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ABOUT THE METHOD OF NON-LINEAR UNSTEADY FLUID FORCES MEASUREMENT

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Abstract - For research of aeroelastic behaviour of bodies at non-linear aerodynamic interaction it is necessary to measure not only the first, but also high harmonics of aerodynamic loading on vibrating bodies. In the presented work, various ways of the non-stationary aerodynamic forces determination are analysed. Substantiation of the mentioned forces measurement through a electrical current in the vibrator mobile coil is handed. That is valid under condition of the proportional - differential feedback utilising in the oscillation control system of the tested bodies.

Keywords high harmonics of the aerodynamic force, electrodynamic vibrator, feedback vibration regulator

INTRODUCTION

For dynamic stability analysis of the cross-flown elastic bodies it is necessary to measure the non-stationary aerodynamic loadings caused by their vibrations. These loadings arise not only with frequency equal vibration frequency of bodies but, in the case of non-linear interaction with a flow, they can contain high harmonics [1]. The information on these loadings allows finding not only flutter border, but also level of stresses in the limit cycle.

Besides it, there are non-stationary aerodynamic loadings caused by periodic non-uniformity of an inlet flow, by flow separation from body surfaces, which arise even on motionless bodies [2].

Thus, there is necessary to have a possibility of aerodynamic loading spectrum measurement on vibrating bodies.

Practical examples of such bodies are the flying devices and their separate elements, tube bundles of heat exchangers, blades of turbomachines, elements of ships and underwater structures.

For measurement of aerodynamic loadings it is possible to fix the bodies to dynamometer, but at vibrations of such systems there will be large inertial forces, on background of which it is difficult to measure the aerodynamic loadings. An exit from this situation is the application of fast-response pressure gauges [3]. They allow determining pressure spectra on body surfaces in a wide range of frequencies. However, it is difficult and expensive to prepare a body with enough of sensors. According to method [4] it is possible to measure the aerodynamic loadings by an electrical current in the mobile coil of the electrodynamic vibrator, with the same one that excites the body vibrations. This technique has been further developed in [5 - 7], the method is rather simple, but intended for measurements of the aerodynamic loading first harmonic. There is also known technique of aerodynamic loadings measurement through a set of such body movement parameters, as displacement, velocity and acceleration [8]. Very large measurement accuracy of these parameters is required in this case.

Purpose of the work - development of a technique [4 - 7] for the aerodynamic loading spectrum measurement on vibrating bodies in wider band of frequencies including high harmonics.

1. MATHEMATICAL MODEL OF EXCITATION SYSTEM FOR BODY VIBRATIONS

During experiments the test body is placed in a measuring room of a wind tunnel and supported by elastic elements located outside [6] (fig. 1). These elements form a parallelogram and provide forward body displacements. The electrodynamic vibrator is applied to excite the demanded vibrations of this system. The given device is intended not only for excitation of the body vibration but also for determination of the non-stationary aerodynamic force, acting on it in vibration direction.

As the tube model is rigid enough, it is possible to present such vibrating system as oscillator (fig. 2) with modal mass m, stiffness coefficient C, mechanical damping coefficient r. The mobile coil of the vibrator has active resistance R and inductance L. This coil is placed in a ring gap of a constant magnet with an induction B. The length of the mobile coil wire, inserted in this gap, is l.

In fig. 2 depicted systems can make forward displacements $x(t)$, which correspond to bend vibrations of an elastic suspension. The tube model displacements are registered with the help of the non-contact gauge.

