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EFFECT OF FINITE MEDIUM SPEED OF SOUND ON CORIOLIS MASSFLOWMETERS

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Abstract: A theory is presented for the interaction between a vibrating tube and the enclosed flowing fluid with finite speed of sound. From this the influence of the speed of sound on the massflow measurements is derived. This model is valid for compressible mediums and for flow velocities which may be quite high (» 100m/s) but still subsonic. Such velocities can easily be reached in a Coriolis mass flowmeter (CMF) for gas application. The exact theoretical calculation is complex and the results can not be written in a simple one line equation. Nevertheless, we approximate the exact formula to estimate the deviation between true mass flow and the CMF reading. This deviation can be described with mainly two terms:

The first term is proportional to $(kd)^2 = \left(\frac{2 p f}{c} d\right)^2$, *with f being the resonant frequency c being speed of sound of the medium, and d being the diameter of tube. This term contributes most to the observed deviation. The second term depends on density and velocity of fluid. A comparison of this model with experimental data shows a good agreement.*

Keywords: eigenfrequency, speed of sound, phase shift, time lag, compressibility

1. INTRODUCTION

Coriolis mass flowmeters (CMF) have proven to record the total mass flow to better than 0.1 % for water at moderate flow velocities (< 20 m/s). Each Coriolis instrument is calibrated with water and gets its own calibration factor before it is implemented into a system. The calibration factor represents the sensitivity of the CMF. The sensitivity is mainly given by geometrical data and material properties of the measuring tube. However, theory and experiments have revealed, that the calibration factor has also some dependence on speed of sound of the fluid. In general, if we do not change the calibration factor of the instrument, then the CMF will slightly overestimate the mass flow for mediums with low speed of sound, such as gases. Here we construct a model to characterize the deviation, which is induced to the mass flow determination due to low speed of sound and/or high velocity of the medium.

2. MODEL

2.1. Description

We consider a straight tube of length L, wall thickness h, and inner diameter d. The tube is conveyed by a compressible (speed of sound < infinity) fluid or gas at velocity U. The velocity U shall be subsonic and the fluid or gas is considered inviscid. Since we take into account the speed of sound of the fluid, it is not possible to use the Kirchhoff-Love beam theory and we have to work out a shell theory specialized to bending vibration of long tubes. Shell theory is needed since the beam theory is essentially one dimensional and does not allow any surfacial loading (as fluid loading) inside a tubular cross-section. In principle, this shell theory with circumferential wave number $m = 1$ merges the Kirchhoff-Love beam theory as the ratio L/R tends to infinity. The solution is build for velocities U, which can reach values close to the speed of sound of the fluid or gas. Furthermore, we take into account the case where the velocity U might be in the order of wl, which is the speed of a particle passing the length of the tube in one period of the eigenfrequency. We search eigenfrequencies and phase shifts of this system.

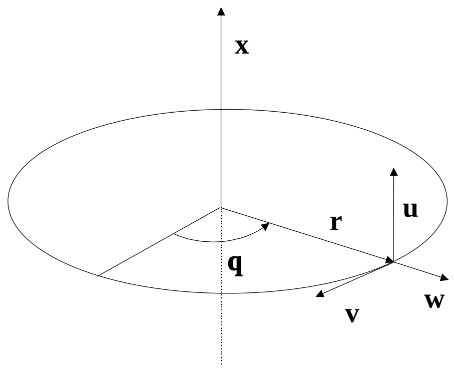

Figure 1. This figure shows the orientation of the vectors in the cylindrical and the circumferential coordinate system. x indicates the direction of the tube.

2.2. Fundamental equations and assumptions

This theory is based on two fundamental equations: First, on the Helmholtz equation describing the hydrodynamic of a fluid in a tube and, secondly, on the Kirchhoff shell equations describing displacements of a vibrating tube.

By solving the fundamental Helmholtz equations we can describe the pressure p, which is invoked by the moving fluid to the fluid-shell interface. The Helmholtz equation in cylindrical coordinates is

(1) $$\frac{\partial^2 \tilde{j}}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{j}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \tilde{j}}{\partial \Theta^2} + \frac{\partial^2 \tilde{j}}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 \tilde{j}}{\partial t^2} - \frac{2U}{c_0^2}\frac{\partial^2 \tilde{j}}{\partial x \partial t} - \frac{U^2}{c_0^2}\frac{\partial^2 \tilde{j}}{\partial x^2} = 0,$$

with $\tilde{j} = j_{(t,x)}$: Velocity potential of the fluid, U: Velocity of the fluid along the tube, c_0: speed of sound of the medium, r: Distance from tube axis, x: Distance along the axis of the tube, Θ: Angle (Figure 1). For the hydrodynamic system, we have to consider two boundary conditions. The first boundary condition indicates, that there is free penetration of fluid particles to a volume inside the tube or free leaving from this volume.

