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NOVEL APPROACH TO THE COMPUTER-AIDED MODELLING OF MEASUREMENT PROCESSES FOR UNCERTAINTY ESTIMATION

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Abstract: The aim of this paper is to present a methodology to the modelling of measurement processes for uncertainty estimation [1]. According to the "Procedure for the Uncertainty Management" (PUMA) described in the supplement of ISO 14253-1 [2] an iterative course of action is derived. In this connection the efficiency of modelling is considered additionally.

The trail version of the novel application software "Model Assistant", developed within the cooperative research project MST-UNCERT, is used for graphical modelling.

A practical stepwise procedure for efficiency modelling has been derived and exemplary applied from this concept.

Keywords: modelling, measurement uncertainty.

1. Introduction

From various points of view measurement uncertainty is technologically and economically important. For manufacturers of measuring instruments a science-based measurement uncertainty declaration can be a decisive advantage in competition. For users of measuring instruments in the industrial measurement technique the precise evaluation of measurement uncertainty of a defined measuring task is an elementary parameter of quality assurance.

The most difficult problem in modern uncertainty evaluation is to develop an appropriate model of the measuring process suitable for the evaluation of the measurement uncertainty [1]. The evaluation of measurement uncertainty is based on both, the knowledge about the measuring process and the quantities which influence the measurement result. The knowledge about the measuring process is represented by the model equation which expresses the interrelation between the measurand and the input quantities. In modern uncertainty analysis [1], the knowledge about the input quantities is expressed by appropriate probability density functions (PDF), whereas the measurement process is represented by a so-called model equation. It expresses the mathematical interrelation between the measurand Y and the input quantities $X_1,...X_N$ [3].

A model may serve to evaluate the original measuring process or to draw conclusions from its behaviour.

Different procedures provided with the actual state of the art in modelling are existent [3-8]. Nevertheless, no publication has been ascertained which keeps in mind:

- how to find the appropriate detailedness of the model,
- the number of influence quantities to analyse,
- when an adequate model is achieved,
- how to verify a model.

Some solutions regarding this problems have already been suggested. This paper presents a versatile approach to the modelling of measurements that is based on the successive decomposition of the measuring system. Influence quantities along the measuring chain are detected step by step and integrated in the model, based on the available knowledge of its associated uncertainty [4]. In this regard the economical aspect of quality inspections is considered. It poses a big challenge to set up the whole functional principle of complex measuring systems. But for the limited purpose of measurement uncertainty evaluation the effort on system analysing and modelling will considerably be reduced if a systematic structured procedure is applied as a course of action for technicians.

Existing modelling procedures are lacking an exit condition for the detailedness of the model. It is the ambition of this paper to extend existing procedures by an approach to the exit condition of modelling and to verify it by a practical appliance.

2. ITERATIVE MODELLING PROCEDURE

The presented approach enables the construction of models for evaluating measurement uncertainty also for complex measuring systems by using a combination of different pragmatic and theoretic principles, whereas the modelling concept is based on step-by-step decomposition of the measuring chain.

2.1. Step-by-step decomposition

Decomposition in this context refers to the process of breaking a complex problem down into easily-understood and achievable parts. Its advantages are the structuring and the reducing of complexity of the measuring process. For the purpose of uncertainty evaluation the step-by-step decomposition is used to create an appropriate model of the measuring process. One of the most important criterions of modelling is how to find the appropriate level of detailedness. If the model is not decomposed sufficiently, the evaluation of the measurement uncertainty would not be correct. If the model is detailed too much, the effort for measurement uncertainty estimation is inefficient high. As an answer to this problem an exit condition of the model decomposition is required.

The initial situation of modelling is the black-box model. Within this basic decomposition step the measuring process is described as black box (Fig. 1).