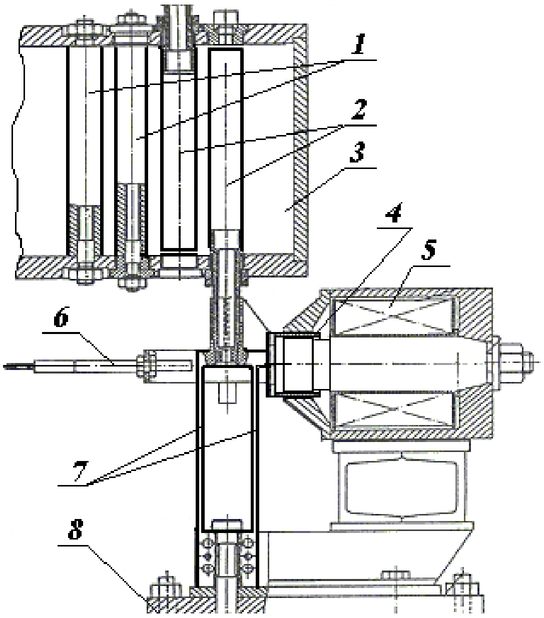

Fig. 1. Design of a tube elastic suspension

1-motionless tube models 2 - vibrating tube models, 3 – measuring room, 4 - vibrator mobile coil, 5 - actuating coil, 6 – moving sensors, 7 - elastic parallelogram, 8 - base.

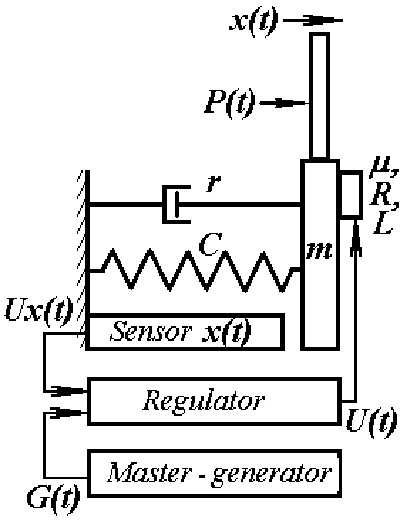

Fig. 2. The analytical model of vibration system

1.1. Equations for movement and current description of electromechanical system

The force arising on moving coil during passing a current *i(t)* according to Ampere's law is

$$F = \mu \cdot i(t), \tag{1}$$

where $\mu = Bl$.

On the other hand, there is induced an electromotive force in the moving coil corresponding to Faraday's law

$$E = \mu \cdot \dot{x}(t). \tag{2}$$

If we adjust the master-generator voltage *U(t)* on the mobile coil, we can operate movement of the oscillatory system. In order to make the control easier it is possible to apply a feedback for which displacement sensor signal can be used

$$U_x(t) = K_x x(t), \tag{3}$$

where K_x - transformation coefficient of the displacement sensor. The displacement sensor and master-generator voltages $U_x(t)$, $G(t)$ are attached to the input of a regulator which amplifiers the difference of these signals. The voltage of the master-generator is formed by required tube displacement $x_r(t)$:

$$G(t) = K_x x_r(t), \tag{4}$$

It is known, that in order to make the difference between real and required movement parameters of the oscillatory system small, the regulator should amplifier the entrance signal not only proportionally (P - regulator) but also differentially (PD - regulator) [9]. The output voltage of the PD-regulator will be

$$U(t)=K_d\frac{d[G(t)-K_x x(t)]}{dt}+K_p\cdot[G(t)-K_x x(t)], \tag{5}$$

where K_p - proportional amplification coefficient, K_d - amplification coefficient of differentiation.

The differential equations describing movement of oscillatory system and change of the electrical current in the mobile coil, is possible to write down in similarity with [10] as:

$$\begin{cases} m\ddot{x}(t)+r\dot{x}(t)+Cx(t)-\mu i(t)=P(t) \\ \left[K_d\Delta\dot{x}(t)+K_p\Delta x(t)\right]\cdot K_x+\mu\dot{x}(t)+Ri(t)+L\dfrac{di(t)}{dt}=0 \end{cases} \tag{6}$$

where $\Delta x(t) = [x(t) - x_r(t)]$, *P(t)* - aerodynamic (external) force acting on the tested tube in the direction of the least elastic elements rigidity.

