Modal Analysis of Coriolis Mass Flowmeter

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Abstract: Coriolis Mass Flowmeter (CMF) is true mass flowmeter, by measuring the Coriolis effect of a vibrating flow pipe. Dynamitic Analysis of CMF plays an important role in its design and application. Finite element modal analysis of CMF Assembly and flow pipe individually is presented in the paper. CAD Model of CMF Assembly is built in AutoCAD, and then imported into FEM software Msc. Marc, and rotated to generate solid FEM model. Lanczos method is adopted in the calculation. Effects of type of the flow pipe's support are investigated. It's presented the FEM calculations are agreed with the theory analysis of Euler beam.

Key words: Coriolis mass flowmeter, CMF, modal analysis, natural frequency, mode

1. Forewords

Coriolis mass flowmeter, as a direct true mass flowmeter, which eliminated influences of temperature, pressure, and fluid states etc. on the precision, can directly and precisely measure mass flow of gas, liquid, even multiphase liquid, high viscosity fluid, and pasty media. It is also featured lower pressure loss, self-drained, sterilized and so on.

Coriolis mass flowmeter or CMF is based on the well-known Coriolis effect, generated when vibrating the measure tube conveying the flow. There's a variety of layouts to implement Coriolis mass-flow metering. Single straight tube CMF, for its supreme merits, as minimum maintenance, small size and weight, found welcome around the industries. One typical single straight layout is illustrated below.

Illustrated in Fig 1, is an embodiment of single straight tube CMF, composed by measuring tube, supporting (or balance) tube, case, electromagnetic exciter, vibration sensors, and flanges.

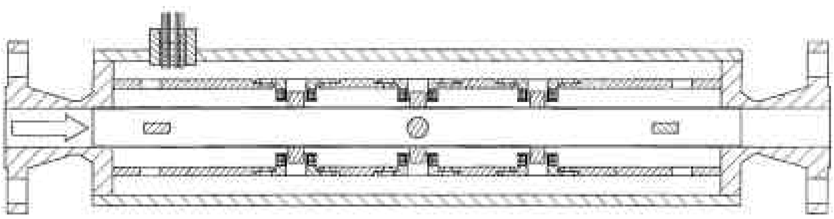


Fig 1 structure of an embodiment of single straight tube CMF

The electromagnetic mean or the exciter, being arranged at the mid point of the tube, oscillates the measuring tube. Vibration sensors are placed at the end portions of measuring tube to detect the vibration of the tube. In the absence of fluid flow, the phase of vibration at each sensor location will be approximately the same.

However, when fluid is passing through the oscillating tube, then additional Coriolis acceleration and Coriolis inertia force generate in the flow and act upon the tube. The Coriolis acceleration of an element of fluid mass is proportional to the angular speed and velocity, $\vec{a}_c = 2\vec{w} \times \vec{v}$. And Coriolis force is: $\mathrm{d}F_c = -\mathrm{d}m \cdot \vec{a}_c = -2\vec{w} \times \vec{v} \cdot \mathrm{d}m$. Because the transient rotation directions of upstream and downstream portion of the measuring tube oppose to each other. So the Coriolis acceleration and force act upon the two portions oppose to each other similarly. The opposed Coriolis force superimpose on the measuring tube and result in distortion of the tube, and change of vibration phase, there will tend to be a lag in the phase of the upstream sensor and a lead in the phase of the downstream sensor. From the phase difference, a measure of the mass flow rate can be obtained.

$$\Delta q = \frac{K}{E} q_m \qquad (1)$$

Where K is a constant, E is elastic modulus of the measuring tube; q_m is mass flow rate of the fluid. The formula can be achieved from the vibration equation of rigidly supported beam. Mass flow rate of the fluid is proportional to the phase difference of the upstream and downstream sensors.

