

# Effect of gas type on the thermal properties of small sonic nozzles

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## Abstract

A previous study (Bignell and Choi, Flow Meas. & Instrum. 13 (2002) 17-22) of the effect of temperature on the coefficient used to characterise small sonic nozzles is reviewed. Adiabatic cooling of the gas stream in the throat causes the body of nozzles to be cooled but a heater and temperature control system allow the temperature of the nozzle to be held constant. Using a gas flow standard that can operate in continuous mode, measurements were made of nozzle coefficients at different temperatures using air, argon, nitrogen and carbon dioxide. The nozzle coefficient changes with the gas type and linearly with the temperature of the body of the nozzle. The first of these changes is explained by changes in the real gas correction factor for argon but not for carbon dioxide. The temperature changes are much greater than those due to the area, the discharge coefficient and the real gas correction factor. It is necessary to invoke the properties of the thermal boundary layer to explain these changes, which are found to be greater for gases having a higher specific heat ratio.

*Keywords:* sonic nozzle; critical flow; standards; boundary layer

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## 1. Introduction

Whilst the discharge coefficient is undoubtedly the most scientifically relevant quantity in describing the behaviour of nozzles, what is actually measured is a combination of factors. We can define  $N$  the nozzle coefficient, using the measured mass flow rate  $Q_m$ , given by [1] as

$$Q_m = A_* C C_R (p_N r_N)^{1/2} = N (p_N) (p_N r_N)^{1/2} \quad (1)$$

where  $p_N$  and  $r_N$  are the upstream pressure and density of the gas,  $A_*$  is the area of the throat and the quantities  $C$  and  $C_R$  are the discharge coefficient and the real gas correction factor. The reason for this is that for small nozzles the diameter of the throat can be difficult to measure with low uncertainty and the relative uncertainty in  $A_*$  is twice that for the throat diameter. The throat, or the section of minimum area, is also not simple to locate in the geometry of a small nozzle.

Because the discharge coefficient varies from about 0.95 to nearly unity as the pressure changes from atmospheric to several MPa the nozzle coefficient has a pressure dependency. The value of  $C_R$  also depends on pressure to an extent depending on the type of gas.  $C$  and  $C_R$  are also temperature dependent and so is the area  $A_*$ , but to varying degrees.

Sonic nozzles are usually calibrated by passing a quantity of gas in a measured time. The quantity of gas may be determined either by weighing it or by a measurement of its volume, pressure, temperature and, if necessary, its relative humidity. In both cases however there is no chance for the nozzle conditions to approach a steady state. This can be quite different

from the conditions that apply when the nozzle is actually used to do a calibration.

A calibration method described by Bignell and Choi for small gas flows [2] allows the flow through the nozzle to be continuous. Measurements can be made continually at intervals corresponding to the time it takes the nozzle being calibrated to pass 25 L of gas under the conditions of the calibration. The value of the nozzle coefficient can be monitored over a long time using this equipment and changes in nozzle coefficients have been observed and reported in [3]. Further more exact measurements have been made where the temperature of the nozzle has been controlled over a range of temperatures. This work, using air as the gas through the nozzle, has been reported in [4] and the current work is an extension of these measurements to other gases.

## 2. Experimental arrangements

The volumetric gas flow standard has already been described in the literature [2] but a short description will be given here for completeness. It uses a spherical volume of approximately 25 L, determined accurately by a water-fill technique and divided into two by a flexible diaphragm that can travel from one extreme position to the other, thus sweeping out the 25 L volume. The walls of the spherical volume are perforated to allow the gas to enter one of two collection spaces. When the diaphragm reaches an extreme position this is detected by a small pressure change across the diaphragm and, through electronics, causes the solenoid valves to reverse the direction of flow through the volume. Pressure, time, temperature and relative humidity are measured to

enable the mass flow rate to be calculated from an equation of state of the gas used. A diagram of this equipment is shown in Figure 1.

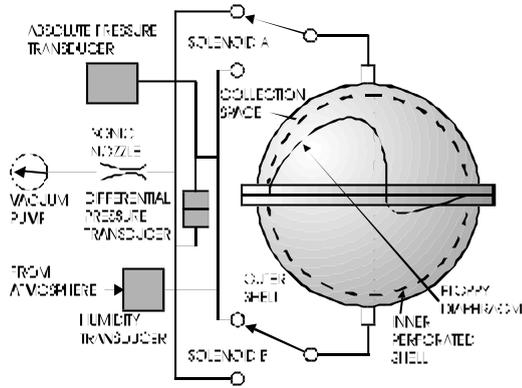


Figure 1. The volumetric gas flow standard used in the measurements.