1.2. Movement stability of electromechanical system

The considered electromechanical system can become unstable by influence of the feedback at some conditions. For determination of the working stability region, solution of the equations (6) at *P(t)* =0 and *G(t)* =0 will be searched in the form

$$x(t) = xe^{\lambda t}, \qquad i(t) = ie^{\lambda t}. \tag{7}$$

The amplitudes of moving *x* and current *i* are complex because they include phase shifts. After substitution (7) in (6) we shall receive

$$\begin{cases} \left(m\lambda^2 + r\lambda + C\right)x - \mu i = 0 \\ (K_x K_d\,\lambda + K_x K_p + \mu\lambda)x + \left(R + L\lambda\right)i = 0 \end{cases}. \tag{8}$$

The solution of this equation system will be not trivial, if its determinant is equal to zero. After the determinant calculation and appropriate modification we shall receive the following characteristic equation:

$$\begin{aligned} &mL\lambda^3 + \left(Rm + rL\right)\lambda^2 + \left(Rr + CL + \mu^2 + \mu K_x K_d\right)\lambda + \\ &+ (RC + \mu K_x K_p) = 0 \end{aligned} \tag{9}$$

The system will be steady, if all roots of the equation (9) will have negative real parts. For this purpose, according to Raus-Hurvitz criterion, the following inequalities should be carried out

$$\begin{cases} K_p > -\dfrac{RC}{\mu K_x} \\ K_p \dfrac{mL}{Rm + rL} - K_d < \dfrac{1}{K_x \mu}\left(Rr + \mu^2 + \dfrac{rCL^2}{Rm + rL} \right) \end{cases}, \quad (10)$$

where $\Omega^2 = C/m$ - undamped natural frequency of a mechanical system.

We shall take the following parameters for an example of electromechanical system: m = 0,25 kg, C = 90000 N/m, r = 0,042 kg/s, R = 4 om, L = 0,0007 H, B = 1 T, l = 15 m, K_x = 1400 V/m.

In fig. 4 there are shown stability borders of this vibration system. The maximal amplification $Kp \approx 61$ for the proportional regulator ($Kd = 0$). The application of differentiation allows to increase considerably the proportional part of amplification.

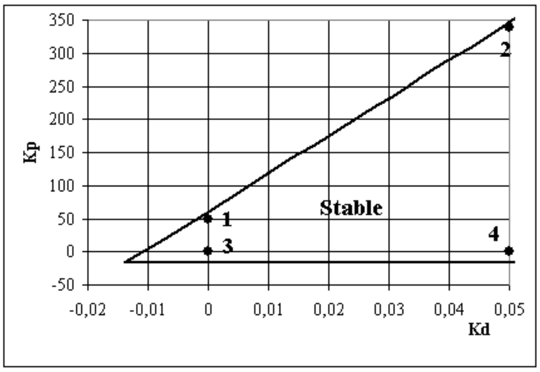

Fig. 3. Stability borders of vibration system

We shall choose four variants of regulator adjustment in the stability field of this diagram:

1. *Kp1* = 55, *Kd1* = 0 (P - regulator)
2. *Kp2*= 340, *Kd2* = 0.05 (PD - regulator)
3. *Kp1* = 0, *Kd1* = 0 (without a regulator)
4. *Kp2* = 0, *Kd2* = 0.05 (D - regulator).

The properties of our electromechanical system for these regulation variants will be analysed later.

1.3. Moving of the vibration system and coil current

Let's consider behaviour of the electromechanical system that is under action of one of the aerodynamic force spectral component *P(t)* and of the harmonic signal generator *G(t)* in the case, when their frequency are equal. Let's consider, that $G(t) = K_x x_r e^{j\omega t}$, $P(t) = Pe^{j\omega t}$, where amplitudes P, x_r are complex, j - imaginary unit. As the electromechanical system is linear and in the field of stability, we shall search its movement and current in the shape of steady vibration with frequency ω:

$$x(t) = xe^{j\omega t}, \qquad i(t) = ie^{j\omega t}. \quad (11)$$

Substituting the accepted expressions (11) in (5), we receive the following system of equations

