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MEASUREMENT UNCERTAINTY FACTORS OF ACOUSTIC FLOW-METERS

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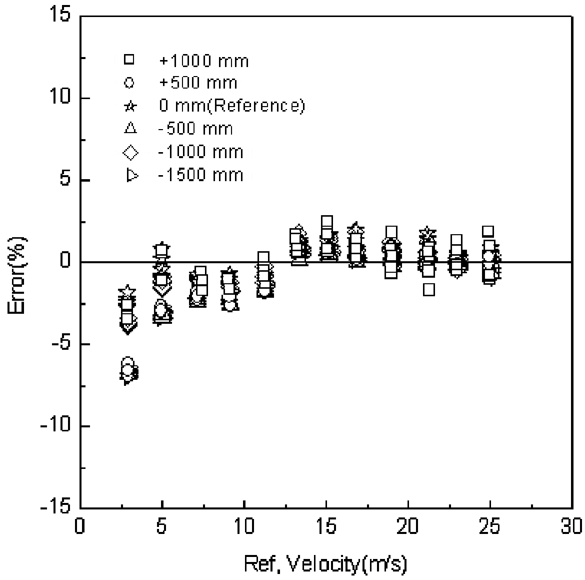
Abstract

This paper addresses uncertainty factors associated with the acoustic flowmeter developed in KRISS. Repeated experimental attempts have been made to investigate their effects on the measurement accuracy of the flowmeter and its robustness in various experimental conditions. The attempts have enabled us to sort out the two major uncertainty factors of the acoustic flowmeter; the measurement errors of the acoustic pressure in the pipe and the acoustic wave reflection characteristics (the ratio of the incident wave to reflected one). Their effects are in details discussed. These findings are shown to lead to a new method that can present more accuracy and robustness in measuring the flow velocity in the pipe. The proposed method decomposes the measured acoustic pressure into the incident and reflected wave components and then estimates the flow velocity by choosing one or both of the decomposed wave components. Their choice depends on the acoustic wave reflection characteristics inside the pipe. Since the Mach number is estimated by using the least squares method, it is found to be the best-fitted result. In order to make those points clear, experimental results will be demonstrated in this paper, including the simulation results that were not possible to implement experimentally.

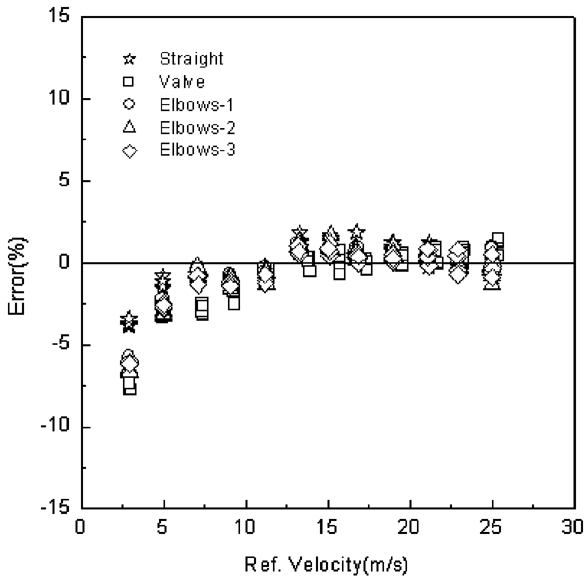
1. Introduction

Fundamentals underlying the acoustic flowmeter are based on the spatial wave-number change of two oppositely propagating plane waves along the pipe. Such wave-number change depending on the Mach number of flowing fluid in the pipe has led us to the new measurement of the mean flow velocity [1]. This idea has been also exploited in other works [2,3] that present different methods of measuring the flow velocity in the pipe. Although those researches have encouraging success in developing the theoretical frameworks for measuring the flow velocity in the pipe, experimental results are not yet sufficient to examine their potential uncertainty factors for the acoustic flowmeter.