The nozzles used were from 0.7 mm to 2 mm throat diameter and were mounted in a special holder shown in Figure 2. The nozzle fits snugly into a copper sleeve wound with 41 μm copper wire. This coil serves as both a heater and a resistance thermometer so that with a suitable arrangement of equipment as indicated in Figure 3, the temperature of the nozzle can be both measured and controlled.

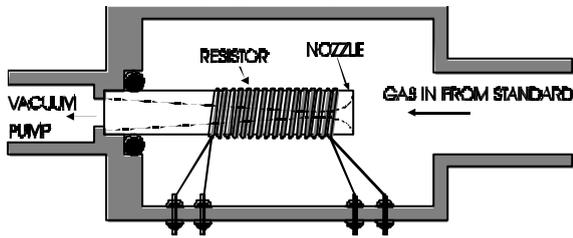


Figure 2. The nozzle holder and copper winding.

The computer measures the resistance of the coil by comparing the voltage across it with that across the standard resistor and hence calculates the temperature using data from a calibration of the coil as a resistance thermometer. It then increases or decreases the voltage supplied to the circuit so as to maintain the required set-point temperature.

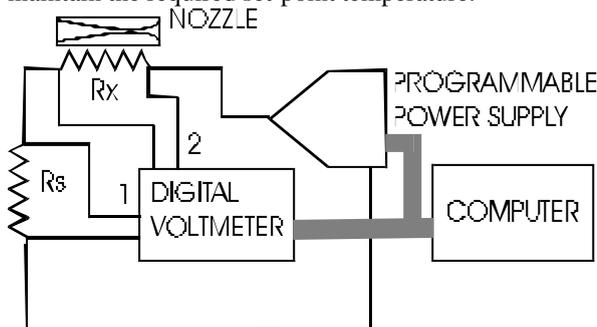


Figure 3. The temperature control system.

### 3. Some preliminary results

This section is a summary of the results presented in [4]. Using the temperature measuring capability of the equipment just described the changes in temperature of a series of nozzles of different diameters as they cool due to the gas flow through them was measured. These results are shown in Figure 4. The nozzle is cooled due to the increase in the kinetic energy of the gas stream at the expense of the temperature of the gas. For a virtually zero inlet velocity, and throat velocity of  $v_{th}$  (the velocity of sound) we have

$$\frac{1}{2} Q_m v_{th}^2 = Q_m c_p \Delta T \quad (2)$$

where  $\Delta T$  is the temperature drop and  $c_p$  is the specific heat at constant pressure. This leads to temperature drops down to  $-30^\circ\text{C}$  in the throat.

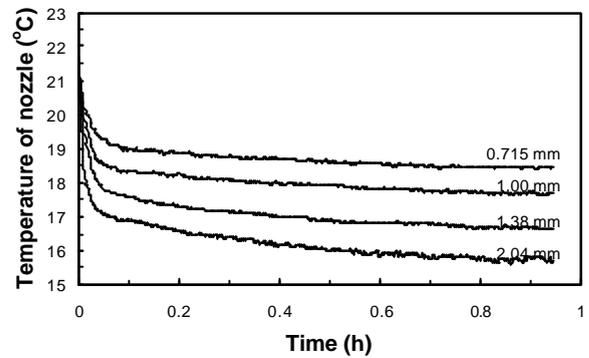


Figure 4. The measured temperature of four nozzle bodies.

This temperature change leads to changes in the nozzle coefficient as can be seen from Figure 5 where the temperature change for a particular nozzle is plotted with the temperature change for that nozzle.

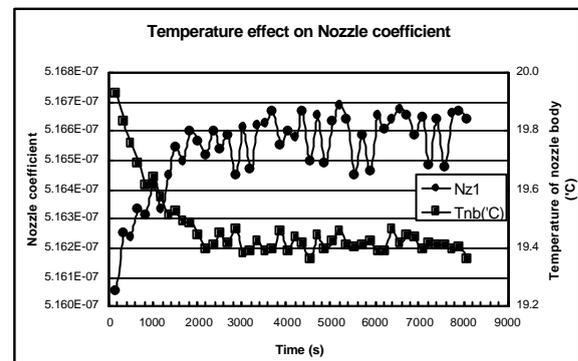


Figure 5. Measured nozzle coefficient and temperature.