$$\begin{cases} a_{11}x_s + a_{12}i_s = P \\ a_{21}x_s + a_{22}i_s = K \cdot x_{rs} \end{cases}, \quad (12)$$

where

$$a_{11} = m(\Omega^2 - \omega^2 + j\omega r/m), \qquad a_{12} = -\mu,$$

$$a_{22} = R + j\omega L, \qquad a_{21} = K + j\omega\mu,$$

$$K = \left(K_p + j\omega K_d\right) \cdot K_x \quad (13)$$

Hereinafter index “*s*” designates flow, and “*0*” - its absence. Solving the given system by the Kramer’s rule, we receive the following determining relations for complex amplitudes of the electromechanical system movement and current:

$$x_s = \frac{a_{22}P - a_{12}K \cdot x_{rs}}{D}, \quad (14)$$

$$i_s = \frac{a_{11}K \cdot x_{rs} - a_{21}P}{D}, \quad (15)$$

where the determinant $D = a_{11}a_{22} - a_{12}a_{21}$.

At absence of a flow ($P = 0$) and the generator non-zero signal ($x_{r0} \neq 0$) in expressions (14) and (15) we shall receive

$$-\frac{a_{12}K}{D} = \frac{x_0}{x_{r0}}, \quad (16)$$

$$\frac{a_{11}K}{D} = \frac{i_0}{x_{r0}}. \quad (17)$$

From the formula (14) follows obvious fact, that at a zero signal of the generator, the system is not motionless and makes some displacements under action of measurable force *P*. These displacements should be small enough to neglect additional aerodynamic forces caused by these displacements. If we want to decrease the just mentioned undesirable displacements, we must have large value of the determinant D. That is achieved by increasing the feedback amplification coefficient *K*.

1.4. Analysis of balanced forces

It is visible from the first equation of the system (12), that the part of the aerodynamic force *P* is balanced by the inertia, rigidity and mechanical damping forces *P(x)*, dependent on the system movement, and by force *P(i)* of the vibrator dependent on the electrical current. It is possible to find relation of these forces at a zero signal of the generator (x_r =0) by substitution (14) and (15) into the first equation of the system (12)

$$\frac{a_{11}a_{22}}{D} - \frac{a_{12}a_{21}}{D} = 1. \quad (18)$$

where $\dfrac{a_{11}a_{22}}{D} = \dfrac{P(x)}{P}$, $\quad -\dfrac{a_{12}a_{21}}{D} = \dfrac{P(i)}{P}$.

The modules and phases of these components are given in a fig. 4 – 7 in dependence on non-dimensional frequency. *P1(x)/P* and *P1(i)/P* correspond to variant 1, *P2(x)/P* and *P2(i)/P* – to the second variant of regulator adjustment, and so on.

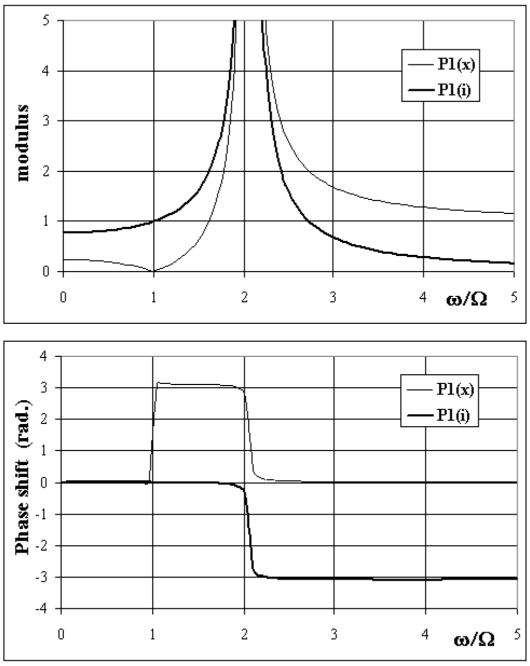

Fig. 4. Relation of balancing forces for P - regulator (Kp=55, Kd=0).

