

Theory Conversion Method And Uncertainty Analysis

For Critical Flow Coefficient Of Venturi Nozzle

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Abstract Without actual flow standard equipment of certain type of flow media or on the occasion of requiring no high measuring accuracy, critical flow coefficient of Venturi nozzle may be disposed by other methods instead of being actual flow calibrated through this flow media. The flow coefficient calibrated by other media, being enough analyzed and computed can be directly applied to a flow measurement of the other media. This article mainly researches on conversion method and analysis and computation of uncertainty of critical flow coefficient when Venturin nozzle works in critical flow state. In this article, on the premise of ensuring enough accuracy, the theory method of conversion between air and the natural gas, aiming at the flow coefficient of critical flow Venturin nozzle, is practical and feasible by experiments, furthermore this method may be popularized to other flow media on some conditions.

Keywords: Flow Measurement; Critical Flow Coefficient; Conversion; Uncertainty; Venturin Nozzle

1. Foreword

After through some fluid medium, for example air calibration, its flow revise coefficient C_D is certain constant for medium A. However, when the fluid medium while using actually changes, such as turning into medium B, because fluid medium is different, the physical property parameters of the fluid are inconsistent, its external condition is certainly different such as temperature pressure ,etc. while reaching the same flow state (such as the turbulence / turbulence),Or under the same external condition of work, the flow state of the fluid will change, and hydrodynamics theory have proved[1]that fluid medium have greater influence on fluid field

characteristics of spray nozzle throat, influence the flow of flowing through spray nozzle too. Fluid medium impact on flow will reflect in change of flow revising coefficient. Therefore draw a question, spray nozzle after air calibration, under prerequisite of lacking flow standard device of natural gas or not passing calibration of natural gas actual flow, can apply to measure the flow of the natural gas medium directly? Essence of this question is how about convert different medium and flow revise coefficient of spray nozzle under the different condition of work?

2. Flow revises coefficient C_D

The influence factor of the actual flow deviates from the flow of theory, including no isentropic process, border layer effect, three-dimension flow effect(spray nozzle geometry form ,etc) impact on flow , express with flow revise coefficient C_D . There are two kinds of methods to confirm flow revise coefficient C_D : experiment law and theory computing method. Some kind of method is to confirm C_D according to these kinds of influence.

The influence of border layer effect express with flow out coefficient C_d , no isentropic flow effect express

with efficiency factor C_η of quality flow, three-dimension flow effect(the influence of geometry form) express with geometry convergence factor C_c . These three kinds of factors act on

together and Influence flow revise coefficient C_D synthetically. So, C_D can show [2] like below:

$$C_D = C_c \cdot C_d \cdot C_\eta \quad (1)$$

2.1. Calculate C_d

Theory analysis and experiment prove, C_d generally has a form[3]:

$$C_d = f(R_{ed}) = a - bR_{ed}^{-n} \quad (2)$$

Among the formula, R_{ed} is Reynolds number, a, b, n separately is constant value determine by R_{ed} and geometry form of spray nozzle.

2.2. C_η

Mass flows effectiveness factor C_η uses the following algorithms[4]:

$$C_\eta = k\tau^{-1}M_t r_* \left(1 + \frac{k-1}{2}M_t^2\right)^{\frac{1}{2}} \quad (3)$$

Among formula, $r_* = P_*/P_0$ is pressure ratio, and

r_* Meet the following equations:

$$r_*^{(1-k)/k} + \frac{k-1}{2}\beta^4 r_*^{2/k} = \frac{k+1}{2} \quad (4)$$

k : isentropic index

β : ratio of flow channel diameter under two

kinds of states

$$\tau = \psi^* = k \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \quad (5)$$

Among above formula, ψ is flow factor, ψ^* is ψ when the speed of spray nozzle throat reaches the velocity ($M_t=1.0$), M_t is Mach.

2.3. Geometry convergence factor

C_c is only measure of the actual fluid deviates from one-dimension flows. Calculates theory of C_c

is very complicated, can quote C_c provided by J.R.Kliegel and functional relation picture [5] of.

$$R/\left(\frac{d}{2}\right) . R \text{ is the radius of camber of spray nozzle}$$

throat, d is the diameter of spray nozzle throat. Contrast this picture, after the geometry form of the spray nozzle is confirmed, $R/\left(\frac{d}{2}\right)$ confirm, can

find the value of C_c from the picture.