(2) $$j_{(r,x)} = 0 \qquad \text{for} \qquad x = 0,1 \qquad x = x/L$$

The second boundary condition is the compatibility condition, which postulates that at the fluid-shell interface the normal velocity of a fluid particle is the same as the velocity of the wall:

(3) $$\frac{\partial \tilde{j}}{\partial r} = \frac{\partial \tilde{w}}{\partial t} + U\frac{\partial \tilde{w}}{\partial x}$$

The second fundamental set of equations are the Kirchhoff shell equations (4), which describe the displacements of a tube:

(4a) $$\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{1-v}{2}\frac{1}{R^2}\frac{\partial^2 \tilde{u}}{\partial \Theta^2} - \frac{r(1-v^2)}{E}\frac{\partial^2 \tilde{u}}{\partial t^2} + \frac{1+v}{2}\frac{1}{R}\frac{\partial^2 \tilde{v}}{\partial x \partial \Theta} + \frac{v}{R}\frac{\partial \tilde{w}}{\partial x} = -\frac{q_1(1-v^2)}{Eh}$$

(4b) $$\frac{1+v}{2}\frac{1}{R}\frac{\partial^2 \tilde{u}}{\partial x \partial \Theta} + \frac{1-v}{2}\frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 \tilde{v}}{\partial \Theta^2} + 2\frac{h^2(1-v)}{12}\frac{1}{R^2}\frac{\partial^2 \tilde{v}}{\partial x^2} - \frac{h^2}{12}\frac{1}{R^4}\tilde{v} - \frac{r(1-v^2)}{E}\frac{\partial^2 \tilde{v}}{\partial t^2} +$$
$$+ \frac{1}{R^2}\frac{\partial \tilde{w}}{\partial \Theta} - \frac{h^2}{12}\frac{2-v}{R^2}\frac{\partial^3 \tilde{w}}{\partial x^2 \partial \Theta} - \frac{h^2}{12}\frac{1}{R^4}\frac{\partial^3 \tilde{w}}{\partial \Theta^3} = -\frac{q_2(1-v^2)}{Eh}$$

(4c)
$$\frac{v}{R}\frac{\partial \tilde{u}}{\partial x} + \frac{1}{R^2}\frac{\partial \tilde{v}}{\partial \Theta} - \frac{h^2}{12}\frac{2-v}{R^2}\frac{\partial^3 \tilde{v}}{\partial x^2 \partial \Theta} - \frac{h^2}{12}\frac{1}{R^4}\frac{\partial^3 \tilde{v}}{\partial \Theta^3} + \frac{\tilde{w}}{R^2} + \frac{h^2}{12}\frac{\partial^4 \tilde{w}}{\partial x^4} +$$
$$+ 2\frac{h^2}{12}\frac{1}{R^2}\frac{\partial^4 \tilde{w}}{\partial x^2 \partial \Theta^2} + \frac{h^2}{12}\frac{1}{R^4}\frac{\partial^4 \tilde{w}}{\partial \Theta^4} + \frac{r(1-v^2)}{E}\frac{\partial^2 \tilde{w}}{\partial t^2} = \frac{(q_3 + \tilde{p}.n)(1-v^2)}{Eh}$$

with q_i: components of driving loads acting at the shell, h: Thickness of tube, $\tilde{u} = u_{(t,x)}$: Axial displacement, $\tilde{v} = v_{(t,x)}$: Circumferential displacement, $\tilde{w} = w_{(t,x)}$: Lateral displacement (see Fig.1), E: Young's module, r : Density of tube, n: Outward normal to volume occupied by the fluid, $\tilde{p} = p_{(t,x)}$: Acoustic contact pressure at fluid-tube-interface.

By solving the Kirchhoff equations we find eigenfrequencies and phase shifts of the mechanical system.