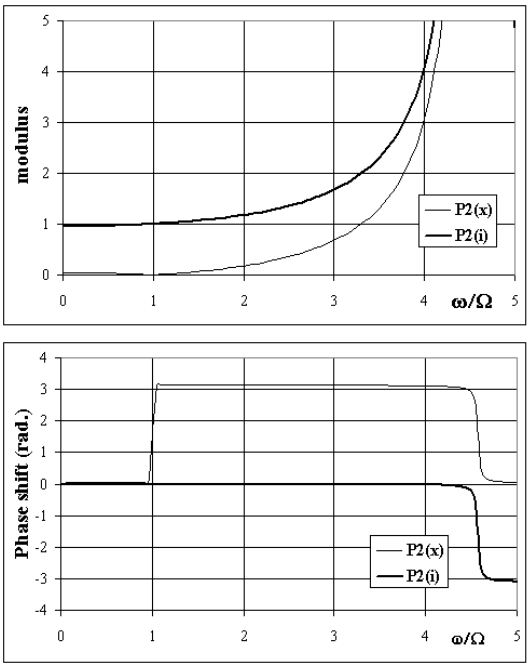

Fig. 5. Relation of balancing forces for PD - regulator (Kp=340, Kd=0,05).

The introduction of a proportional feedback is similar to rigidity increase of the oscillatory system, therefore, the electromechanical resonance is shifted to the region of higher frequencies. For example in the case of P - regulator the resonant frequency is more than 2 times higher than mechanical resonance frequency (see fig. 4). It is possible to see that up to the electromechanical resonance the aerodynamic force is mainly balanced by the vibrator force, and in the above resonance area – by the forces of inertia, rigidity and mechanical damping.

The increase of amplification in the feedback loop due to the PD - regulator application enhances the electromechanical resonance frequency even more than P – regulator (see fig. 5). In this case it is possible to measure the first 3-4 harmonics in the fore-resonance area by a current in the vibrator's mobile coil. The current amplitude and phase depend on the aerodynamic force frequency rather monotonous in this area.

At absence of a feedback ($K = 0$), the forces depending on the system displacements are prevailing (see fig. 6), and measurement of high harmonics of aerodynamic forces is possible to perform according to method described in [8]. The vibrations are not induced by a vibrator but by aerodynamic forces in this method. For limitation of these self-induced vibration amplitudes, the system damping is adjusted by change of active resistance R. It is necessary to note to this technique that the system displacements are much higher than at PD - regulator application. These displacements can cause additional aerodynamic forces reducing measurement accuracy.

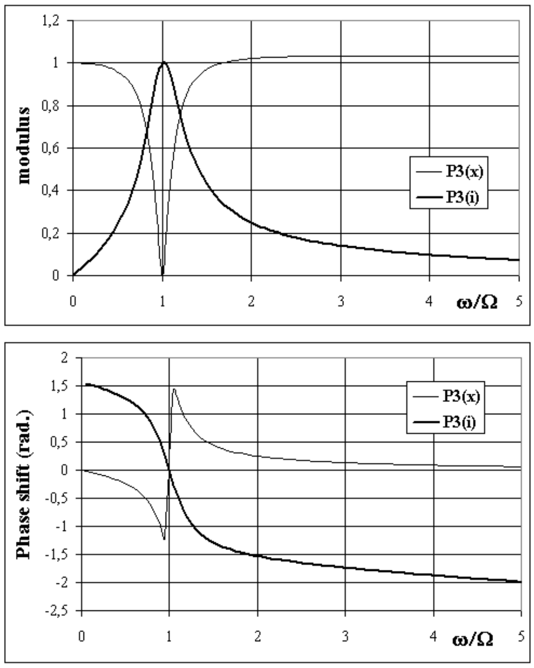

Fig. 6. Relation of balancing forces without a regulator (Kp=0, Kd=0).