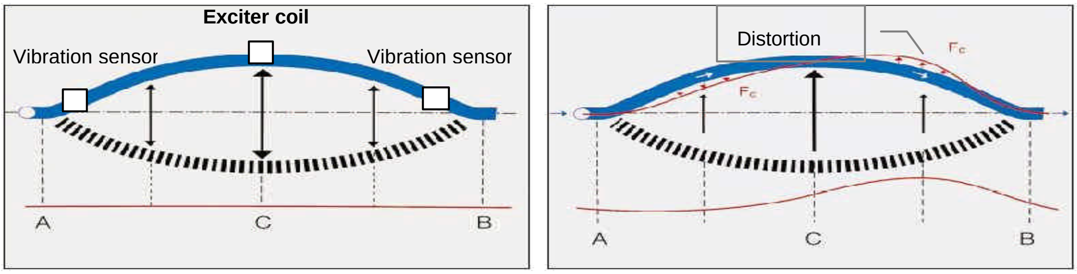


Fig2 Principles of CMF

Modal analysis is necessary to learn more about the mechanism of the CMF. It also found applications in: determining of vibration sensors location, calculation of strain and stress, fatigue analysis, structure optimization, circuit design, signal processing, state monitoring and failure prediction, and so on.

2. Theoretical analysis of measure tube vibration

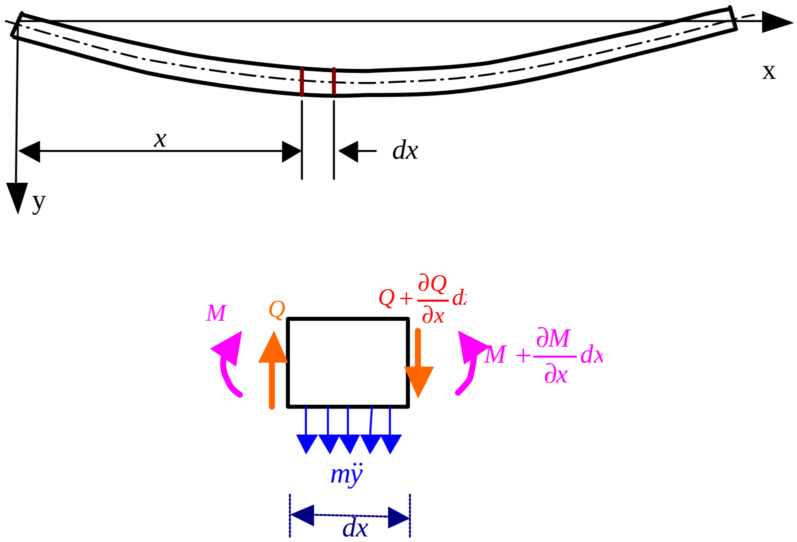


Fig3 bending vibration[tm8]

Measure tube is conventionally welded or brazed with outer tube at the two ends, subjected to vibrating force, and then can be simplified as a beam model rigid supported at the ends. Rotating effects and shearing deformation are omitted, as an Euler beam, from the equilibrium conditions of an element of the beam, dx, shown in FIG3, we can get:

$$-\frac{\partial Q}{\partial x} = m\frac{\partial^2 y}{\partial t^2}, \quad \frac{\partial M}{\partial x} = Q, \quad M = EI_x \frac{\partial^2 y}{\partial x^2} \tag{2}$$

Q—shearing force; *M*—bending moment; *m*—mass of a unit of length; *y*—deflection of the beam at x? EI_x —flexural rigidity.

And then we can get the differential equation of bending vibration, as following:

$$\frac{\partial^2}{\partial x^2}(EI_x \frac{\partial^2 y}{\partial x^2}) + m\frac{\partial^2 y}{\partial t^2} = 0 \tag{3}$$

Suppose the solution as the following form:

$$y(t,x) = W(x)T(t) \tag{4}$$

$$T(t) = A\sin wt + B\cos wt \tag{5}$$

Here, *W*(*x*) is the vibration amplitude of the beam at x, and *T*(*t*) is the motion of any point, represent that vibration is harmonic or can be decomposed to a series of sine wave response. Solving the equation and get general solution is:

$$W(x) = Ce^{kx} + De^{-kx} + Ee^{jkx} + Fe^{-jkx} \tag{6}$$

Here, $k = \sqrt{\frac{w}{a}}$ and $a^2 = \frac{EI_x}{m}$, *C, D, E, F* is constants decided by the boundary conditions. *W*(*x*) represents the mode shape.