Synthesize above-mentioned three factor computing technologies, can provide the calculation formula that the flow revise coefficient C_D .

$$C_D = C_c C_d k\tau^{-1}M_t r_* \left(1 + \frac{k-1}{2}M_t^2\right)^{\frac{1}{2}} \quad (6)$$

3. Conversion method of the flow revises

coefficient under different fluid medium

Assume flow revise coefficient of spray nozzle given under the flow medium A, B are C_D and C_{D0} . The Physical parameter of medium A is $(R_0, \mu_0, \gamma_0, k_0)$, the Physical parameter of medium B is (R, μ, γ, k) . Assume known C_{D0}, p_0 and T_0 (p_0 is bandstop pressure and temperature of In front 1 of the spray nozzle), demand to convert out C_D , while spray nozzle work at medium A and bandstop pressure and temperature of In front 0 of the spray nozzle is p, T .

Have by formula (1):

$$\frac{C_D}{C_{D0}} = \frac{C_c C_d C_\eta}{C_{c0} C_{d0} C_{\eta0}} = \frac{C_c}{C_{c0}} \cdot \frac{C_d}{C_{d0}} \cdot \frac{C_\eta}{C_{\eta0}}$$

For some spray nozzle given, have under medium A and B $C_c = C_{c0}$

Order $g(k) = C_\eta / C_{\eta0}$,

$$H(\gamma, k) = C_d / C_{d0}, f(\gamma, k) = C_D / C_{D0} \quad (7)$$

Then, above formula turn into:

$$C_D = C_{D0} \cdot f(\gamma, k) \quad (8)$$

$$f(\gamma, k) = g(k) \cdot H(\gamma, k) \quad (9)$$

According to formula (3) conclude:

$$g(k) = \frac{\left(\frac{2}{k_0+1}\right)^{k_0/(k_0-1)} \cdot r_*}{\left(\frac{2}{k+1}\right)^{k/(k-1)} \cdot r_{*0}} \quad (10)$$

According to formula (2) conclude:

$$H(\gamma, k) = \frac{C_d}{C_{d0}} = \frac{a - bR_{ed}^{-n}}{C_{d0}} = \frac{a}{C_{d0}} - \left(\frac{a}{C_{d0}} - 1\right) R_{ed}^{-n} \cdot \left(\frac{R_{ed}}{R_{ed0}}\right)^{-n}$$

(11)

$$\text{Because } R_{ed} = \frac{4Q}{\pi d \mu}$$

$$Q = C_D A p / \sqrt{RT}$$

$$\text{So } \frac{R_{ed}}{R_{ed0}} = \frac{Q}{Q_0} \cdot \frac{\mu}{\mu_0} = \frac{C_D}{C_{D0}} \cdot \frac{p}{\sqrt{RT}} \cdot \frac{\sqrt{R_0 T_0} \mu}{p_0 \cdot \mu_0} \quad (12)$$

Order among above formula

$$\alpha = \sqrt{R_0 T_0} \mu / (p_0 \mu_0 \sqrt{R}) \quad (13)$$

Take formula (8) into formula (12) conclude:

$$\frac{R_{ed}}{R_{ed0}} = \frac{p}{\sqrt{T}} \alpha \cdot g(k) H(\gamma, k) \quad (14)$$

Take formula (14) into formula (11) conclude:

$$H(\gamma, k) = \frac{a}{C_{d0}} - \left(\frac{a}{C_{d0}} - 1\right) R_{ed0}^{-n} \cdot \alpha^{-n} \cdot g^{-n}(k) p^{-n} \cdot T^{\frac{n}{2}} \cdot H^{-n}(\gamma, k)$$

$$\text{order } A = a/C_{d0}, B = \left(\frac{a}{C_{d0}} - 1\right) R_{ed0}^{-n} \alpha^{-n} g^{-n}(k)$$

Then, above formula turn into:

$$H(\gamma, k) = A - B p^{-n} T^{\frac{n}{2}} H^{-n}(\gamma, k)$$

(15)

The equation (15) is the equation of $x = \varphi(x)$ type,

So, order $x = H(\gamma, k)$, formula (15) turn into:

$$\varphi(x) = A - B p^{-n} T^{\frac{n}{2}} x^{-n} \quad (16)$$

$$\varphi'(x) = n B p^{-n} T^{\frac{n}{2}} x^{-(n+1)} \quad (17)$$

$$|\varphi'(x)| = |n B p^{-n} T^{\frac{n}{2}} x^{-(n+1)}| \leq |n B p^{-n} T^{\frac{n}{2}}| \cdot |x|^{-(n+1)} \quad (18)$$

