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NATURAL GAS ENERGY FLOW (QUALITY) UNCERTAINTY ESTIMATION USING MONTE CARLO SIMULATION METHOD.

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ABSTRACT

In the natural gas market, open access along with gas brokering and marketing has resulted in multiple gas contracts through one physical measuring point. Accurate metering of natural gas has become more important than ever as de-regulation subjects pipeline companies to competition. A more competitive market is driving the need for real-time accurate electronic flow measurement. Modern electronic natural gas metering systems (ENGMS) introduced additional, though necessary, complexity in the estimation and verification of the reported results. Additionally, it becomes more and more important to be able to verify these results. The application of Monte Carlo simulation as a combined energy flow measurement uncertainty estimation method seems to offer specific advantages over the more complex, traditional uncertainty estimation methods while at the same time fully conforms with the method of the ISO/GUM, the authoritative document for uncertainty evaluation. Since Monte Carlo simulation relies on randomness, it seems to capture more naturally and more related to the underlying physics of measurement uncertainty. Typical comparisons of estimated uncertainties by the model and the conventional method (RSS) have been carried out. In general the Monte Carlo method gives slightly higher estimated uncertainties. This is due to the fact that the simplified conventional methods inevitably neglect correlations between the variables.

Keywords: Uncertainty estimation, Monte Carlo simulation, Natural gas, Energy, complex systems.

1 INTRODUCTION

During the last few years, there have been substantial developments in the field of uncertainty analysis leading to the publication by the International Standards Organisation (ISO) of the Guide to the expression of Uncertainty in Measurement, in which an international widely accepted methodology has been established. GUM has become the authoritative document for all aspects of uncertainty evaluation and terminology. Thus, the ISO/5168, 1978, which covers flow measurement uncertainty estimation issues, is currently under revision to conform to the GUM.

There is a growing realisation within the natural gas industry that it is necessary to have more reliable methods of estimating the uncertainty of the natural gas flow rate measurement and, more particularly, the energy flow measurement. All interested parties require faster and more reliable (accurate) information from the measuring point to make quick and appropriate business decisions and to minimise imbalances. All these aspects are specifically addressed and emphasised in the Directive 98/30/EC of the European Parliament and of the Council of the European Union of 22 June 1998 concerning common rules for the internal (EC) market in natural gas. Further details are also included in two subsequent reports on harmonisation requirements. Coherent technical as well as commercial trading rules is considered to be critical to the proper functioning of the internal EC market. A harmonisation of gas metering and accounting methods is demanded.

Orifice meters account for 80 plus percent of installed process plant meters. The traditional simple orifice meters used for natural gas flow measurements have now been replaced by modern electronic orifice metering systems. They employ a whole set of advanced electronic equipment ranging from the latest smart pressure and temperature transmitters, on line densitometers, relative density analysers, and flow computers to on line gas chromatographs for the continuous measurement of gas energy content, flow rate and gas quality.

Although these systems have offered to a certain extent the necessary fast transmission of information, they have created, due to their complexity, difficulties in the proper estimation of combined measurement uncertainties. The difficulty of reliable measurement uncertainty estimation increases almost exponentially with complexity of the measurement system. The computational operation of such modern electronic gas metering systems is considered in the methodology reported in this paper. In a N. Gas metering system and virtually in any other metering system, two main types of sensors are actually used:

- Direct sensors (physical sensors) that 'directly' sense the variable in question.
- Model based sensors (Lu,Y.Z., 1996) or 'virtual sensors' that infer or estimate the variable in question from other direct measurements based on models or functional relationships. These models constitute additional measurement equations.

It may be said that virtually all the existing sensors are of the second kind since even direct sensors give an output (in voltage or current) that is somehow functionally related to the measurand. Inevitably, the complexity generated due to these interdependencies of direct and model measurements are sometimes extremely difficult to be accounted for in a combined measurement uncertainty analysis.

The traditional analytical methods in orifice flow measurement uncertainty estimation, as specified in ISO/5168, 1978, requires the consideration of all the interdependencies (partial derivatives) amongst all the respective variables. This renders the 'complete' estimation of the combined measurement uncertainty a difficult if not an impossible task. Clause 7.1.5 in ISO 5168 (1978) includes numerous examples of partial derivatives that actually cannot be calculated directly and numerical methods are the only alternatives. ISO-5168 does not take into account other interdependencies that could be generated within a modern electronic orifice metering system due to different measurement models and computational methods employed.

Conventional numerical methods are based on Taylor series expansion usually up to 2^{nd} order. They are not only inefficient in terms of programming effort but also limited due to their intrinsic linear interpolation characteristics. For example, in systems with large uncertainties, considerable non-linearity may exist even within the specific uncertainty intervals of the dependent variables and the covariance terms may also be significant.