Usually, the measuring tube is welded in the CMF, and can be simplified as rigidly supports, then, there is:

when $x = 0$, $y(0,t) = 0$, or $W(0) = 0$; $y'(0,t) = 0$, or $W'(0) = 0$; when $x = L$, $y(L,t) = 0$, or $W(L) = 0$; $y'(L,t) = 0$, or $W'(L) = 0$.

And then we get the mode frequency, or the *i*-th order of nature frequencies the measuring tube or beam is:

$$w_i = [\frac{(2i+1)p}{2L}]^2 \sqrt{\frac{EI_x}{rA}} \tag{7}$$

3. Finite element modal analysis

3.1 CAD modeling and FEM modeling

CMF, as axial symmetry structure, can adopt simple 2D model in FEM. To improve the analytical accuracy, sophisticated 3D model should be used.

When only build the 3D model of measuring tube, as a simple solid model, you can directly create it in Mentat, the GUI preprocessor and postprocessor for Marc. But it's not a convenient tool to construct a complicated solid model, so it's recommended to construct the 2D or 3D model in other CAD softwares and then imported into Marc/Mentat. The straight tube CMF, for its axial symmetric feature, can be build by revolving a 2D drawing in Mentat.

? Preparing the CAD drawings and importing into Marc

Usually AutoCAD drawings, as a versatile CAD type, supported by most CAD software, and can be easily reuse in FEA. You can also start drafting and revising the assembly drawing in AutoCAD from scratch. Omit or delete some trivial structures, and then import the DWG type 2D drawing into Marc.

? Geometry checking

CAD drawings imported may not be fully compatible with the FEA software. Before other operations, Geometry checking should be done to ensure the geometrical or structural integrity and perfection.

Check regions, which represent the body of a part, whether are closed region, if not, connect the broken lines. Eliminate duplicated curves, combine and delete short curves. Welding portion treated as rigid fixing.

? meshing the 2D regions individually

Allow the program automatically meshing each region, then adjust the local meshing manually. Save the elements of different parts as different element set, so as to conduct operations on them conveniently.

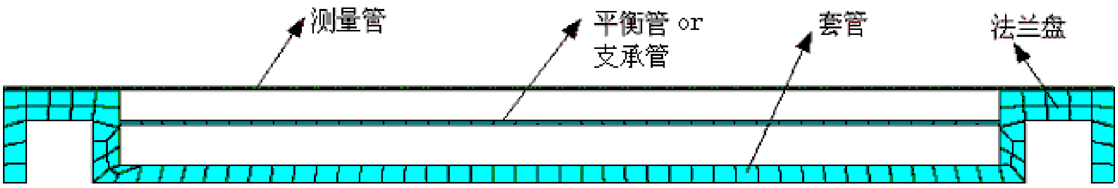


Fig4 2D mesh of the whole CMF

? Rotate to generate the solid model

Rotate the 2D mesh about the symmetrical axial, and then get the 3D mesh. The material attributes and geometrical attributes of 2D elements will automatic pass to their corresponding 3D elements.

In the 2D FEA, triangular and quadrangular elements (four-noded isoparametric elements) are often used. Three-noded triangular elements can well approach the boundary of a region when meshing it, but can not well approach the real strain distribution for its poor transformation performance. Four-noded isoparametric elements are adopted in the 2D model as shown in FIG. 4.

After rotation, a three-dimensional, eight-node element is generated. Define the element type as element Type 7 (3D, eight-node, first-order, isoparametric element), or namely, arbitrarily distorted brick.

? Combine the redundant nodes of adjacent region

The redundant nodes of adjacent region should be merged, where the parts of CMF are welded together. Such as, the binding portions between measuring tube and supporting tube, or supporting tube and flanges etc.

? applying loads and constraints:

For modal analysis, applying loads are not needed. In the later section, different constraints, influencing on modes, are discussed.

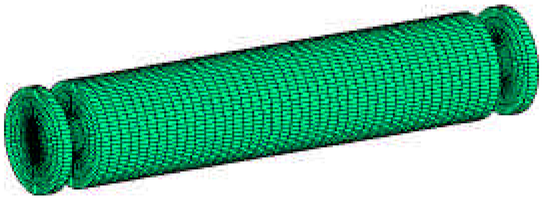


Fig5 3D mesh of the whole CMF

The mesh of the 2D axial symmetrical model is showed in Fig4, the generated 3D mesh through revolving is showed in Fig5.