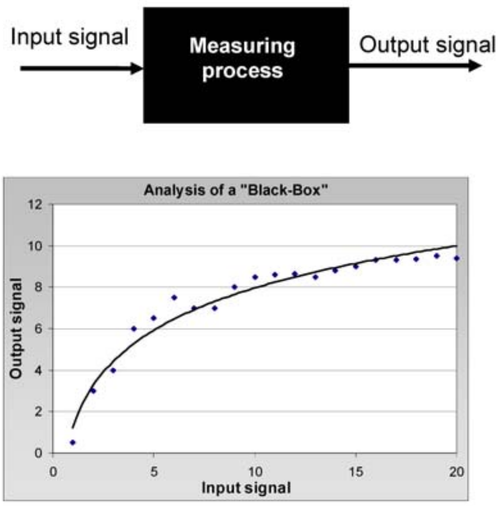

Fig. 1. Black-box principle with functional interdependency of input and output signal (example) [9].

For models of that kind repeated measurements directly deliver values of the measurand. It is not necessary to measure other quantities that are functionally dependent on the measurand. In several cases this low level of model detailedness is sufficient for measurement uncertainty estimation.

The principal point of decomposition is how to fractionise an extensive measurement system into its functional elements. There the extent and the dimension of several components are of extreme importance. It is the aim to find technologically expedient and economically justifiable components. In accordance with the following rules the systematic decomposition of measurement systems into several functional components is being simplified and more practicable:

- Components are created with the objective of reutilisation.
- A component should be self-contained.
- A component should be clearly delimitable.

2.2. PUMA-procedure

A practical, iterative, GUM-based [1] course of action for uncertainty estimation is given in the “Procedure for the Uncertainty Management” (PUMA) described in the supplement of ISO 14253-1 [2] (Fig. 2).

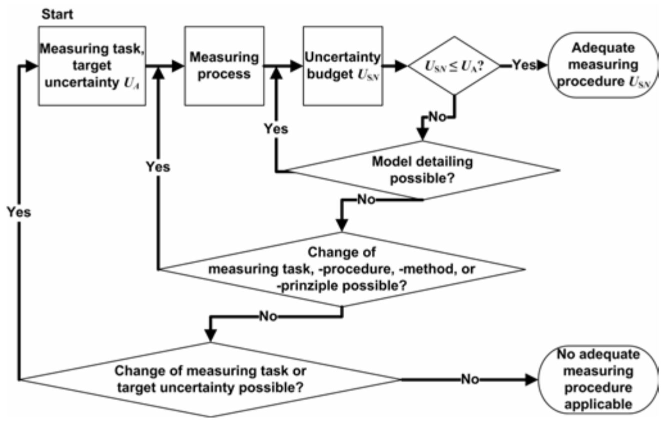

Fig. 2. Iterative method for uncertainty estimation – Procedure for the Uncertainty Management (PUMA) [2].

The procedure is based on worst-case estimates of uncertainty components. The iterations regulate the amount of the intentional too-high estimated uncertainty contributions. These worst-case estimates are imperative to prevent wrong decisions based on measurement results. The amount of too-high estimated uncertainty contributions depends on the economical assessment of the total company situation.

According to this approach the measuring process model is being detailed step by step.

2.3. Exit condition of the model decomposition

It is important to know the state of a suitable model or with other words: When is the condition of a sufficiently detailed model fulfilled? Two cases are distinguished as solution approach of that problem:

1. From the tolerance of the inspection feature to be measured follows a target value of the measurement uncertainty, e. g. by using the “Golden Rule of Measurement”, which demands an uncertainty-tolerance ratio of 1:10 or by using effects of erroneously deciding conformity caused by measurement uncertainty [10]. Based on the target value for the first modelling step – the black-box model – the measurement uncertainty is being estimated. This estimated measurement uncertainty of the first decomposition step is being compared with the target value of the measurement uncertainty. If the target value is not exceeded, an adequate model is being found. Otherwise, another step of decomposition should follow. The principle of worst-case estimates [2] is being used for the characterisation of model quantities. After every decomposition step the achieved measurement uncertainty is compared with the target value of the measurement uncertainty.