According to the convergence principle of iterative

method: if make $|\varphi'(x)| < q < 1$ establish, then

equation $x = \varphi(x)$ will certainly have real root exist, and equation can solve solution by iterative method:

$$x_{l+1} = \varphi(x_l) \quad (l = 0, 1, 2, \dots) \quad (19)$$

When taken iterative method to solve solution of $x = \varphi(x)$, the choice of initial value is no influence on solving equation, only influence the convergence speed. So the equation (15) should meet the condition:

$$|n B p^{-n} T^{\frac{n}{2}}| \cdot |x|^{-(n+1)} < q < 1$$

That is to say: $|n B p^{-n} T^{\frac{n}{2}}| < q |x|^{(n+1)}$

$$|x| > \left[\frac{|n B p^{-n} T^{\frac{n}{2}}|}{q} \right]^{\frac{1}{n+1}} = w$$

But $x = R(\gamma, k)$ can not use the negative number,

so the range of x is:

$$x > w = \left[\frac{|nBp^{-n}T^{\frac{n}{2}}|}{q} \right]^{\frac{1}{n+1}} \quad (20)$$

Among above formula, $q \in (0,1)$ is a controlling amount.

According to ahead analysis and derive, draw conversion formula of C_D under two fluid medium:

$$C_D = C_{D0} f(\gamma, k) \quad (21)$$

$$f(\gamma, k) = g(k) H(\gamma, k) \quad (22)$$

$$g(k) = \frac{\left(\frac{2}{k_0+1}\right)^{k_0/(k_0+1)}}{\left(\frac{2}{k+1}\right)^{k/(k+1)}} \cdot \frac{r_*}{r_{*0}} \quad (23)$$

$H(\gamma, k)$ is solution of below formula:

$$H(\gamma, k) = A - Bp^{-n}T^{\frac{n}{2}}H^{-n}(\gamma, k) \quad (24)$$

Among formula

$$A = a/C_{D0}, B = \left(\frac{a}{C_{D0}} - 1\right) R_{ed0}^{-n} \alpha^{-n} \cdot g^{-n}(k) \quad (25)$$

$$\alpha = \sqrt{R_0 T_0} \mu / (\sqrt{R} p_0 \mu_0) \quad (26)$$

r_* and r_{*0} are separately Critical pressure ratio

under k and k_0 , both meet the following equations:

$$r^{(1-k)/k} + \frac{k-1}{2} \beta^4 r^{\frac{2}{k}} = \frac{k+1}{2} \quad (27)$$

(27)

$$\beta = \frac{d}{D} \quad (28)$$

(28)

4. Uncertain degree of C_D conversion formula

Known by formula (21), flow medium influence on fluid revise coefficient C_D embody mainly in function $f(\gamma, k)$.

4.1 Assume function $f(\gamma, k)$ is no error

The uncertain degree of C_D depend on the uncertain degree ΔC_D of C_D under the primitive calibration condition of the spray nozzle.

4.2 Assume the uncertain degree of $f(\gamma, k)$

is $\Delta f(\gamma, k)$

The uncertain degree of C_D is determined together by ΔC_D and $\Delta f(\gamma, k)$.

Assume relative error of $f(\gamma, k)$ is:

$$\delta_1 = \Delta f(\gamma, k) / f(\gamma, k) \times 100 \% \quad (29)$$

The relative error of C_{D0} is δ_2

$$\delta_2 = \Delta C_{D0} / C_{D0} \times 100 \% \quad (30)$$

(30)

Then, the error of C_D is:

$$\delta = \sqrt{\delta_1^2 + \delta_2^2} \quad (31)$$

(31)

5. Experiment and conclusion

Take Venturin nozzle as an example, calibrate in the air medium, the condition of calibration is $p_0, T_0, \mu_0, R_0, \gamma_0, k_0$, Venturin nozzle revise

coefficient is C_{D0} . Then change into the condition of work of natural gas medium p, T, μ, R, γ, k . According to ahead calculation formula (21)~(28), calculate out C_D of natural gas medium and corresponding condition of work, after compare C_D and flow revise coefficient C'_D of natural gas medium calibrated by actual flow., convert precision in $\pm 2\%$. The results of a lot of experiments are

identical basically. Prove this computing method is feasible in some range of convert precision.

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