To describe velocity potential and the beam-like movements of the clamped-clamped tube we use the physically meaningful assumptions

(5) $\tilde{j} = j_{(x)} \cos(m\Theta)\, e^{-ivt}$, $\tilde{u} = u_{(x)} \cos(m\Theta)\, e^{-ivt}$, $\tilde{v} = v_{(x)} \sin(m\Theta)\, e^{-ivt}$, and $\tilde{w} = w_{(x)} \cos(m\Theta)\, e^{-ivt}$

where m is the mode number. The time dependence for these functions (5) are assumed to be e^{-ivt}. We consider only beam-type motions ($m = 1$). Therefore, there is no disturbance of the cross sectional shape of the tube. As we consider clamped-clamped condition for the tubes, we get the following boundary condition for the displacements:

(6) $u_{(x)} = v_{(x)} = w_{(x)} = w'_{(x)} = 0$ for $x = 0, 1$ with $x = x/L$

If we set $m = 1$ then circumferential and axial coordinates (v, u) can be expressed as a function of lateral coordinates

(7) $v = -w$ and $u = R\dfrac{\partial v}{\partial x} = Rv' = -Rw'$

and the coordinates $v_{(x)}$ and $u_{(x)}$ can be expressed as a function of $w_{(x)}$. For $m = 1$ the lateral displacements $w_{(x)}$ can be described by the trial function

(8) $w_{(x)} = C_1 X_{1(x)} + C_2 X_{2(x)}$,

where X_1 and X_2 are beam functions for clamped-clamped condition and C_1 and C_2 are modal amplitudes. The beam functions are eigenmodes of a beam, which depend on the boundary conditions of the tube. The tube parameters define the beam functions completely, whereas C_1 and C_2 have to be found during the following calculations.

2.3. Determining eigenfrequency and phase shift

First, the Helmholtz equation (1) for the above defined boundary conditions (2) and (3) yields the velocity potential j of the fluid along the tube. The velocity potential j describes the contact pressure p at the fluid-tube-interface, which is

(9) $$p = -r_0\left(-iwj + \frac{U}{l}\frac{\partial j}{\partial x}\right) = p(C_1, C_2).$$

The contact pressure is not only a function of C_1 and C_2, but also it is a function of system parameters

(10) $$p = p(w, r_0, U, L, c_0)$$

For $U = 0$ the contact pressure is purely real and becomes complex as the fluid begins to flow. Then the complex contact pressure p can be written as

(11) $$p = \mathrm{Re}(p) + i \cdot \mathrm{Im}(p)$$

Since the formulation of the contact pressure is quite cumbersome, we will not illustrate the detailed formula for *p*, here.

To describe the lateral movement $\mathrm{w}_{(x)}$ of the tube, we solve the three Kirchhoff equations (4a-c) for the clamped-clamped conditions (6) while including the hydrodynamic contact pressure p (11). As we replace $\mathrm{w}_{(x)}$ with the beam function (8) and as we apply Galerkin averaging, we get a linear system of two equations in C_1 and C_2.

(12) $$G \cdot \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0, \qquad \text{with} \qquad G = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}.$$

k_{ij} are complex expressions depending on system parameters. In particular, k_{ij} depends on the velocity U, on the speed of sound of the fluid c_0, and on the eigenfrequency w of the system. Natural frequencies w in the current system can be found by putting $\det(G) = 0$.

To find the phase shift between any two points of the tube one of the coefficients, C_1 or C_2, in equation (10) can be put to 1 and equation (8) becomes

(13) $$\mathrm{w}_{(x)} = X_{1(x)} + C_2 X_{2(x)}$$

C_2 is determined by solving one of the two equations in formula (12). With this, we have found the lateral displacement for any point on the tube as a function of system parameters. The amplitude of the lateral displacement at any point of the tube is

(14) $$A_{(x)} = \sqrt{\mathrm{Re(w)}^2 + \mathrm{Im(w)}^2}$$

and the phase shift between the driving force any point on the tube is

(15) $$j_{L(x)} = \arg tg\left(\frac{\mathrm{Im(w)}^2}{\mathrm{Re(w)}^2}\right)$$

Finally, the phase shift between any two points x_+ and x_- along the tube is

(16) $$j_{L(x_+, x_-)} = arctg\,\frac{\mathrm{Im}\left(\mathrm{w}_{(x_+)}\right)}{\mathrm{Re}\left(\mathrm{w}_{(x_+)}\right)} - arctg\,\frac{\mathrm{Im}\left(\mathrm{w}_{(x_-)}\right)}{\mathrm{Re}\left(\mathrm{w}_{(x_-)}\right)}$$

and the time lag becomes

(17) $$t = \frac{j_{L(x_+, x_-)}}{w}.$$

The time lag t is depending on mass flow, as well as on tube and fluid parameters. Therefore, we can determine the mass flow through