2. The second criterion for the exit condition of modelling uses the difference between achieved measurement uncertainties in subsequent decomposition steps. Also in this case the principle of worst-case estimates [2] is being used for the characterisation of model quantities. If the effect of a decomposition step is not essential or, with other words, if the difference between achieved measurement uncertainties in two subsequent decomposition steps is smaller than a special limit value, the exit condition of a sufficiently detailed model is met.

If a sufficient detailed model is found and the resulting measurement uncertainty is still higher than the target value, main uncertainty sources have to be eliminated or the measurement equipment has to be improved.

3. GRAPHICAL VISUALISATION OF THE MODEL

Within the cooperative research project MST-UNCERT the application software "Model Assistant" was developed. The software intends a stepwise user-guide for graphical modelling. The graphical visualisation and connection of basic modelling elements is realised (Fig. 3). Quantities of the model are being characterised conforming to the GUM [1]. Components of a model called modules can be saved and reused. It is possible to export models as xml-file and to use them in other programs for uncertainty calculation. On the basis of these models the uncertainty calculation can be realised according to GUM [1] or by means of "Monte-Carlo Simulation" [11, 12].

In this paper the "Model Assistant" is used for the exemplary demonstration of the stepwise procedure for efficiency modelling.

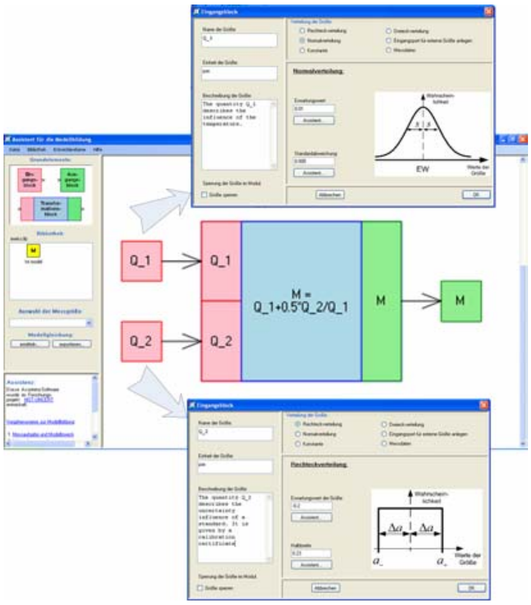

Fig. 3. "Model Assistant" with a simple model in the second decomposition step. For characterisation of the input quantities Q_1 and Q_2 separate dialog windows are used.

4. EXPERIMENTAL EXEMPLIFICATION OF THE PROCEDURE

The presented modelling procedure based on decomposition was tested using a typical measuring task. The measurand was the radius of a 2D-calibration standard with given expanded measurement uncertainty $U_{P=95\%} = 0.25$ µm. The measuring equipment consists of a 2D-coordinate measuring machine (CMM) and an optical sensor with CCD-camera and image processing system. For instance as tolerance a maximal permissible error (MPE) for the optical coordinate measuring machine is given with MPE=2 µm. The measurement was realised with magnification of 10 diameters. This has required a stepwise measuring procedure by moving the CMM axes between several measuring steps (Fig. 4).