To examine the uncertainty factors, experimental attempts have been carried out for more than one year, especifically for the accuracy analysis of the developed acoustic flowmeter by changing not only the acoustic pressure patterns along the measurement positions but also the flow velocity profiles. In practice, the change of the acoustic pressure distribution along the pipe section can be achieved by installing a series of straight pipes of different length to the intake part of the pipe. An idea behind these tests was to generate the different acoustic pressure distributions within the measurement section by changing the positions of the reflected acoustic wave from the open end of the pipe. Fig. 1 (a) shows the experimental results obtained from six experimental setups having the different lengths of straight pipes installed to the upstream of the measurement section. The measurement errors, i.e. the normalized difference between the KRISS reference and measured flow velocities, are seen to be less than 2 % for the range of velocity U 7 m/s. But, such measurement error is shown to get larger for the low velocity range of U 5 m/s. The measurements for different pressure distributions are observed to yield variations over a finite range. Those variations may indicate the measurement resolution of the developed flowmeter. Their mean values show a ‘curved’ bias from the zero value in the range of flow velocity U < 20 m/s. Such biased feature of the mean values may be classified as one of the systematic errors for the developed flowmeter.



(a) For different acoustical patterns



(b) For different flow velocity profiles

Fig.1. Relative flow velocity errors between the nozzle-typed reference flowmeter and the developed acoustic flowmeter measured from different experimental conditions.

Of course, the influence of different flow velocity profiles inside the pipe has been also examined by installing a gate valve, a single elbow, closely coupled double elbows in plane (U-shaped), and closely coupled double elbows out of plane. Fig. 1 (b) shows the flow velocity measurement errors obtained from the different velocity profiles. The relative measurement errors are seen to be less than 2 % in case of flow velocity U >= 8 m/s regardless of different velocity profiles over the cross-section area of the pipe. It is obviously quite a big advantage for the acoustic flowmeter that exploits the uniformly distributed acoustic plane wave over the across-sectional area to measure the flow velocity. But, as the flow velocity decreases in the range of less than 5 m/s the relative measurement errors are observed to increase. The measurements for different velocity profiles are seen to yield variations over a finite range, which may indicate the measurement resolution of the developed flowmeter. Still, their mean values show a ‘curved’ bias from the zero value in the range of flow velocity U < 12 m/s. This biased feature of the mean values may be classified as one of the systematic errors for the developed flowmeter.

As the first stage of developing the prototype model and theoretical backgrounds for the acoustic flowmeter, the experimental results shown in Fig. 1 have been very encouraging to us. The research team has moved to the second stage for refining the developed results to a target level for real production. Here is a fundamental issue in the second stage: “How can we break through the current measurement accuracy in the acoustic flowmeter ?” This issue has encouraged us to re-examine the experimental results by considering possible measurement error factors, such as the acoustic pressure measurement errors of installed microphones, the acoustic property of the pipe-work (the ratio of the incident wave pressure to the reflected one, i.e. the reflection characteristics in the pipe), the measurement error of the speed of sound in the pipe, etc. The estimation method of the mean flow velocity using the measured acoustic pressures along the pipe has reconsidered in a sense of acoustical theory, such as approximate formulations and their effects on the resultant errors. Detailed contents are introduced in the following sections.

2. Uncertainty Factors of Acoustic Flowmeter

In this work, multiple equi-spaced acoustic pressure measurements [1] along the pipe are used to monitor the phase change for the incident and reflected wave components in the pipe. A single tone acoustic source, whose electrical signal is generated by the stabilized function generator, is driven to generate the standing wave that consists of the incident and reflected acoustic wave components in the pipe. The acoustic pressure related to the standing wave is measured by using the array of multiple equi-spaced microphones. The measured acoustic pressures at the three neighboring positions satisfy the following spatial recursive conditions [1,4]

$$P_{n+1} \cdot e^{jk^+ \Delta} + P_{n-1} \cdot e^{jk^- \Delta} = (1 + e^{j(k^+ + k^-)\Delta}) \cdot P_n \qquad (1)$$

