitude and phase [5]. We'll give an example of syntheses of the random process with a set normal density of the distribution of probability

$$P_{y(t)}(v) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-v^2}{2\sigma^2}}.$$

This expression describes the density of a strifefissions of probability of the stationary central normal process with dispersion σ. We'll choose this very process as a model of the optimum signal. Having placed the density, which is set to syntheses, in the equation and using expression for tabular integral, we have

$$P_a(z) = \frac{|z|}{2\sigma^2} e^{\frac{-z^2}{2\sigma^2}}, |z| \le \infty.$$

So, the process $x(t)$ will have normal distribution with $\psi(t) = \sin \pi \cdot t, \ 0 \le t < 1$ if density of pulse amplitude values is portioned under the law $P_a(z) = \frac{1}{2} P_R(|z|)$, where $P_R(|z|)$ is the law of the Relay's distribution. But the distribution of duration of pulses T_i and moment τ_0 can be arbitrary.

For signals with the square-wave form of oscillations, having conducted a differentiation in generalised sense, it's easy to make sure, that $P_a(z) = P_y(|z|)$.

5. CONCLUSION

The offered generalized model of the measuring signal (4) can serve as a model of both periodic and random signals. It is suitable for analog and digital signals. The classification of models, which is offered on this basis, reflects the whole variety of measuring signals. It can be regarded as the universal model of any known measuring signals. On the basis of this model dynamic systems can be synthesized, which are suitable for practical realization by both hardware and program devices.

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Author: Yury K. Rybin, Ph.D., Assoc. Prof., Department of Computer-aided Measurement Systems and Metrology, Electro Physical Faculty, Tomsk Polytechnic University, Pr. Lenina 30, Tomsk, 634050, Russia, phone: +7-3822-417527; fax: +7-3822-420449,
e-mail: rybin@galore.tomsk.ru