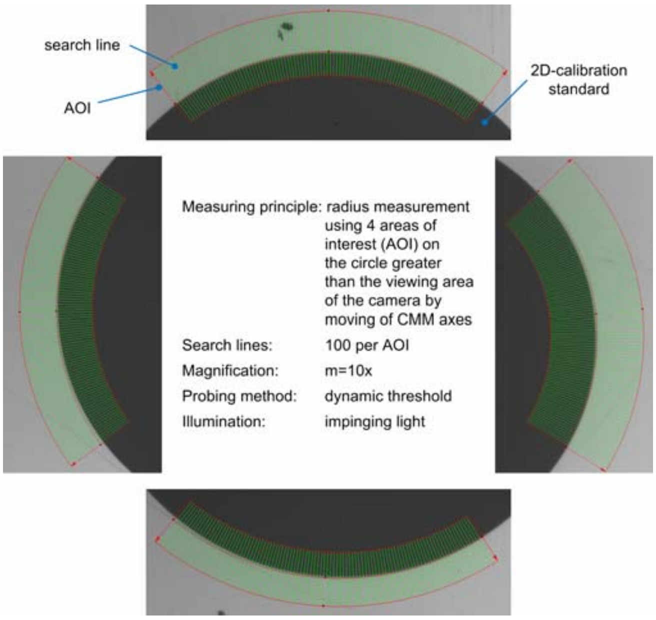

Fig. 4. Principle of the radius measurement using a 2D-calibraion standard.

The exemplification of the presented procedure demands using the decomposition method for evaluating measurement uncertainty. The first decomposition step is based on repeated measurements that directly deliver values of the measurand. The deviation of these values represents effects of all influence quantities of the measuring process.

In repeated measurements (Fig. 5) a standard deviation (1) of $s=u_c$=0.02 µm for optimal measuring conditions and parameter settings was calculated.

$$s = u_c = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2 \qquad (1)$$

Under unfavourable measuring conditions and parameter settings for example in case of measurements realised by non-experts a standard deviation of $s=u_c$=0.15 µm was achieved.

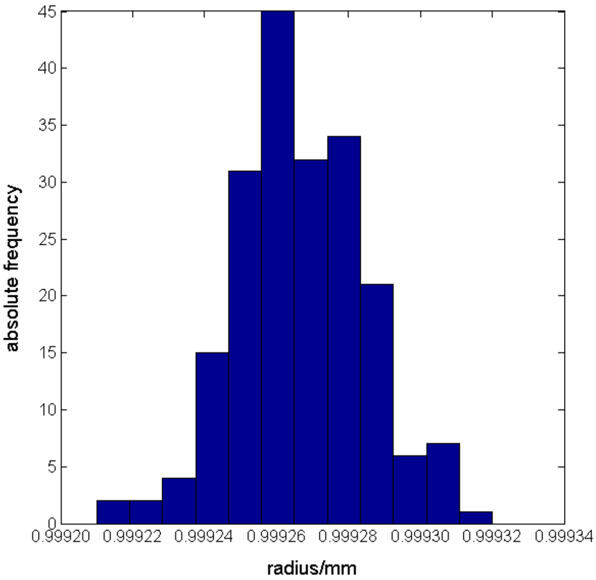

Fig. 5. Histogram of 200 measurement values for the radius in case of optimal measuring conditions and parameter settings.

The expanded measurement uncertainty $U_{\mathrm{P}} = u_c \cdot k$ for the coverage probability of $P = 95\%$ varies between $U_{\mathrm{P=95\%}} = u_{\mathrm{c}} \cdot k \approx u_{\mathrm{c}} \cdot 2 = 0.04...0.30$ µm for this initial decomposition step.

The given MPE=2 µm for the measuring task corresponds with a tolerance *T*=2·MPE=4 µm. On the basis of the first exit condition for the decomposition a target value of the expanded measurement uncertainty results to *U*=*T*/10=0.4 µm. Comparing the estimated expanded uncertainties with the target value the first exit condition is succeeded.

For exemplification of the second exit condition the next decomposition step is necessary. In the second step the measuring process is decomposed into three clearly delimitabled and self-contained components (Fig. 6): The coordinate measuring machine (CMM), the probing system and the unit under test (UUT).

The calibration certificate of the CMM gives a maximal permissible error (MPE) of *1.6 µm* for 2D-measurements. Corresponding to [1] a type-B evaluation of measurement uncertainty is realised. A rectangular probability density function (PDF) with half-width=*1.6 µm*=MPE is used to characterise the influence contribution of the CMM.

The manufacturer of the optical probing system specifies MPE=0.6 µm for 2D-measurements. This component is characterised as rectangular PDF too.