In equation (1), P_n denotes the complex-valued phase vector of the measured acoustic pressure at position $n \cdot \Delta$ (Δ = gap between two neighboring microphones) and $\{k^+, k^-\}$ do the wave numbers corresponding to the positive-going (the same direction as fluid flow) and negative-going (the opposite one) acoustic waves, respectively. The set of phase vectors $\{ P_n ; n = 0, 1, \cdots, 5\}$ are very accurately obtained from the estimation of the discrete Fourier coefficients that are calculated using the integer periods of sampled acoustic pressure signals (exact 500 periods chosen to calculate the coefficients and 48 samples per period also chosen in this work). In equation (1), the positive-going wave number $k^+ = (k_0 - j\alpha)/(1+M)$ and the negative-going one wave number $k^- = (k_0 - j\alpha)/(1-M)$ ($k_0 = 2\pi f/C_0$, f = frequency, C_0 = speed of sound, α = acoustic wave attenuation factor, M = Mach number). In this work, since the ratio of α / k_0 is found to be 0.0014, the attenuation factor is neglected in estimating the Mach number. Furthermore, since a range of Mach number M is considered to be less than 0.1 (flow velocity $U \leq 34.3$ m/s), the approximated ones of $1/(1+M) \cong 1-M+M^2-M^3$ and $1/(1-M) \cong 1+M+M^2+M^3$ are chosen by neglecting the forth and higher order terms. Under those approximate conditions, the spatial recursive form in equation (1) is simplified as

$$P_{n+1} \cdot e^{-j(M+M^3) \cdot k_0 \Delta} + P_{n-1} \cdot e^{j(M+M^3) \cdot k_0 \Delta} = 2 \cdot \cos((1 + M^2) \cdot k_0 \Delta) \cdot P_n \qquad (2)$$

To exploit the least squares based parameter estimation method, the Mach number on the right-hand side in equation (2) was replaced by the mean value of previously estimated Mach numbers, i.e. $(1+M^2) = (1+M_m^2)$ (M_m = the mean value of previously estimated Mach numbers). Very interestingly, equation (2) means that the n-th phase vector P_n multiplied by the $2\times\cos((1+ M_m{}^2)\cdot k_0\Delta)$ are equal to the sum of the (n+1)-th phase vector negatively rotated by the angle of $(M+M^3)\cdot k_0\Delta$ and the (n-1)-th phase vector positively rotated by the same angle of $(M+M^3)\cdot k_0\Delta$. This graphical understanding leads to the least squares based estimation of the Mach number-dependent rotation angle $(M+M^3)\cdot k_0\Delta$ by minimizing the squared distance between the left-hand and right-hand phase vectors of equation (2). Since the six equi-spaced microphone array is chosen in this work, the four sets of spatial recursive forms $\{ (P_0,P_1, P_2), \cdots, (P_3,P_4, P_5) \}$ are used to estimate from the least squares based Mach number estimation. Experimental results shown in Fig.1 were obtained by this Mach number estimation.

2.1 Phase Vector Estimation Errors

At the onset of this work, it became apparent that the measurement errors of the phase vectors $\{ P_n ; n = 0, 1, \cdots, 5\}$ do make direct effect on the estimation of the Mach-number-dependent rotation angle in equation (2), i.e. $(M+M^3)\cdot k_0\Delta$. In practice, it is not possible to examine experimentally how much the estimated phase vector errors affect the estimated Mach number directly by varying unwanted acoustic noise components inside the pipe. Once the experimental setup is completed, its acoustic noise characteristics are fixed, including flow induced noises and turbulent noises. To overcome this experimental limit, this work has chosen to use a computer simulation technique by adding the Gaussian noise to the estimated phase vectors. Let the complex-valued phase vectors be $\{P_n ; n = 0, 1, \cdots , 5\}$. Then the noise added phase vectors $\{ P_{N, n} \}$ are defined as

$$P_{N,n} = P_n + (n_{R,n} + jn_{I,n}) \qquad (3)$$

in which the terms $\{ n_{R,n}, n_{I,n}; n = 1, 2, \ldots, N_n \}$ denote the normally distributed, zero-mean Gaussian noises with the equal-valued variance σ. The added complex noise is seen to vary the magnitude and phase of the phase vector. A series of 2,000 noise contaminated phase vectors are used to estimate the Mach number. To examine their effects on the flow velocity estimation, this work has chosen the relative noise variance that is defined as $\sigma = 0.01\times$ RNL$\times P_{rms}$ (P_{rms} = the root mean squared value of acoustic pressure over the one wavelength, RNL = relative noise level in percentage). Fig. 2 shows the standard deviations of estimated flow velocity errors for the different RNL's.