3.2 Modes extraction method

Marc's dynamic analysis capability allows you to perform the following calculations: Eigenvalue Analysis, Transient Analysis, Harmonic Response, and Spectrum Response. The paper Only focus on Eigenvalue Analysis to extract eigenvalues and eigenvectors, that is, to calculate the mode frequencies and deflections, After the modes are extracted, they can be used in a transient analysis or spectrum response calculation.

Marc uses either the inverse power sweep method or the Lanczos method to extract eigenvalues and eigenvectors. The inverse power sweep method is typically used for extracting a few modes while the Lanczos method is optimal for several modes.

4. Finite element Analysis of the CMF

4.1 mode frequencies and mode shapes of the measuring tube

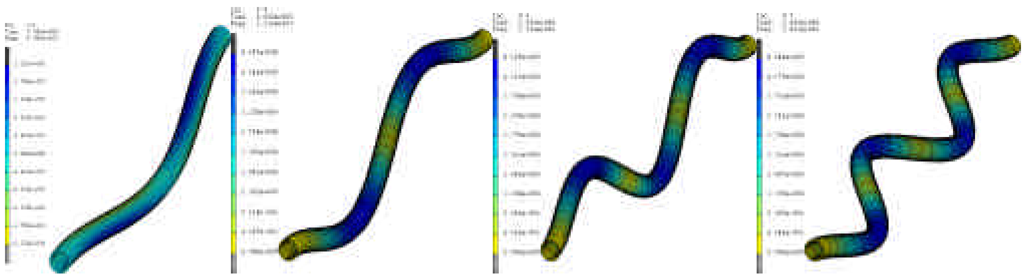
Suppose the measuring tube parameters for computation are as follows:

Geometric properties: length 396mm, wall thickness 1mm. Diameter 12mm

Material properties: isotropic material, Young's modulus: 2×10^{11} N/m^2, Poisson's ratio: 0.3, mass density: 7.8×10^3 kg/m^3

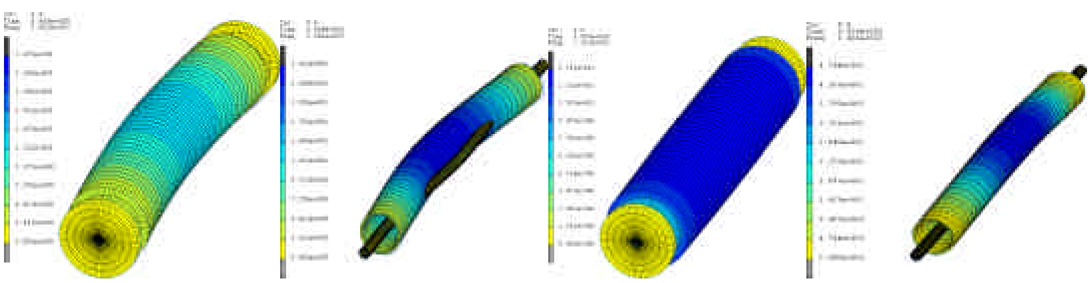
Constraints: fix all the nodes of the ends of the measuring tube.

The first 4 orders of CMF's modes are showed in Fig6. It coincides with the theory result of Euler beam model of CMF.



(a) (b) (c) (d)

Fig6 mode frequencies and mode shapes of the measuring tube[tm35]



(a) (b) (c) (d)

Fig7 mode frequencies and mode shapes of the CMF

4.2 mode frequencies and mode shapes of the CMF

Measuring tube parameters for computation are as follows:

Geometric properties: case, overall length 500mm, wall thickness 8mm, diameter 96mm; support tube, wall thickness 2mm, diameter 44mm, wall; flange, thickness 10mm, diameter 96mm; neck, length 30mm; Sizes of measure tube are same as above.

Material properties: isotropic material, Young's modulus: 2×10^{11} N/m^2, Poisson's ratio: 0.3, mass density: 7.8×10^3 kg/m^3

Constraints: fix all the nodes of the ends of the two flanges.