We can use only the differentiating regulator (D - regulator) (see fig. 7). The resultant effect is the same as if instead of an inlet sensor of the feedback loop was used a velocity-meter. However, it is difficult in this case to measure the low-frequency aerodynamic forces by a vibrator current.

1.5. Determination of aerodynamic loading

If we measure reaction of the electromechanical system to aerodynamic force, it is possible to determine complex amplitude P on the basis of the relations (14) - (17), and that will be performed in three ways.

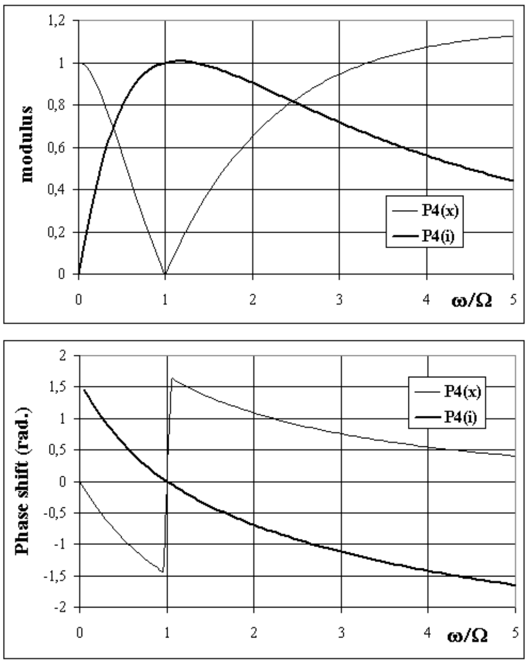

Fig. 7. Relation of balancing forces for D - regulator (Kp=0, Kd=0,05).

1. Substituting expression (16) in (14), we shall receive coupling of the aerodynamic force with the oscillatory system displacements in a flow and without a flow:

$$P = -\frac{D}{a_{22}}\left(x_0 \frac{x_{rs}}{x_{r0}} - x_s \right). \tag{19}$$

2. From the formula (14) we shall find x_{rs} and substitute it in the expression (15) for current. Then, taking into account (16) and (17), we shall receive expression for the complex amplitude of force

$$P = \mu\left(\frac{i_0}{x_0} x_s - i_s \right). \tag{20}$$

The received expression coincides with the one given in [4]. The dynamic calibration for determination of μ is described in [5].

3. With the help of substitution (17) in (15) we shall receive coupling of aerodynamic force with current of the vibrator's mobile coil in a flow and without a flow:

$$P = \frac{D}{a_{21}}\left(i_0 \frac{x_{rs}}{x_{r0}} - i_s \right). \tag{21}$$

1.6. Calibration

The unknown multipliers before brackets in (19) and (21) remain for finding. If we add to the oscillatory system a known force P_c and measure the corresponding system displacement x_c and current i_c at absence of the generator signal (x_r = 0), we shall find the required multipliers from (14) and (15):

$$\frac{D}{a_{22}} = \frac{P_c}{x_c}, \tag{22}$$

$$\frac{D}{a_{21}} = -\frac{P_c}{i_c}. \tag{23}$$

From (18) it is possible to see, that opposite value of *P(x)/P* is qualitatively similar to (22) and opposite value *P(i)/P* to (23). Therefore, it is possible to say that the phase dependences (22) and (23) are similar to those depicted in fig. 4-7.

Taking into account, that at the research of aerodynamic interaction between bodies it is necessary to have not less than two oscillatory systems, this circumstance is used for their calibration. For this purpose the oscillatory systems need to be connected by an elastic element, the coupling of which deformation with force is known (fig. 8). As such elastic element is used dynamometer that has a natural frequency few times higher than is the top measured frequency.

From calibration it is necessary to find the frequency dependences of the P_c/x_c and P_c/i_c at the generator zero signal.