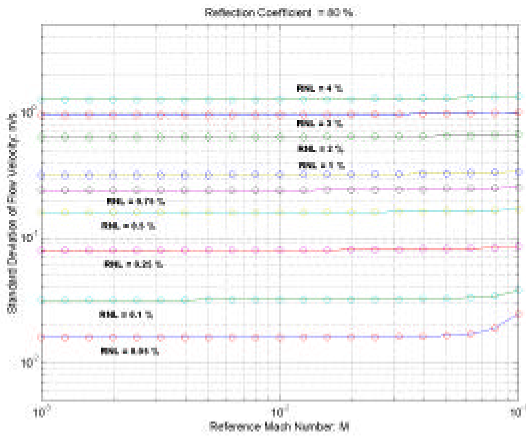


Fig. 2 Standard deviations of estimated flow velocities for the different noise levels (RNL = relative noise level in percentage).

The standard deviation for the relative noise level of RNL = 0.5 %, whose value is 0.16 m/s (about 1.6 % velocity measurement error at Mach number M = 0.03), is shown to be close to the variation of estimated flow velocity given in Fig. 1. If the relative standard deviation for the phase vectors estimated from acoustic pressure signals is larger than 0.5 %, the estimated flow velocity yields larger error as shown in Fig. 2. The relative noise level considered in Fig.2 is quite small, i.e. the case of RNL = 0.1 % is equivalent to the signal-to-noise ratio (SNR) = 20 dB. In fact, the standard deviation for the estimated flow velocity is directly related to the measurement resolution of the developed flowmeter itself. The standard deviation of 0.16 m/s obtained from real flow velocity measurements is quite large in comparison to a target value of 0.05 m/s in the second stage. This has led to the reappraisal on the current version of the Mach number estimation method. An improved method will be discussed in the following section.

2.2 Effects of Reflection Coefficient on Estimated Mach number Errors

It was observed from the robustness tests for different acoustic fields that highly biased Mach number estimation results were obtained when two anechoic terminators were installed between the acoustic pressure measurement sections. Those results were never observed before adding the anechoic terminators. The anechoic terminators were found to reduce in a mount of 90 % the magnitude of the reflected acoustic waves from the intake part of the pipe opening and the contracted throat of the reference nozzle. In a sense of acoustics, the anechoic terminators change the ratio of the incident wave generated by acoustic source located at the end of the measurement section to the reflected wave coming outside the measurement section, i.e. the acoustic wave reflection characteristics. It means that any acoustic flow measurement method of exploiting both incident and reflected waves together cannot avoid such highly reflection-dependent Mach number estimation, specifically in the acoustic field with low reflection. It is readily understood that the relatively small reflection wave becomes very sensitive to the acoustic noise inside the pipe. This noise contaminated reflected wave yields highly fluctuating Mach number in real measurement. Fig. 3 illustrated the standard deviation of estimated flow velocities obtained for the relative noise level of RNL = 0.5 %.

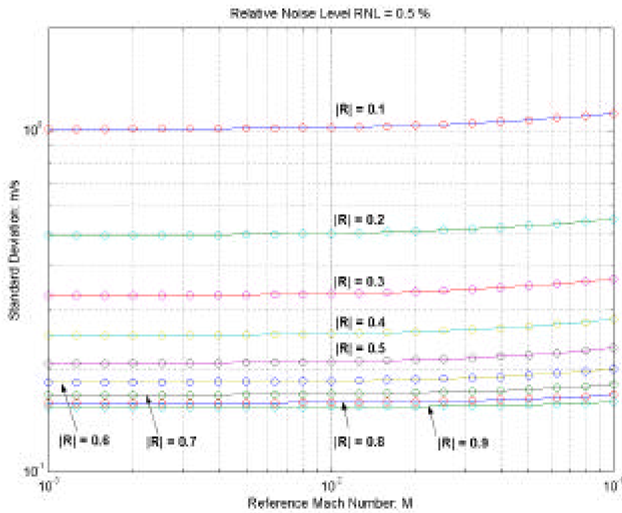


Fig. 3 Standard deviations of estimated flow velocities for different acoustic reflection coefficients |R|.