A PC based model of a modern Electronic Natural Gas Metering Station (ENGMS) has been developed using MATLAB/SIMULINK. The model and many aspects of the computational methodology are based on a typical design. Energy and mass flow rate uncertainties are estimated using Monte Carlo simulation methods. The model also allows the study of error propagation through the system and the visualisation of not so obvious (obscured) correlations between various parameters in a complex system.

2 UNCERTAINTY ESTIMATION METHODS - CONVENTIONAL VERSUS MONTE CARLO SIMULATION.

2.1 Conventional Methods

The functional relationship (measurement model/equation) between N input variables (X_1, X_2, … X_N) and the output quantity Y being measured (measurand) in a flow measurement process is:

$$Y = f(X_1, X_2, X_3 ..., X_N) \quad (1)$$

Xi includes corrections (or correction factors) and quantities that account for other sources of variability, such as that due to different observers, instruments, samples, laboratories and times at which observations are made e.t.c. Thus, the general functional relationship f in (1) expresses not simply a physical law but a measurement process. Some of these variables are under the direct control of the measurement's 'operator', some are under indirect control, some are observed but not controlled and some are not even observed. As Moffat, R.J., 1982 said, a measurement aiming to obtain the result Y as a function of one or more variables will probably be designed as a *partial derivative measurement*. One or more of the controlled variables are varied hoping that the rest will remain constant throughout the measurement process.

$$Y = f(X_1, X_2, X_3; \quad x_5, x_6 ..., x_N) \quad (2)$$

This is illustrated in Eqn (2) where every variable to the left of the semicolon is treated as a controlled, or at least observed (measured) variable and all the other terms are parameters which are not expected to change, or important during the measurement. The fact that when the controlled variables are held constant and the results still display scatter is because by definition the model/equation is only an abstract representation of the real world. Even finite element models with hundreds of thousands of elements provide a very simple representation of how the physical system actually behaves. While appearing deterministic, such modelling uncertainty can also be characterised using probabilistic analysis method.

The methodology conventional estimation methods can be illustrated using a simple measurement equation with Y as a continuous function of X_1 and X_2.
Y is approximated using a polynomial approximation or nth order Taylor's series expansion about the means of variable X_1 and X_2

$$f\left(X_1, X_2\right) = f\left(x_{1i}, x_{2i}\right) = f\left(x_1, x_2\right) + \frac{\partial f}{\partial X_1}\left(x_{1i} - x_1\right) + \frac{\partial f}{\partial X_2}\left(x_{2i} - x_2\right) + W \tag{3}$$

where x_1, x_2 are mean and W is the remainder.

$$W = \frac{1}{2!}\left[\frac{\partial^2 f}{\partial X_1^{\,2}}\left(x_{1i} - x_1\right)^2 + \frac{\partial^2 f}{\partial X_2^{\,2}}\left(x_{2i} - x_2\right)^2 + 2\frac{\partial^2 f}{\partial X_1 \partial X_2}\left(x_{1i} - x_1\right)\left(x_{2i} - x_2\right)\right] \tag{4}$$

As the partial derivatives are generally computed at $X_1=x_1$ and $X_2=x_2$, they are the same for all i=1,…,N and all the higher terms are normally excluded (W=0). This is acceptable provided that the uncertainties in X_1 and X_2 are small and hence all x_{1i} and x_{2i} are close to x_1 and x_2 respectively. Thus the square terms approach zero more quickly than the first order terms. Additionally, *if $f(X_1,X_2)$ is a linear function, then the second order partial derivatives in 4 are identically zero and so W=0.* Both linearity and 'small' uncertainty are most controversial in fluid flow measurements.

The standard deviation is the *combined standard uncertainty* in Y and can be obtained by (Taylor, J.R.,1997):

$$u(Y) = \sigma(Y) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_i - y\right)^2} = \sqrt{\left(\frac{\partial f}{\partial X_1}\right)^2 \sigma\left(X_1\right)^2 + \left(\frac{\partial f}{\partial X_2}\right)^2 \sigma\left(X_2\right)^2 + 2\frac{\partial f}{\partial X_1}\frac{\partial f}{\partial X_2}\sigma\left(X_1 X_2\right)} \tag{5}$$

This equation gives the uncertainty as a standard deviation *whether or not the measurements of X_1 and X_2 are independent or normally distributed.* It is based on a first order Taylor series approximation of Y. The standard deviations $\sigma(X_1)$ and $\sigma(X_2)$ are referred to in the GUM as the standard uncertainties associated with the input estimates x_1 and x_2. The partial derivatives are the *sensitivity coefficients*, which give the effects of each input quantity on the final results (or the sensitivity of the output quantity to each input quantity). The calculation of sensitivity coefficient of each input quantity can be carried out using either analytical or numerical methods.