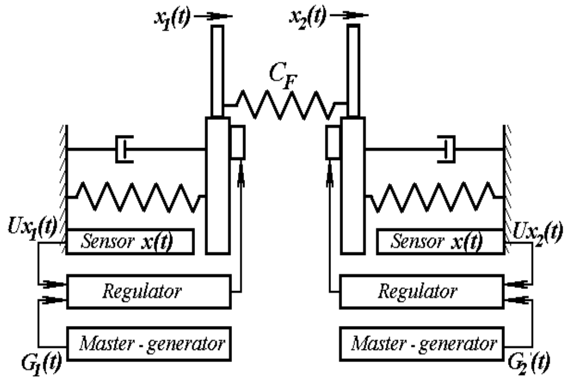

Fig. 8. The scheme of oscillatory system calibration

We shall perform the calibration in following way. The generator signal is set to zero value at the first system. Adjusting the generator amplitude and frequency of the second oscillatory system, we shall add the given force to the first oscillatory system through the dynamometer. Then vice versa.

If the dynamometer is sufficiently stiff and the regulator has large amplification, then there are not necessary large displacements and vibrator currents of the both systems.

In order to keep properties of the oscillatory system after calibration, it is necessary to attach a weight to the oscillatory system, which mass is equal to the dynamometer reduced mass. This mass can be found from rigidity and natural frequency of dynamometer.

1.7. Comparison of different ways of aerodynamic force determination

In all three ways the aerodynamic force is determined from the difference of measurements in a flow and without it. For reduction of the force determination error it is necessary to have minimal vibrator current without the flow (17). From fig. 9 follows, that it can be achieved, if required harmonic vibration will be raised on frequency of the mechanical resonance ($\omega \approx \Omega$).

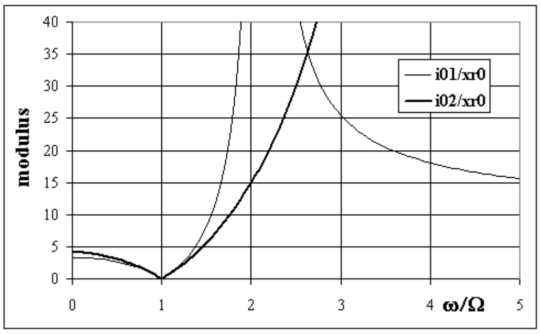

Fig. 9. The module i_0/x_{r0} dependence on generator signal frequency for the 1st and 2nd variants of regulation

The expressions (19) and (21) can be more simplified, if the generator signal will be the same ($x_{rs} = x_{r0}$) in the flow and at absence of it. The first terms in (19) and (21) will be equal to zero at a zero signal of the generator in the flow and therefore there is no sense to measure without a flow.

In the first way (19) at the large amplification of a regulator there will be small displacements that will lower accuracy of aerodynamic force determination. At reduction of amplification there is a narrowed frequency range, where the calibrating dependences are monotonous and without sharp changes of phases. If the generator is not used in the first way, the first term is zero and we come to the technique [8].

As it was said above, at zero signal of the generator, the vibration system makes some displacements under action of measurable force P. Therefore, in the second way (20) it is necessary to measure the relation of a current to displacement (i_0/x_0) at all frequencies without a flow. However, the frequency dependence of this relation has sharp changes both in the modulus, and in phase, and is similar to fig. 9.

In the third way (21) of aerodynamic forces spectrum measurements it is necessary to measure the coil current spectrum only in the flow. It is enough to measure the coil current on the generator frequency without a flow. In the case of applying PD - regulator there is possible to receive the monotonous calibrating dependences in wider frequency a range.

CONCLUSIONS

The electromechanical system behaviour for excitation of the given vibrations of a tested body was estimated in the article. These estimations gave us possibility to find out three ways for determination of aerodynamic load spectrum acting on a cross-flown body. The comparison of the mentioned methods shows, that for increase of accuracy of the aerodynamic load spectrum determination the most suitable methods are those ones measuring the mobile coil current in the vibrator. Thus the best results gives using of a proportional - differential feedback of the vibration regulator. The calibration for various ways of measurement was offered.

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