It is obviously observed from Fig. 3 that the standard deviation of estimated flow velocities very depends on the magnitude of the reflection coefficients, i.e. |R|. As the reflection coefficient decreases from |R| = 0.8 to |R| = 0.3 their corresponding standard deviation increases about two times. Furthermore, when the reflection coefficient is close to 0.1 its deviation is 6 times larger than that of |R| = 0.8. The reflection coefficients at 500 Hz was found to be close to 0.8 in the real experiments given in Fig. 1. The standard deviation for the reflection coefficient |R| = 0.8 is seen to be equal to that of RNL = 0.5 % shown in Fig. 2. This dependence of the estimated flow velocity on the acoustical reflection characteristics is expected to be as small as possible. To realize this robustness to the various acoustical field characteristics in real pipe systems, this work proposes a new idea that both incident and/or reflected waves are exploited only for the acoustic field with the larger reflection, i.e. |R| > 0.3. But, when measured acoustic reflection coefficient is smaller than 0.3, the incident wave component is recommended to be used to estimate the Mach number. More detailed estimation method will be discussed in following section.

3. New Mach Number Estimation Method

In the previous section, the estimated flow velocity using the acoustical flowmeters was shown to very depend on not only the errors in estimating the phase vectors from the measured acoustic pressure signals but also on the acoustic reflection characteristics, i.e. the ratio of the incident wave to the reflected one. Ideally, any acoustic flowmeter is expected to be less sensitive to measurement noises and to be independent of the sound field characteristics. To achieve this aim, we have made the reappraisal on the method of estimating the Mach number in the pipe. This work exploits the fact that the decomposition of the standing wave into the incident and reflected wave components are possible as reported in the previous work[5]. Let the position of the first microphone be at the origin, i.e. x = 0. The phase vector measured at the position $x_n = n\cdot\Delta$ is described as

$$P_n = P_I \cdot e^{-jk^+ \cdot nD} + P_R \cdot e^{jk^- \cdot nD} \tag{4}$$

In equation (4), P_I and P_R denote the incident and reflected wave components at the origin (n = 0) and Given a set of multiple phase vectors {P_n; n = 0, 1, 2, ⋯ , N-1}, the best fitted incident wave components {P_I, P_R} are obtained by solving the following normal equation using the least squares method [5]

$$\begin{bmatrix} 1 & 1 \\ e^{-jk^+ \cdot D} & e^{-jk^+ \cdot D} \\ . & . \\ . & . \\ . & . \\ e^{-jk^+ \cdot (N-1)D} & e^{-jk^+ \cdot (N-1)D} \end{bmatrix} \begin{bmatrix} P_I \\ P_R \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \\ . \\ . \\ . \\ P_{N-1} \end{bmatrix} \tag{5}$$

Let the N-by-2 matrix be **A**, the incident and reflected wave vector be $\mathbf{x} = [P_I, P_R]^T$ and the measured phase vector be $\mathbf{y} = [P_0, P1, \dots , P_{N-1}]^T$. The best fitted incident and reflected wave vector, in a sense of the least squared method, is obtained using the singular value decomposition [5], i.e. $\mathbf{x} = [\mathbf{A}^{H\cdot}\mathbf{A}]^{\#} \cdot \mathbf{A}^H\mathbf{y}$. (the superscript H denotes the Hermitian operator). In order to obtain the new incident and reflected wave vector from the currently measured phase vector, the averaged Mach number M_m over the previously estimated ones is used to calculate the positive-going and negative-going numbers $k^+ = (k_0 - j\alpha)/(1+ M_m)$ and $k^- = (k_0 - j\alpha)/(1- M_m)$ in equation (5). Since the acoustic property of a flowing fluid is not rapidly changed over the sampling period of each phase vector (one second chosen in this work), the new incident and reflected wave vector is obtained very consistently.

It is quite straightforward to decompose the measured phase vector (4) into the incident and reflected components as

$$P_{n,I} = P_n - P_R \cdot e^{jk^- \cdot nD}, \quad P_{n,R} = P_n - P_I \cdot e^{-jk^- \cdot nD} \tag{6}$$

The decomposed incident and reflected wave components at each measurement position are normalized in respect to those of the first microphone x = 0, i.e.

$$H_{n,I} = \frac{P_{n,I}}{P_I} = e^{-a \cdot nD} \cdot e^{-jk_0 \cdot nD/(1+M)}, H_{n,R} = \frac{P_{n,R}}{P_R} = e^{a \cdot nD} \cdot e^{jk_0 \cdot nD/(1-M)} \tag{7}$$