(18) $\dot{m} = k' \cdot t$ with $k' = k'_{(Tube\ parameters,\, c_{Fluid},\, U_{Fluid},\, w,\, Fluid\ parameters)}$

In the elementary theory of CMF, k' (sensitivity) depends on tube parameters only. In our model, k' depends also on fluid properties and velocity of the fluid in a complex way. While using this model, we can determine the deviation of the mass flow, which is introduced by low speed of sound and/or by high velocities of the fluid. Unfortunately, the code to determine the time lag with this model is quite cumbersome and can not be expressed by a simple one-line-equation. Therefore, we further develop a formula to approximate the deviation, which is introduced to the mass flow by speed of sound and velocity of the fluid.

3. FORMULA TO APPROXIMATE MASS FLOW DEVIATION

The model outlined in Section 2 describes the time lag between any two point on the tube. For this system time lag is generally considered to depend on tube parameters and on mass flow, only. Thus, mass flow becomes proportional to the measured time lag. In contrast, the model outlined above indicates, that time lag depends on system parameters others than mass flow as well; i.e. speed of sound of fluid and fluid velocity. Since the main contribution to the time lag still comes from the mass flow ($\dot{m}$), we can characterize the time lag as follows:

(19) $t = t(\dot{m}) + \Delta t$ with $\Delta t = \Delta t(c_F, v_F, r_F, r_T, w, d, L)$,

$\dot{m}$: mass flow, c_F : speed of sound of fluid, v_F : velocity of fluid, r_F : density of fluid, r_F density of tube, w : frequency, d : inner diameter and L length of vibrating tube.

The relative influence of Δt to the total time lag is defined as *delta,*

(20) $$delta = \frac{\Delta t}{t} \cdot 100\,\%\,.$$

Delta can be approximated by the following equation

(21) $$delta = a_1 \cdot (kd)^2 + a_2 \cdot \frac{A_f \cdot r_f}{A_f \cdot r_f + A_t \cdot r_t} \left(\frac{2}{L \cdot w}\right)^2 v_{Fluid}^2$$

with $kd = \frac{2 \cdot p \cdot f}{c_{Fluid}} \cdot d$, $A_{f,t}$: cross area of fluid and tube and a_i being constants to be determined. We will briefly discuss the two terms in equation (21):

1. For low speed of sounds the first term is largely dominant. This term is increasing with decreasing speed of sound of the fluid. For low speeds of sound (e.g. 300 m/s) this term can increase the total time lag t by several percents.
2. The second term depends on tube parameters as well as upon velocity and density of fluid. The coefficient a_2 is in the order of unity. Note that this term is independent on the speed of sound of the fluid. This term origins from changing eigenfrequencies as fluid velocity is increased ($f = f_{(v)}$).

By using the simple approximation (21) we can increase the accuracy of mass flow determination for Coriolis type mass flow meters. For fluids with moderate velocities and with high speed of sound (> 1000 m/s) *delta* is small and below 0.05%. *Delta* only becomes important for gases at high flow velocity and/or at low speed of sound. For the later case, *delta* typically reaches values of permille to percents.

4. COMPARISON WITH EXPERIMENTAL DATA

We have many experimental data available to estimate the influence of the speed of sound on the accuracy of the mass flow determination. The measurements have been performed with different CMF with nominal diameters ranging from 25 mm to 100 mm. Although the behaviour of different instruments is slightly different, they show an increasing deviation, which is proportional to $(kd)^2$. The experimental data suggest that the deviation is in the order of

$$delta\_experimental \cong 12.4\ ...\ 15.2 \cdot (kd)^2$$

On the other hand, the model outlined in Section 2 indicates a theoretical dependence on $(kd)^2$ of

$$delta\_model \cong 13.0 \cdot (kd)^2 ,$$

which is in good agreement with the experimental results.

5. CONCLUSION

Based on the theoretical and experimental studies we can increase the accuracy of Coriolis massflow meters by simply taking into account the correction shown in formula (21). Although this correction can be neglected for most fluids, it becomes relevant for gases with low speed of sound and/or at high velocities. For the latter case the correction may be in the order of percents of the totally determined mass flow.

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