The temperature stabilising circuit was developed to enable nozzle coefficients to be measured at a series of constant temperatures rather than on the fly as in Figure 5. Using the same set of four nozzles as in

Figure 4 and with air as the gas the nozzle coefficients were measured over a range of temperatures. To present these more clearly they have been plotted against temperature as normalised nozzle coefficients, that is the measured value at a particular temperature divided by the value at 20 °C. The results of this are shown in Figure 6.

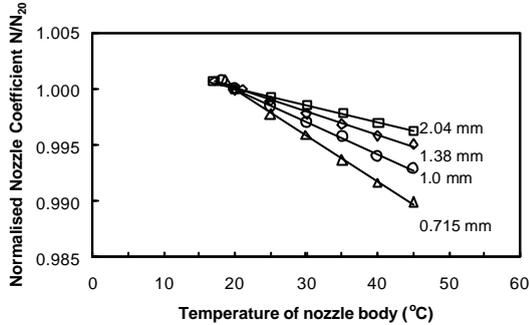


Figure 6. Normalised nozzle coefficients measured using air for four throat diameters, plotted against temperature.

#### 4. Effects due temperature

There are a number of reasons that we might expect the nozzle coefficients to change with temperature. Since  $N$  is defined equal to  $A_* C C_R$  we can differentiate to separate out the temperature dependencies due to these components. Thus

$$\frac{1}{N} \frac{dN}{dT} = \frac{1}{C_R} \frac{\partial C_R}{\partial T} + \frac{1}{A_*} \frac{\partial A_*}{\partial T} + \frac{1}{C} \frac{\partial C}{\partial T} \quad (3)$$

The most straightforward of these is the effect of temperature on the area. If the coefficient of linear thermal expansion of the material is  $I$  then the change in the diameter of the throat due to a change in temperature from  $T_0$  to  $T$  will be given by

$$d(T) = d(T_0)[1 + I(T - T_0)]. \quad (4)$$

For the area of the throat  $I$  is replaced by  $2I$  which for a brass nozzle is  $3.8 \times 10^{-5} \text{ K}^{-1}$ . Clearly the area and hence  $N$  increases as the temperature but the effect in Figure 6 shows a decrease in nozzle coefficient with temperature.

The real gas correction factor depends only very slightly on temperature. The discharge coefficient given by

$$C = a - b \text{Re}^{-0.5} \quad (5)$$

where  $a$  and  $b$  are constants, has a temperature dependence through the Reynolds number. This depends on the area of the throat and on the viscosity of the gas, a temperature dependent quantity that can be represented by

$$\frac{m}{m_{ref}} = \left( \frac{T}{T_{ref}} \right)^w \quad (6)$$

where  $m_{ref}$  is the value of the viscosity at the reference temperature  $T_{ref}$  and  $w$  is a constant for the particular gas. The Reynolds number is given by [1] as

$$\text{Re} = \frac{4Q_m}{p d m} \quad (7)$$

By substituting for  $m$  and  $d$  using Eqs. (4) and (6) we can get a temperature dependent expression for  $\text{Re}$ , and hence for  $C$ . But since  $Q_m$  depends on  $C$  we need to recalculate  $Q_m$ , that is, use an iterative solution to work out the dependence of  $C$  on temperature. When this is done the middle curve of Figure 7 is obtained. This Figure also shows the pure thermal expansion effect and the measured effect. It is clear that there is a large discrepancy between the experimental results and the calculated ones.

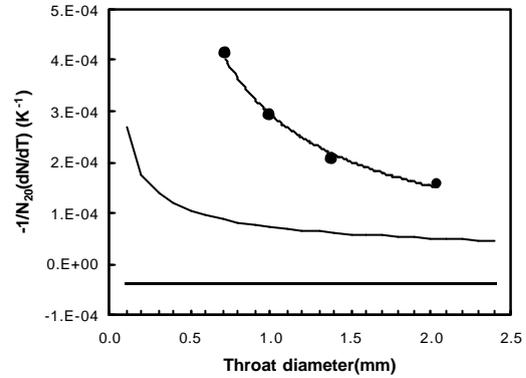


Figure 7. Measured and calculated values of the change in  $1/N_{20}(dN/dT)$  with temperature. The bottom straight line is due to thermal expansion, the middle curve due to the change in the discharge coefficient, and the top curve has been fitted to the experimental values.