The term expanded uncertainty is used in GUM to express the % confidence interval about the measurement result within which the true value of the measurand is believed to lie and is given by:

$$U(Y) = \sqrt{\left(tB_Y\right)^2 + \left(tS_Y\right)^2} = t\sqrt{B_Y^2 + S_Y^2} = tu(Y) \tag{6}$$

For a level of confidence of approximately 95% the value ot t is 2. . Coleman, H.W., and Steele, W.G., 1995, presented a detailed analysis of the subject. The result of a measurement is regarded as only an approximation or estimate of the value of the specific quantity subjected to measurement. The uncertainty of the measurement result can be obtained by two methods.

Type A evaluation of uncertainty is based on any valid statistical method. Calculation of the standard deviation of the mean, least squares curve fitting, analysis of variance (ANOVA) for identification and quantification of random effects are all valid methods.

Type B evaluation of uncertainty is based on scientific judgement and can be carried out by means other than the statistical analysis of a series of observations (data) using all relevant information available. These may include previous measurement data, manufacturer's specifications, data provided by calibration reports, experience with or general knowledge of behaviour and properties of relevant materials and instruments.

Type B evaluation introduces a Bayesian sense or sense of subjectivity. GUM seems to offer this compromise between objective and subjective probabilities and the two major statistical theories are combined for improved uncertainty estimation. Random and systematic uncertainties are treated identically. The term random and systematic uncertainty can actually be misleading when applied in a sense that does not take into account the use made of the corresponding quantity. Depending on how that quantity appears in the measurement equation could condition the type of uncertainty. Component of uncertainty arising from random effects could affect in a systematic way the measurement results. Specific examples of such cases can be found in the new ISO/CD-5167, 1999.

2.2 Monte Carlo Simulation

With the availability of digital computers, numerical experiments have become an increasingly popular method to analyse physical engineering systems. *Simulation* is generally defined as the process of replication of the real world based on a set of assumptions and conceived models of reality, Kottegoda, N.T., and Rosso, R., (1998). Monte Carlo simulation was devised as an experimental probabilistic method to solve difficult deterministic problems since computers can easily simulate a large number of experimental trials that have random outcomes. When applied to uncertainty estimation, random numbers are used to randomly sample *parameters' uncertainty space* instead of point calculation carried out carried out by conventional methods. Such analysis is closer with the underlying physics of actual measurement processes that are probabilistic in nature. Nicolis, G. (1995) pointed out that in nature the process of measurement, by which the observer communicates with a physical system, is limited by a finite precision. As a result, a 'state' of a system must in reality be understood not as a point in phase space but rather as a small region whose size reflects the finite precision of the measuring apparatus. So on the probabilistic view we look at our system through a 'window' (phase space cell), whereas in the deterministic view it is understood that we are exactly running on a phase space trajectory which is clearly an unrealistic assumption in view of the presence of the errors. So the application of Monte Carlo simulation in the uncertainty estimation of different states of a system seems to offer a more realistic approach.

Some advantages of Monte Carlo simulation have been described by Basil, M., and Jamieson, A.W., (1998). Among others, it can handle both small and large uncertainties in the input quantities. Complex partial differentiations are not necessary. It also accounts, if there are known joint distributions, for <u>input</u> covariances or dependencies, Gilks, et.al. (1996).

3. ENERGY FLOW MEASUREMENT PROCESS WITH ORIFICE METERS – THE MONTE CARLO UNCERTAINTY ESTIMATION.

In every purchase, sale, transportation and exchange contract, general terms and conditions with reference to measurement are clearly spelled out. These terms and conditions address four typical areas, a) Definitions of terms, b) Gas quality specifications, c) Method of measurement and d) Measurement equipment. All directly or indirectly point to the importance of measurement and the associated uncertainty. The cost of purchased gas is calculated by:

$$\$ = E_{Flow} * P \qquad (7)$$

$\boldsymbol{E_{Flow}}$: is the energy flow in Kcal/Month and

P: is an agreed Gas energy price in $/Kcal.

It is of interest to note that Energy flow rate measurement is associated with an uncertainty but there is not any kind of uncertainty in terms of money exchange!. The reliable estimation of measurement uncertainty becomes necessary for the selection of the appropriate metering system, its design optimisation and subsequently for the continuous evaluation of its performance, i.e.:

- A priori specification of reasonable desired measurement control limits;
- A posteriori uncertainty estimation of what actually produced.

These limits define the tolerance interval for an indisputable energy purchase transaction between the parties involved. The energy flow rate is calculated by:

$$E_{Flow} = Q * GCV \qquad (8)$$

where Q is the volumetric flow rate in Rm^3/Month and GCV is the Gross or Superior Calorific Value in Kcal/Rm^3

Both Q and GCV are referred to some reference conditions.

The flow rate measurement can be characterised as a quantitative measurement while the measurement of the GCV is a qualitative measurement since it is based on the determination of the gas composition by mole fraction that virtually describes its quality.