On the calibration certificate of the UUT, the 2D-calibration standard, the expanded measurement uncertainty is given with $U_{P=95\%} = 0.25$ µm. A normal or Gaussian PDF with standard deviation $s = u = U_{\mathrm{P=95\%}}/2 = 0.125$ µm is used to describe the error of the UUT.

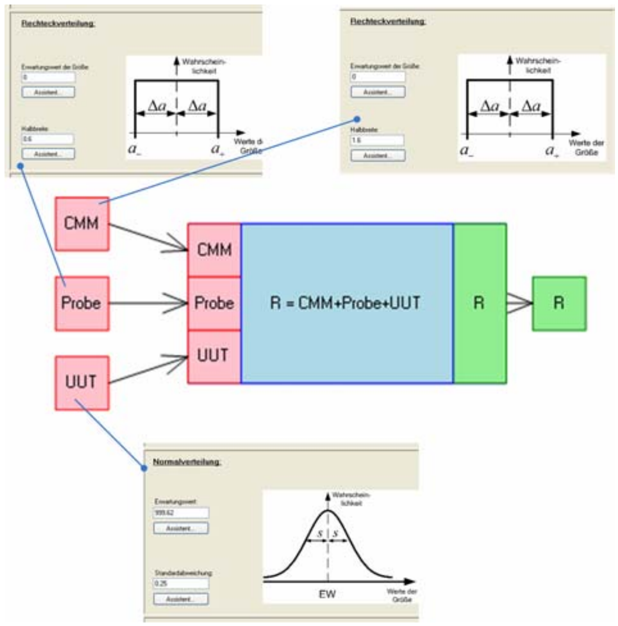

Fig. 6. Model of the second decomposition step with the components coordinate measuring machine (CMM), probing system (Probe) and the unit under test (UUT) using the "Model Assistant".

Each of this three components delivers uncertainty contributions higher than the combined uncertainty of the black-box measurement. The causes of that effect are on the one hand the worst-case error declarations given by the manufacturers and on the other hand not estimatable correlations between the influences.

According to [1] the expanded measurement uncertainty in the second decomposition step yields to:

$$U_{P=95\%} = 2 \cdot u_c = 2 \cdot \sqrt{\left(1.6/\sqrt{3}\right)^2 + \left(0.6/\sqrt{3}\right)^2 + \left(0.125\right)^2} = 1.99 \text{ µm}. \quad (2)$$

This means, in the second step the first exit condition is not succeeded. The second exit condition, based on the comparison of the results of two subsequent decomposition steps, is succeeded. The achieved expanded measurement uncertainty in the second decomposition step is substantially higher than in the first step.

Only for the purpose of demonstration the third decomposition is realised. Within this step the probing system is decomposed in optical system and image processing (Fig. 7).

For the optical system the expanded uncertainty is given with $U_{P=95\%} = 0.30$ µm. A normal or Gaussian PDF with standard deviation $s = u = U_{\mathrm{P=95\%}}/2 = 0.15$ µm is used to describe the error of the optical system.

The influence of the image processing according to the described measuring task was estimated by experiments using type-A evaluation of measurement uncertainty [1]. Also a normal PDF with standard deviation $s = u = U_{\mathrm{P=95\%}}/2 = 0.01$ µm is used to characterise this influence quantity.