#### 5. Measurements with other gases

Measurements using gases other than air were done by allowing the gas to flood the inlet chamber containing the relative humidity probe and exiting to excess. As long as the exit contained a positive output flow then the gas drawn into the flow standard must be air-free. Measurements were made with argon, carbon dioxide and nitrogen. The nitrogen supply was inadequate for the 2 mm size nozzles. The results of nozzle coefficient measurements for 2 mm, 1 mm and 0.7 mm diameter nozzles are shown in Figures 8, 9 and 10.

The lines are generally the best straight lines fitted to the points though in one case there are only two points. This was due to the lack of time to take more points though in this case they have been spaced apart in temperature as much as possible to get the best measure of the slope of the line from only two points.

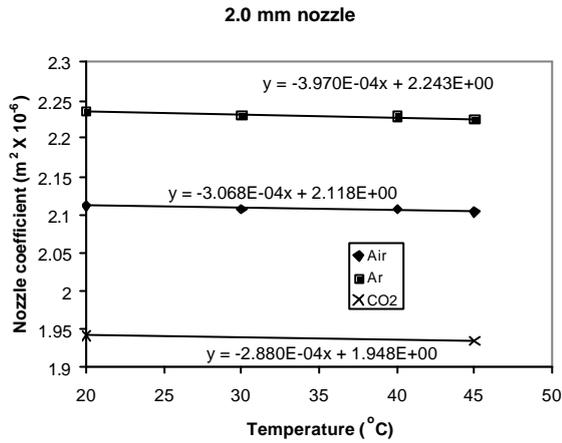


Figure 8. Nozzle coefficients for 2 mm diameter nozzle.

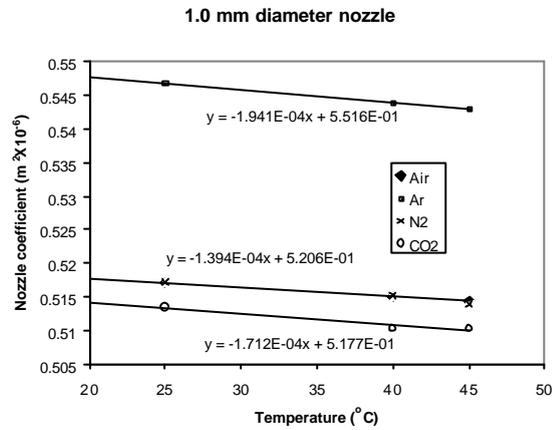


Figure 9. Nozzle coefficients for 1 mm diameter nozzle. The air and nitrogen points are virtually overlapping.

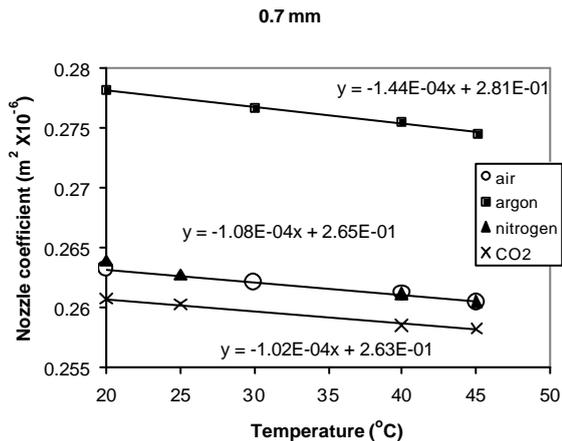


Figure 10. Nozzle coefficients for 0.7 mm diameter nozzle.

From Figures 8, 9 and 10 it is immediately evident that the nozzle coefficient depends strongly on the type of gas and that there is very little difference between the values for air and nitrogen for which two it is almost equal. The coefficient is larger for

argon than for air or nitrogen and these have larger values than for carbon dioxide. At least some of this is explained by the different real gas correction factors (RGCFs) for these gases. Table 1 shows the values of the ratio of the RGCF for argon to that for air and a similar ratio for the values of the measured nozzle coefficients. These calculations include a correction for the change in the discharge coefficient.