The measured instantaneous dry natural gas mass flowrate (in Kg/sec) through a single meter run is given by:

$$q_m = C_d E \varepsilon \frac{\pi d_o^2}{4} \sqrt{2 \Delta P \rho_1} \qquad (8)$$

The respective volumetric flow rate at reference conditions (e.g. 15 ^oC and 1.01325 bar) is:

$$Q = \frac{q_m}{\rho_{ref}} = \frac{C_d E \varepsilon \frac{\pi d_o^2}{4} \sqrt{2 \Delta P \rho_1}}{\rho_{ref}} \qquad (9)$$

where ρ_{ref} is the gas density at reference conditions.

3.1 The Simulation Program

The PC-model developed reports the energy flow rate (MJ/s), mass flow rate (kg/s) and the reference volume flow rate (Rm^3/s). The flow rate is calculated at the upstream conditions (P1, T1) in accordance with ISO-5167. The model consists of two main systems i.e. the flow (quantity) system and the energy (quality) system (Figure 1). :

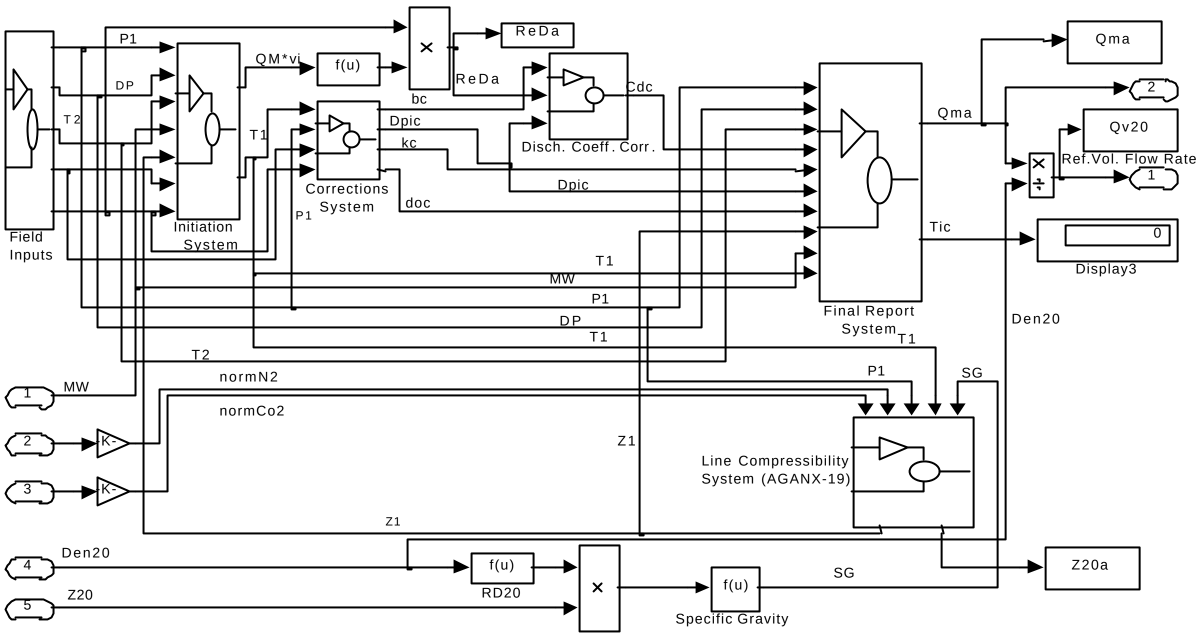

Figure 1. – Flow (Quantity) System.

The other subsystems are

- A field input subsystem that can accept real or simulated input signals;
- An initialisation subsystem that is necessary for the initialisation of the program,
- A corrections subsystem that deals the downstream to upstream corrections (e.g. the conversion of downstream temperature and line density to the upstream values)
- A discharge coefficient subsystem that calculates discharge coefficients (Cd),
- A line compressibility subsystem based on AGANX-19 method.

The energy system includes all the computational modules of a gas chromatograph for the calculation of energy content (Gross Calorific Value) and other necessary parameters such as reference density and reference compressibility according to the relevant ISO–6976 (1996). The final reporting subsystem reports the flow quantity results and other secondary parameters such as sound velocity and statistical parameters (e.g. cumulative means and variances). Variable values can be extracted at any point of the propagation links allowing the study of uncertainty propagation through the system.

The direct measurements that usually take place in a gas metering station is illustrated in Figure 2.