It indicates that the normalized incident and reflected wave components are separated into the magnitude and phase parts. The magnitude part is related to the acoustical attenuation of flowing fluid in the pipe and the phase is directly related to the Mach number. More specifically, the multiple phases obtained from the normalized wave components in (7) are shown to be of the linear form according to the equi-spaced microphone positions. These linear phase responses of the decomposed incident and reflected waves are exploited to estimate the Mach number since the linear-phase- in-position relationship presents the best fitted

Mach number in a sense of the least squares method, as well known. The linear phase responses for the decomposed incident and reflected waves are defined as a linear form

$$\grave{E}_I(n) = S_I(M) \cdot k_0 \cdot nD + O_I, \qquad \grave{E}_R(n) = S_R(M) \cdot k_0 \cdot nD + O_R \tag{8}$$

In equation (8), the subscript I and R denote the incident and reflected wave components. The slopes { $S_I(M)$, $S_R(M)$ } of the linear phase responses in equation (8) are defined as a function of Mach number

$$S_I(M) = -1/(1+M), \qquad S_R(M) = 1/(1-M) \tag{9}$$

and the symbols O_I and O_R denote the offset angles in the reference position x = 0. Our interest is concerned only to the estimation of the slopes of the linear phase responses in equation (9), which include the Mach number.

In this work, two different ways of estimating the Mach number are proposed. The first one is to exploit both incident and reflected phase information { $S_I(M)$, $S_R(M)$ } given in equation (9) in case of the reflection coefficient |R| > 0.3. The Second one is to use only the incident phase-related slop $S_I(M)$ in case of the small reflection coefficient |R| < 0.3 because it enables the reduction of the estimated bias of Mach number caused by the noise sensitive reflected wave in the pipe. The first method of estimating Mach number uses the difference phase of

$$\grave{E}_R(n) - \grave{E}_I(n) = \frac{2}{1-M^2} \cdot k_0 \cdot nD + O_1 \tag{10}$$

Given the measured phases {$\Theta_I(n)$, $\Theta_R(n)$; n = 0, 1, 2, … , N-1} from the N microphones, the best fitted Mach number is obtained by the following equation using the least squares method

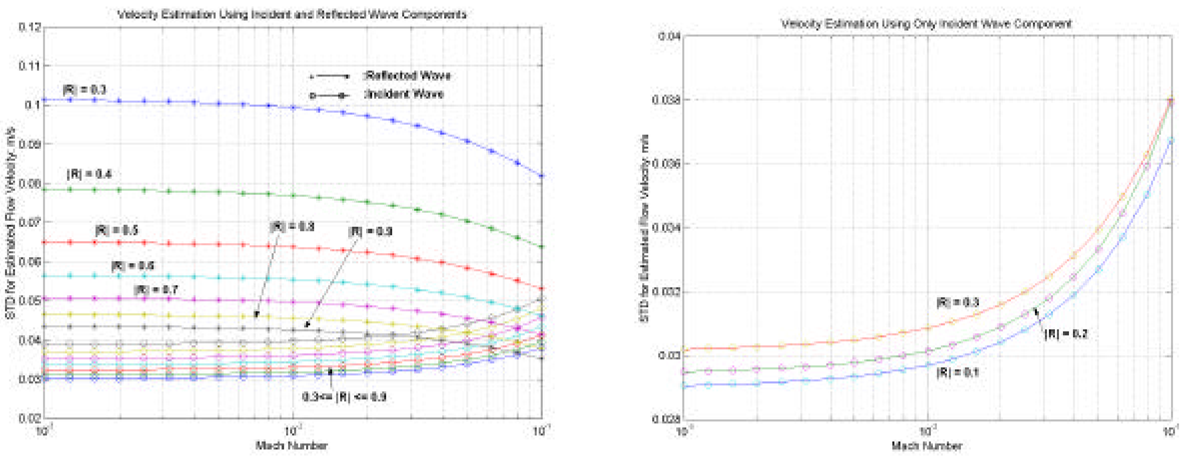
$$S_1(M) = \frac{2}{1-M^2} = \frac{N \cdot \sum_n x_n \cdot y_n - \sum_n x_n \cdot \sum_n y_n}{N \cdot \sum_n n^2 - \left(\sum_n x_n\right)^2} \tag{11}$$