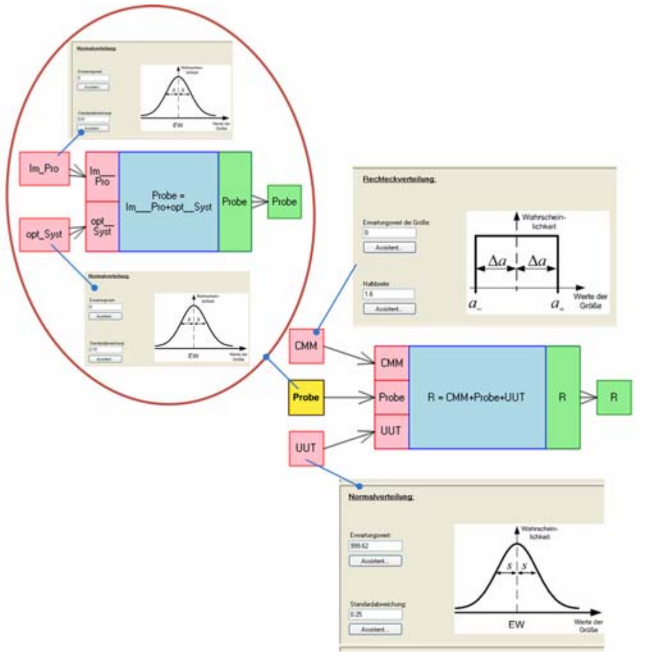

Fig. 7. Model of the third decomposition step with the component probing system (Probe) as submodel using the "Model Assistant".

According to [1] the expanded measurement uncertainty in the second decomposition step yields to:

$$U_{P=95\%} = 2\cdot u_c = 2\cdot\sqrt{\left(1.6/\sqrt{3}\right)^2+\left(0.125\right)^2+\left(0.15\right)^2+\left(0.01\right)^2} = 1.89\ \mu\text{m}. \quad (3)$$

From this follows, also in the third step the first exit condition is not succeeded. In relation to the second decomposition step a lower uncertainty is derived.

Figure 8 visualises the complete results of all three decomposition steps for the exemplified measuring task. It is shown that for the described example the first decomposition step delivers the lowest uncertainty. It is assumed that in several cases this low level of model detailedness is sufficient for measurement uncertainty estimation.

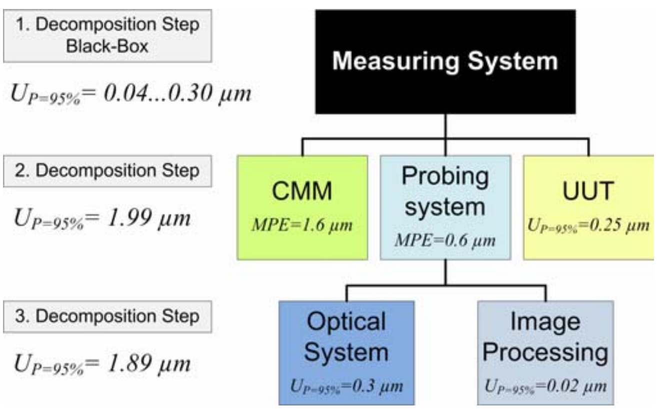

Fig. 8. Decomposition steps of the radius measurement with 2D-CMM and optical sensor system combined with image processing

5. DISCUSSION

The presented approach enables the evaluation of measuring uncertainty also for complex measuring systems by using a combination of different pragmatic and theoretic principles, whereas the modelling concept is based on the idea of the measuring chain. The measurand and other influence quantities are considered as causative signals.

Exemplary, it was shown how influence quantities are built in the model and characterised. Relating to the measurement task it was ascertained, that under special conditions, the empirical combined standard uncertainty can go below the given MPE values of several quantities. For a complex measuring system like the described optical CMM decomposition yields to higher uncertainties because of worst-case estimations and unknown correlations between the influence quantities. Otherwise, the black-box model requires a representative number of measurement repeatings.

6. CONCLUSION

Consequently, a stringent systematic plan of procedures with the ambition to evaluate the measurement uncertainty in technical and economical points of view is given to be used by technicians in measurement technology. It has been shown that the two suggested exit conditions for model decomposition are usable for the special purpose of measurement uncertainty evaluation.

Fundamentally, this procedure is based on the actual state of the art in modelling [3-8].

The advantage of the presented novel approach is to weigh the effort of unnecessary high levels of model detailledness against the amount of the resulting measurement uncertainty.

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