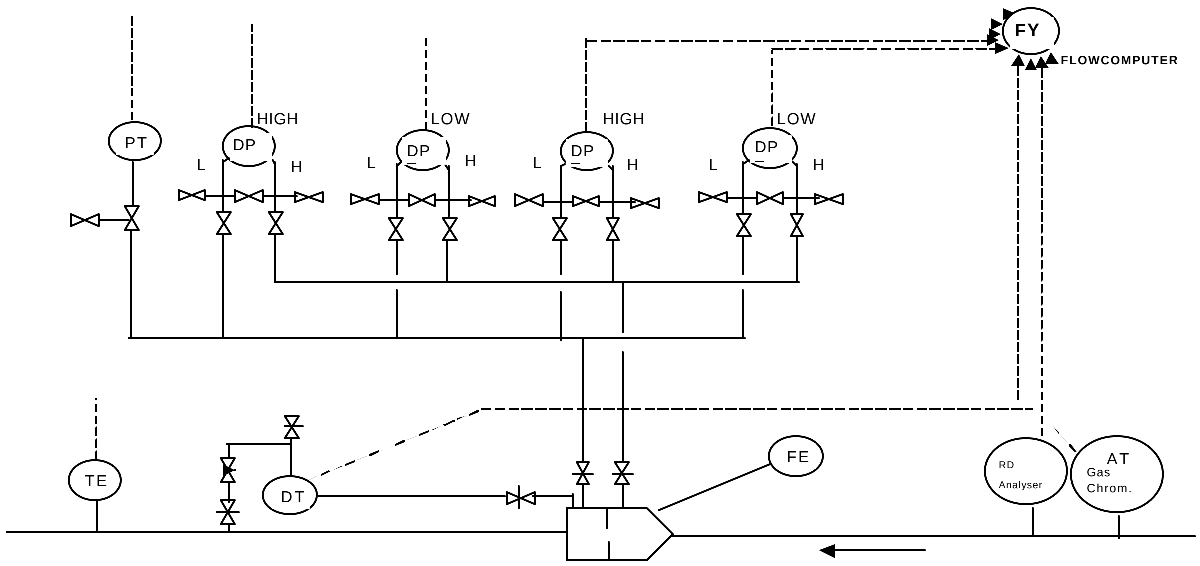

Figure 2. A typical Process and Instrumentation diagram of an orifice metering system (Single run)

There are other direct measurements required. For example, pipe and orifice diameters are calibrated measurements that are occasionally recorded according to a specific calibration program. These occasional calibration measurements are entered as constants in the model based measurement of the 'actual' pipe and orifice diameters taken into account of the real time gas temperature.

$$do = do_C \left(1 + \lambda \left(T_1 - To_C\right)\right) \qquad (10)$$

where , do_C is the direct (calibration) measurement, To_C is the calibration temperature, λ is the orifice material expansion coefficient and T1 is the gas temperature upstream to the orifice plate.

The temperature T_1 is also a model based measurement obtained from the directly measured T2 and other quantities.

The discharge coefficient, given by Stoltz or Reader-Harris/Gallagher equations, is a function of neta ratio, pipe Reynolds number, upstream fluid density (calculated from the state equation is also functionally related to temperature) and velocity. Thus it is not difficult to visualise that the discharge coefficient has a complex dependency to temperature. In fact, all the variables shown in Equation 9 have similar interdependencies through model correlations used in the computation. The upstream temperature T_1 is given by T_2 by the following Joule Thompson equation:

$$T_1 = T_2 \left(\frac{P_3}{P_1} \right)^{K4}$$

where $k4 = \frac{1-k}{k}$ and k is the isentropic exponent.

3.2 Results – Uncertainty calculation.

To estimate the overall measurement uncertainty, all the 'direct' measurements (directly observed and calibrated) are modeled as random variables and are inputs to the simulation program. Their respective uncertainties correspond to those uncertainties obtained from a large number of repeated measurements of the same quantities during respective calibrations. The random variables have a normal distribution since in reality, the result of such calibrations follows normal distributions. The possible input correlations between these variables have not been considered since the instruments could have been calibrated using different standard equipment and under different conditions.

All the variables with their respective uncertainties are shown in Table 1 and plotted in Figures 3 to 6, together with the simulation results and estimation obtained using the traditional root of sum squares method. The combined uncertainties shown correspond to 95% confidence intervals, (coverage factor t=2). The uncertainty of AGANX-19 and the discharge coefficient calculations were considered to be 0.2%, for 0.6% respectively. These uncertainties are also sampled using separate normal distributions. Various tests were carried out to estimate the number of trials that were necessary in order to obtain a 95% coverage for the results.

All the results presented in this paper correspond to those obtained from 1000 simulation cycles (with multiple seeds) of 100.000 points. The sampling means and standard deviations of the input variables estimated from single 100.000 points cycles overlapped with those used as inputs to the simulation.