Table 1

Temperature (°C)	Ratio RGCFs argon/air	Ratios of observed nozzle coefficients		
		2 mm	1 mm	0.7 mm
20	1.058	1.058		
25	1.057		1.057	
30	1.057	1.059		1.056
40	1.057	1.058	1.056	1.055
45	1.056	1.058	1.055	1.054

Thus it is clear that for the argon/air measurement the change in the RGCF explains at least most of the observed change in nozzle coefficient, which like the RGCF ratio, is not very dependent on temperature. This is less true for the smallest nozzle where there is some temperature dependence and the gap between the theoretical ratio and the observed ratio is larger. There is a slight temperature dependency of the theoretical values of the ratio of the RGCFs for nitrogen to carbon dioxide, due mainly to the carbon dioxide, but the observed ratios of the nozzle coefficients are quite different from the theoretical ones. These data are shown in Table 2.

Table 2

Temperature (°C)	Ratio RGCFs N2/CO2	Ratios of observed nozzle coefficients	
		1 mm	0.7 mm
20	1.023		
25	1.024	1.0072	
30	1.024		
40	1.025	1.0097	1.010
45	1.026	1.0071	1.009

Here the observed ratios for the 1 mm and the 0.7 mm nozzles are nearly the same but quite different from the theoretical ones. There is a slight tendency for the observed ratio to decrease with temperature whereas the theoretical ratio of the RGCFs increases with temperature. Johnson, Wright, Nakao, Merkle and Moldover [6] have found that the discharge coefficients of nozzles using carbon dioxide are increased by up to nearly 3% even to values greater than unity, by vibrational non-equilibrium effects. This is sufficient to explain the discrepancy observed here.

The quantity of gas in the thermal boundary layer in a particular section of the inlet portion of the nozzle or in the throat will be proportional to the circumference at that part, that is, for nozzles of the same geometry, to  $d$ , the throat diameter. This is because the geometry of the nozzle is scaled to  $d$  [1]. The total quantity of gas flowing will be proportional

to the area of the throat  $A^*$  or to  $d^2$ . Hence, the fraction of the flux affected by heat transfers with the wall will vary as  $1/d$ . A graph of  $1/N_{20}(dN/dT)$  against  $1/d$  was previously shown to give a reasonable straight line for air. A similar plot is shown as Figure 11 for argon and for air and for both gives a straight line as the above analysis predicts.

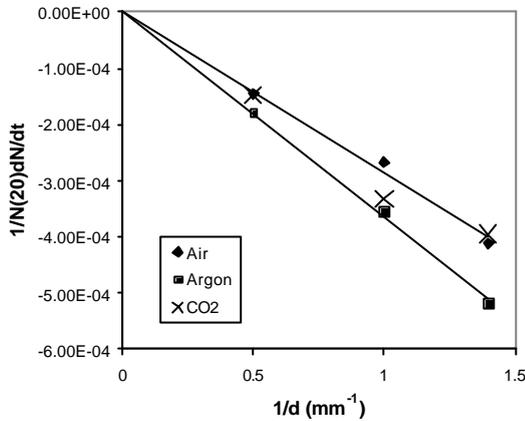


Figure 11. Values of  $1/N(dN/dT)$  plotted against the reciprocal of the nozzle diameter.

The values for carbon dioxide however, are not able to fit into this framework probably for the same reasons that gave the discrepant results of Table 2.

It has been stated by Johnson et al [5] that the effect of the thermal boundary layer is greater for gases that have a higher ratio of specific heats. The three gases used here have specific heats of 1.304 (carbon dioxide), 1.403 (air) and 1.668 (argon) and Figure 12 shows the values of  $1/N_{20}(dN/dT)$  plotted against the specific heat ratios. It would be good to be able to plot more points on this graph. Unfortunately there is no gas with a specific heat ratio of about 1.5 to 1.6 though there are some gases with lower ratios than carbon dioxide.

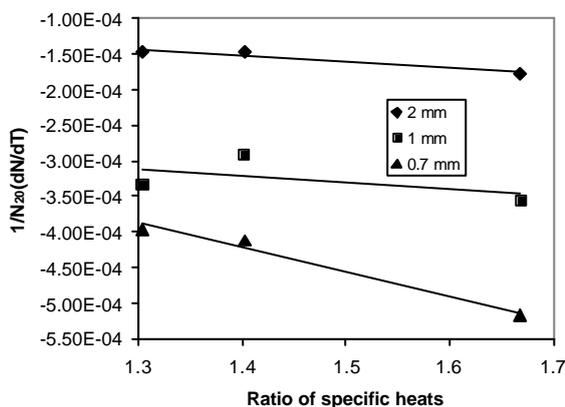


Figure 12. Values of  $1/N_{20}(dN/dT)$  plotted against the ratio of specific heats for the gases used.