Fig7 (a) (b) (c) (d) show the first 4 orders of CMF's modes. Bending of the case, bending of support tube & measuring tube, twist of the whole CMF (3 tubes), bending of the support tube. Deformation is enlarged in order to easily be recognized.

4.3 comparisons of the CMF and the measuring tube

Modes of the CMF and the measuring tube are agreed with each other. The first, and third mode of CMF correspond to the first and second (bending) mode of measuring tube. The relative mode frequency differences are within 0.3%. Whole CMF has more modes than the measuring tube in a definite frequency range, that may influence the working of CMF.

The result show that when low frequency dynamic characteristics of CMF are studied, can be simplified as a measuring tube only, and even as a beam. Table 1 lists some modes of the CMF.

mode frequencies Hz				Mode shape
Orders	the whole CMF	Orders	measuring tube	
1	445.414	1	446.52	1st order bending of measuring tube
2	1082.14			1st order bending of case & support tube
3	1216.75	2	1215.16	2nd order bending of the measuring tube
4	1223.24			Twist of the whole CMF
5	1461.17			1st order bending of support tube
		3	2344.22	3rd order bending of the measuring tube
		4	3801.64	4th order bending of the measuring tube

Table 1 modes comparison measuring tube and the whole CMF

4.4 Influences of support type on mode frequencies

To the rigid constraints single straight tube CMF, except its low sensitivity, it may suffer from the enormous thermal stress, residual stress and assembly stress. That may furthermore results in zero drift, sensitivity changing, or even fatigue and cracks.

Special materials, with lower elastic modulus, lower thermal expansion coefficient, etc., are often used to alleviate the problem. Changing the constraint type is another solution, such as, appending corrugated pipe to the measuring tube, employing disconnected pipe or cantilever pipe. Then the measuring tube may be simplified as beams with different supports.

Next, support or constraint type's influences on mode frequency are discussed. Geometry and material, calculation method are same as above. Consider two rigid support at the ends; one rigid support at an end, one axial free end at another end; two axial free ends; free state (unconstrained); and cantilever state, Showed in Table 2.

Support / Order No.	Free state	2 rigid support	1 rigid support 1 axial free	2 axial free	cantilever
1 (bending)	446.755	446.52	308.321	197.533	70.6445

2 (bending)	1221.35	1215.16	989.764	785.203	439.713
3 (bending)	2366.12	2344.22	2038.01	1748.84	1218.78
4 (bending)	3852.21	3801.64	3201.97 (expansion)	3066.7	2355.05

Table 2 mode frequencies of different support type (unit: Hz)

Compare the mode frequencies of different supports in Table 2, two rigid supports at the end or two axial free ends results in the highest mode frequencies. When axial are free, the mode frequencies will decrease, when one end is completely free, namely, cantilever state, and then the mode frequencies are the lowest.

The reason is the less constraint, the lower the equivalent bending rigidity. For the axial free end, the static axial stress, and most dynamic axial stress are eliminated, the bending vibration is less impeded, so the equivalent bending rigidity decreases.

It should be aware that dynamic axial stress will not be eliminated, for the existence of dynamic axial inertia force[tm62].

Because the axial freedom is not constrained, expansion mode appeared earlier.

For one rigid, one axial free end type model, and the modes do not symmetrical appeared for its asymmetrical constraints.

5. Conclusion

Finite element method can play an important role in dynamic analysis of CMF. For simple model, we can directly build the model in Marc/Mentat. For a complicated model, we can take advantages of 2D CAD drawings, imported to Marc, and then build the 3D model for FEM in it. The 3D models of axial symmetry parts can be generated by revolving the 2D meshes of imported 2D drawing in Marc. Lanczos method is chosen to extract eigenvalues and eigenvectors. The modal analysis of a simple measuring tube model and a complicated model of the CMF assembly as a whole are compared, modes of the assembly are more than the single measuring tube, and the difference of their nature frequencies is quite small. Some modes frequency and shape may be a disturbance to the working mode if not carefully designed and optimized. Support conditions will greatly influence the modes.

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