Pipe diameter, D	300	mm	Gas Composition - Mole%						U_{C1}	0.1%	U_D	0.4%
Orifice diameter,do	160	mm	C1	98.7	iC5	0.057	MEOH	0.0017	$U_{C2,...nC6}$	0.02%	U_{do}	0.07%
Beta ratio,b	0.53		C2	0.383	nC5	0.0048			U_{GCV}	0.6%		
Upstream Pres. P1	49.5	Bara	C3	0.206	nC6	0.003	MW	16.295	$U_{line dens}$	0.2%	reading	
Upstream Tem.T1	17.7	°C	iC4	0.02	N2	0.64	Sound sp	446.6 m/s	$U_{ref. dens}$	0.1%	reading	
GCV	37.1	MJ/Rm³	nC4	0.027	C02	0.015			U_{Cd}	0.6%	reading	

Range	Est.Uncertainties		Conventional method			Monte Carlo Simulation			% Difference in estimated Uncertainty (MCS v Conv.)		
DP mbars	U_{DP} %	U_{Ec} %	UQ_m %	UQ_v %	Uengy %	UQ_m %	UQ_v %	Uengy %	%Diff. UQ_m	%Diff. UQ_v	%Diff. Uengy
600	0.1676	0.0485	0.6441	0.6518	0.8859	0.6548	0.6614	0.8922	1.6	1.4	0.7
560	0.1733	0.0452	0.6443	0.6520	0.8860	0.6554	0.6622	0.8927	1.7	1.5	0.7
520	0.1799	0.0420	0.6445	0.6522	0.8862	0.6562	0.6627	0.8933	1.8	1.6	0.8
480	0.1877	0.0388	0.6448	0.6525	0.8865	0.6568	0.6633	0.8938	1.8	1.6	0.8
440	0.1970	0.0355	0.6453	0.6530	0.8868	0.6575	0.6641	0.8944	1.8	1.7	0.8
400	0.2083	0.0323	0.6461	0.6538	0.8874	0.6584	0.6649	0.8951	1.9	1.7	0.9
360	0.2222	0.0291	0.6471	0.6548	0.8881	0.6598	0.6662	0.8962	1.9	1.7	0.9
320	0.2399	0.0259	0.6485	0.6562	0.8891	0.6616	0.6680	0.8976	2.0	1.8	0.9
280	0.2629	0.0226	0.6506	0.6583	0.8907	0.6642	0.6705	0.8996	2.0	1.8	1.0
240	0.2940	0.0194	0.6538	0.6614	0.8930	0.6680	0.6743	0.9025	2.1	1.9	1.1
200	0.3380	0.0162	0.6590	0.6666	0.8968	0.6741	0.6802	0.9071	2.2	2.0	1.1
160	0.4047	0.0129	0.6683	0.6757	0.9037	0.6846	0.6905	0.9150	2.4	2.1	1.2
120	0.5169	0.0097	0.6873	0.6945	0.9178	0.7062	0.7118	0.9316	2.7	2.4	1.5
80	0.7432	0.0065	0.7373	0.7441	0.9558	0.7585	0.7634	0.9721	2.8	2.5	1.7

Table 1. Energy flow uncertainty comparison results between RSS method and Monte Carlo simulation –(U_x: Uncertainty of x, C_i: Gas composition, GCV: Gross Calorific Value, line dens: Upstream density, ref. dens: Density at reference conditions).

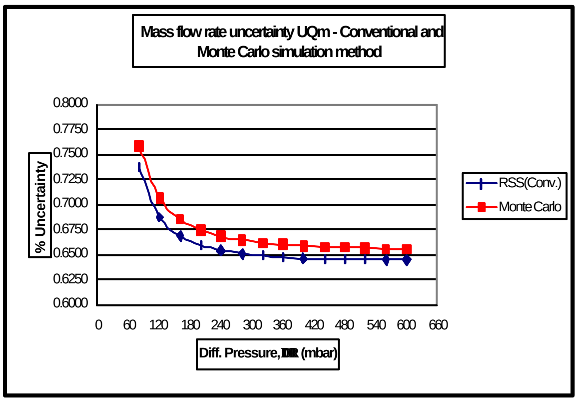

Figure 3. Mass flow rate uncertainty-Comparison using conventional and Monte Carlo simulation methods.

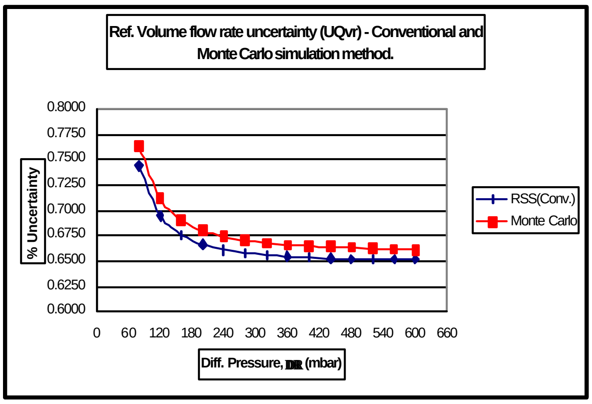

Figure 4. Ref. Volume flow rate uncertainty-Comparison using Conventional and Monte Carlo simulation methods.