in which $x_n = k_0 \cdot n\Delta$ and $y_n = \Theta_R(n) - \Theta_I(n)$. Equation (11) is obviously shown to enable the best-fitted Mach number using the multiple microphones when a large acoustic reflection wave exists in the pipe, i.e. |R| > 0.3 in this work. The second method of using only the incident wave component is also readily obtained in a similar way done in obtaining equation (11). Let the measured phases of the incident wave be $y_n = \Theta_I(n)$ and the position dependent terms be $x_n = k_0 \cdot n\Delta$. Then, the slop of the incident wave component is readily obtained as

$$S_2(M) = \frac{-1}{1+M} = \frac{N \cdot \sum_n x_n \cdot y_n - \sum_n x_n \cdot \sum_n y_n}{N \cdot \sum_n n^2 - \left(\sum_n x_n\right)^2} \tag{12}$$

This result enables to the estimation of the best-fitted Mach number from the slope of the incident wave component.

Fig. 4 illustrates the standard deviation of the estimated Mach numbers for different acoustic coefficients. When the acoustic coefficients |R| are equal to or larger than 0.3, it is shown that the method of using the linear phase responses of the decomposed incident and reflected waves can provide much better estimated Mach number than the previous method in comparison to Fig.3. From Fig.4 (a), it is obviously shown that the larger reflection coefficient gives the more accurate estimation of Mach. Of course, the incident wave components, denoted by the symbol of “o”, are found to yield more stable and accurate Mach number estimation than the reflected one does. Even for the quite small reflection coefficient |R| <= 0.3, the incident wave are also observed to present the very accurate estimation from Fig. 4 (b).



(a) Reflection Coefficient |R| >= 0.3 (b) Reflection Coefficient |R| <= 0.3

Fig. 4 Estimated Mach number for different reflection waves

Fig. 5 shows that the relative difference error between the estimated and reference Mach numbers for the incident and reflected wave components. Similarly in Fig.4, the decomposed incident wave component in Fig. 5 , denoted by the symbol of “o”, yields much better estimated Mach number than the reflected one, denoted by the symbol of “*”. It indicates that the decomposed incident wave leads to more robustness in estimating the Mach number than the previous method discussed in Section 2.

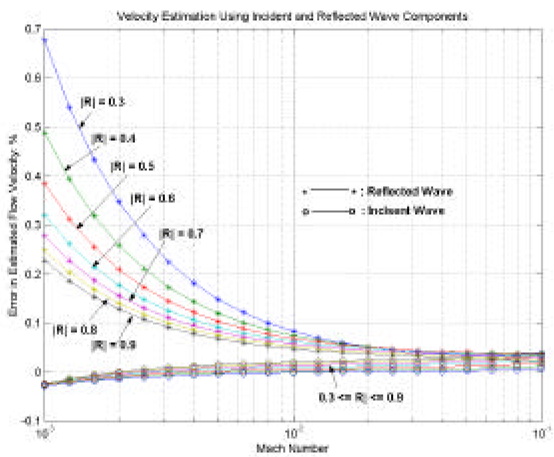


Fig. 5 Relative difference error between estimated and reference Mach numbers.

5. Concluding Remarks

This paper addresses uncertainty factors observed from the acoustic flowmeter developed by KRISS at the first stage. They were examined from repeated experimental attempts to improve its measurement accuracy and robustness by varying experimental conditions. Those attempts have enabled us to sort out the two major uncertainty factors of the acoustic flowmeter; the measurement errors of the acoustic pressure in the pipe and the acoustic wave reflection characteristics. The former one is related to unwanted acoustic noise components inside the pipe, which is shown to limit the relative accuracy of the previously developed acoustic flowmeter to the range of about 2 % that is not satisfied in comparison to the target level of about 0.5 %. The sound field with the low reflection wave component is shown to yield much worse measurement. Those experimental results have enabled the reappraisal on the previously developed acoustic flowmeter.

In Section 3, the new method has been proposed. It is shown to present higher accuracy and better robustness in measuring the flow velocity in the pipe. The new method implements the decomposition of the measured acoustic pressure into the incident and reflected wave components and then estimates the flow velocity by using the linear phase responses of the decomposed incident and/or reflected wave components. The choice is dependent on the acoustic wave reflection characteristics inside the pipe. Since the estimation of Mach number is made using the least squares method, it is found to present the best-fitted flow velocity estimation method. So as to make those points clear, experimental results have been demonstrated in this paper, including the simulation results that were not possible to be implemented in experimental conditions.

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