## 5. Discussion

Johnson et al [5] have pointed out: “the effect of heat transfer is predominantly confined to a thin region

near the wall denoted as the thermal boundary layer. Increased temperatures in this region decrease both the density, and to a lesser extent, the flow velocity thus resulting in a lower mass flux.” This would result in a lower value for the nozzle coefficient in a calibration. They also point out that the sensitivity of the discharge coefficient (and thus the nozzle coefficient) to the wall thermal boundary condition is greater for gases with larger ratios of specific heats and greater at the lower values of Re. We have observed that  $1/N_{20}(dN/dT)$  is greater for smaller nozzles, that is for flows of lower Reynolds numbers. It is also clear from Figure 12 that the effect of a higher wall temperature is greater for gases with a higher specific heat ratio. This effect seems to be greater for small nozzles than for larger nozzles but the results for the 1 mm diameter nozzle are confusing and need to be repeated.

The significance of these findings is illustrated in Table 3, which is based on results discussed in [4]. It shows that there are no metrologically significant changes in  $N$  for nozzles of diameters greater than about 3 mm.

Table 3.

Diameter of throat (mm)	Measured temperature drop (K)	$1/N_{20}(dN/dT)$ (K <sup>-1</sup> )	Change in flow rate (%)
0.715	3.082	$4.03 \times 10^{-4}$	0.125
1.0	3.907	$2.94 \times 10^{-4}$	0.12
2.04	5.239	$1.5 \times 10^{-4}$	0.08

## 6. Conclusion

The greater part of the change in nozzle coefficients due to the use of argon rather than air or nitrogen has been shown to be due to the change in the RGCF. For carbon dioxide this does not explain the change. The expected decrease in the nozzle coefficient with increasing temperature of the nozzle as previously found for air [4] has been demonstrated for argon, carbon dioxide and nitrogen in Figures 8, 9 and 10. The magnitude of this change represented by  $1/N_{20}(dN/dT)$  shown previously for air to be linear with the reciprocal diameter of the throat, behaves similarly for argon. The results for carbon dioxide do not fit this pattern.

Plotting  $1/N_{20}(dN/dT)$  against the specific heat ratio shows the expected increase in this quantity with higher specific heat ratios. It also shows a stronger effect for the 0.7 mm nozzle than for the 2 mm one but the results for the 1 mm nozzle are confusing and need to be repeated.

The results with carbon dioxide are not readily explained by straightforward theory and in some cases appear inconsistent. It is possible (as always) that some of the measurements may be in error, especially for the 1 mm diameter nozzle and these should be repeated. If they are not in error then there

appear to be some interesting phenomena connected with carbon dioxide that could be further investigated. Certainly the work on vibrational relaxation effects [6] is very significant for the carbon dioxide results. Work like this can always be extended to larger and smaller nozzles. There seems little to be gained from the use of larger nozzles but smaller ones may show some interesting effects or at the least an intensification of the observed behaviour.

### References

- [1] ISO 9300: Measurement of Gas Flow by Means of Critical Flow Venturi Nozzles, 1990.
- [2] Bignell, N. and Choi, Y. M., Volumetric positive displacement gas flow standard, in: Flow Meas. Instrum. 12 (2001) 245-251
- [3] Bignell, N., Using small sonic nozzles as secondary flow standards, in: Flow Meas. Instrum. 11 (2000) 329-337.
- [4] Bignell, N. and Choi, Y. M., Thermal effects in small sonic nozzles, in: Flow Measurement and Instrumentation 13 (2002) 17-22
- [5] A. N. Johnson, C. L. Merkle, P. I. Espina, G. E. Mattingly and J. D. Wright, Numerical characterization of the discharge coefficient in critical nozzles, in: NCSL Conference Proceedings Albuquerque, NM, 1998, pp 407-422.
- [6] A. N. Johnson, J. D. Wright, S. Nakao, C. L. Merkle, M.R. Moldover, The effect of vibrational relaxation on the discharge coefficient of critical flow venturis, in: Flow Meas. Instrum. 11 (2000) 315-327.