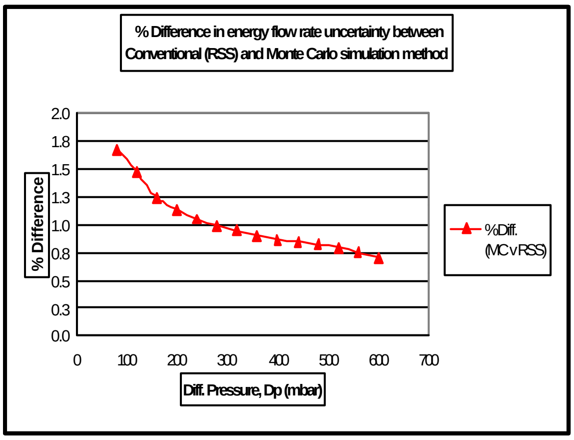

Figure 5. %Difference in gas energy flow rate uncertainty obtained by Conventional (RSS) and Monte Carlo simulation methods.

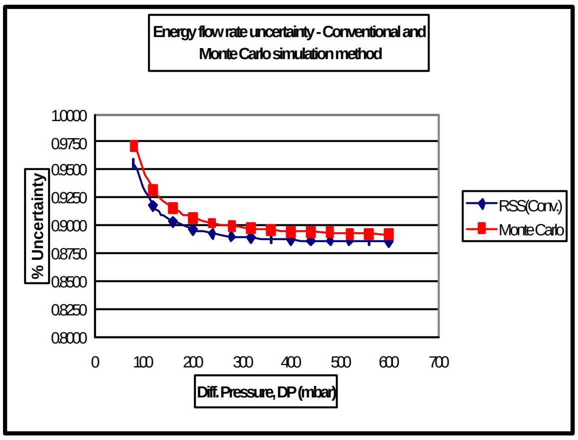

Figure 6. Gas Energy flow rate uncertainty- Comparison using Conventional and Monte Carlo simulation methods.

It can be seen that the estimated uncertainties from Monte Carlo method are between 0.8 to 1.8% higher than the respective estimated using the RSS method. This can be explained by the fact of multiple generated correlations during the estimation, which predominately must be positive.

As mentioned in the previous section, obscure relationships can be readily visualised using this approach. As an example, Figures 7 to 10 show the correlation between viscosity, compressibility factor, upstream pressure and temperature with speed of sound.

In the simulation program, sound velocity was calculated using $c = \sqrt{\frac{kP_1}{\rho}}$. Figure 9 demonstrates that sound velocity has no correlation with pressure while it is highly correlated with temperature. This is explained by the respective density and isentropic exponent variations that de-couple the sound velocity pressure dependence. Density can be obtained from various model-based measurements. Either from, $\rho_1 = \rho_{\mathrm{Re}\,f} \frac{Z_{\mathrm{Re}\,f} T_{\mathrm{Re}\,f} P_1}{Z_1 T_1 P_{\mathrm{Re}\,f}}$ or from state equation, $\rho_1 = \frac{P_1 MW}{Z_1 R T_1}$. Both models are included in the program giving different estimated uncertainties. So the program can be used also for testing of alternative models. Whatever is the case when density enters equation (11) the pressure dependency "weakens" and c is actually given by: $c = \sqrt{\frac{kZ_1RT_1}{MW}}$. The word 'weakens' was used because although k and Z_1 are also model-based measured from pressure and temperature, temperature dependence predominates.

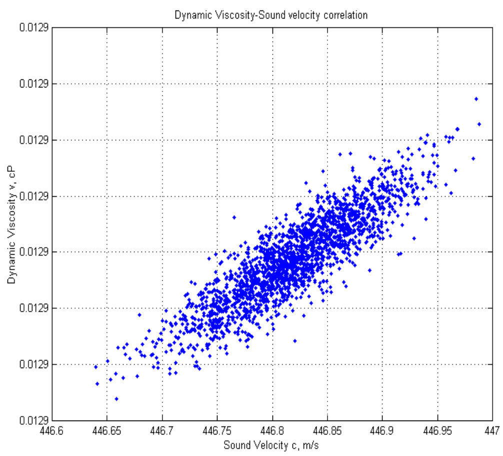

Figure 8. Dynamic viscosity – Sound velocity observed correlation.

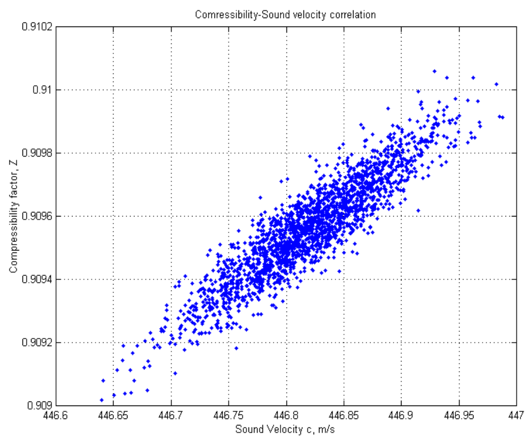

Figure 9. Compressibility factor-Sound velocity observed correlation.

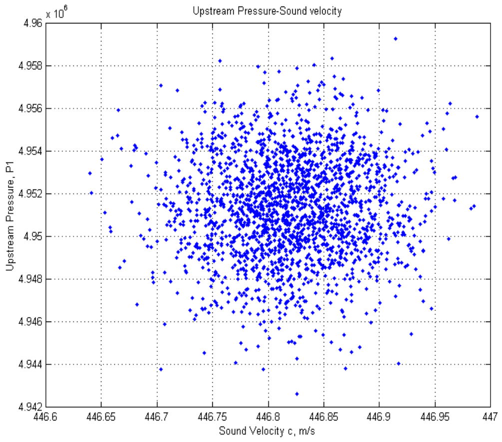

Figure 10. Upstream Pressure-Sound velocity observed correlation.

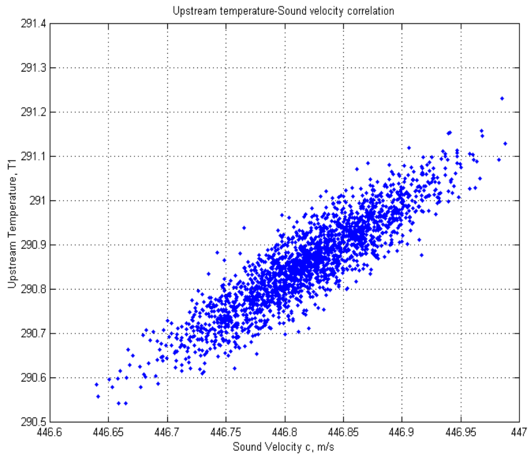

Figure 11. Upstream temperature-Sound velocity observed correlation.

4. CONCLUSIONS

A PC based program has been developed in Matlab/Simulink for the use of Monte Carlo uncertainty estimation methods in a modern electronic Natural gas fiscal metering system. Comparison between Monte Carlo and the traditional RSS method for the energy flow rate uncertainty estimation revealed that the correlation effects between the variables could be significant in terms of a reliable uncertainty estimation result.

Using the velocity of sound, as an example, it was demonstrated that the method can be used to identify complex or obscure correlations that exists between variables.

With Monte Carlo methods the analytical calculation of sensitivity coefficients and complex correlations are no longer required as they are automatically accounted for. Additionally, real signals or untypical distributions can be also examined propagating through the measurement equations and further analysed with the aid of the various Matlab's powerful toolboxes. Such untypical distributions (for example for the gas constituents, Maxwell-Boltzman distributions) could be more representative while it also examined a possible combination of real time measurements signals characteristics and simulated signals.

Further research is necessary to determine the best way of accounting for possible existing correlations in the input variables due to calibrations or other factors. Unfortunately, calibration is a process that is notoriously neglected in the existing literature for uncertainty analysis which is usually limited in the pure mathematical aspect of the matter. Just only a pure mathematical analysis could lead to uncertainty considerations that in reality may have not any physical sense. Calibration must be thoroughly considered since it is an essential process for measurement instruments giving important and importance knowledge on their operation and subsequently on the full operation of a measurement system. It permits the following of the evolution of an instrument, and therefore knowledge of the full range of its abilities, and can remove any disagreements in measurement through reliable uncertainty estimations that are based, in every case, on the existed physical reality.

The subject is quite extensive and as Kline, S.J., 1985 has already mentioned, while a great deal of work has been done on the "trees" of uncertainty procedures, a "forest" called uncertainty analysis exists as a valid part of engineering experimental work.

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Nomenclature

$U_{C1,C2,\dots Cn}$	: Gas Composition uncertainties
U_{GCV}	: Gross Calorific Value uncertainty
$U_{line\ dens.}$	: Line Density uncertainty
$U_{ref.\ dens}$	: Reference density uncertainty
U_{Cd}	: Discharge coefficient uncertainty
U_D	: Pipe diameter uncertainty
U_{do}	: Orifice diameter uncertainty
U_{DP}	: Differential pressure uncertainty
U_{Ec}	: Expansibility factor uncertainty
U_{Qm}	: Mass flow rate uncertainty
U_{Qv}	: Volume flow rate uncertainty
U_{engy}	: Energy flow rate uncertainty
Z_1	: Upstream compressibility factor
$Z_{Ref.}$	: Reference compressibility factor
K	: Isentropic exponent
MW	: Molecular Weight
R	: Gas constant