

# An Approximate Mathematical Model of Ultrasonic Wave Modulated by Von Karman Vortex Street in a Liquid Fluid

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## Abstract

It is known that both phase and amplitude of an ultrasonic wave passing through Von Karman vortex street perpendicular on the bluff body axis, are modulated. Modulation is result of natural fluid flow in the Von Karman vortex street, absorption and dispersion of ultrasonic wave.

This paper is intended to present influence of vortex shedding on the amplitude modulation of ultrasonic wave which is caused by movement of vortices in vortex street. Mathematical model of the amplitude modulation of ultrasonic wave in Von Karman street and influence of ultrasonic wave absorption in vortices on the amplitude modulation have also been presented. Experimental results of testing prototype based on amplitude modulation developed for this research, and comparative analysis with result of vortex meter prototype based on phase modulation of ultrasonic wave have been done also.

Test results obtained on a vortex meter prototype based on amplitude modulation, developed for this research and comparative analysis of these results and results obtained on a vortex meter prototype based on phase modulation of ultrasonic wave have been presented in the paper.

## Introduction

A double row of staggered vortices is formed behind the bluff body, immersed in a fluid flow, if the Reynolds value number lies within a certain range. This effect is known as the Karman vortex street.

Frequency of generated vortices is directly proportional to the average flow velocity and that dependence is linear. Relation between the vortex frequency  $f$ , the mean flow velocity  $v$  and the characteristic dimension of the bluff body  $d$  is expressed in a dimensionless form as the Strouhal number  $Sh$ ,

$$Sh = \frac{f \cdot d}{v} \Rightarrow v = f \cdot \frac{d}{Sh} = k \cdot f$$

A vortex causes changes of pressure in the vortex street and most vortex flow-meter use a pressure sensor for detection of the vortices generated on both sides of the bluff body. Piezoelectric type pressure sensor has generally been used for detection of vortices.

Vortices can also be detected using ultrasound. Ultrasound is used more and more for detection of vortices due to its specific features such as high sensitivity and the effect of wave modulation by Von Karman vortex street.

These features form the basis for design of new type of vortex flow meter .

Amplitude and phase modulation of the ultrasonic sound are directly caused by occurrence of vortices in Von Karman vortex street.

In this paper we are going to present mathematical model of amplitude modulation of ultrasonic wave, simulation and experimental results of calibration of vortex flow meter prototype based on this phenomenon.

Flow through a pipe of diameter  $D$  could be expressed as :

$$Q = v \cdot A = v \cdot \frac{D^2 \pi}{4} = f \cdot \frac{d}{Sh} \cdot \frac{D^2 \pi}{4} = K \cdot f$$

$$K = \frac{d \cdot D^2 \pi}{4Sh}$$

### Amplitude modulation of ultrasonic wave

Analysis of propagation of ultrasonic wave trough Von Karman vortex street perpendicular to the pipe and bluff body axis will be done assuming that the following conditions are satisfied:

- the flow is two-dimensional,
- vortices are circular in shape,
- the flow is potential in vortices and turbulent outside the vortices,
- the shape of vortices does not change in stable flow region,
- Von Karman vortex street is unlimited and vortices generated from both side of bluff body have the same value of circulation , but opposite direction.

Amplitude modulation of the ultrasonic wave is result of interaction between two mechanical fields: We will consider motion of the wave in  $x$ - $y$  plane as shown in fig.1.

Equation describing ultrasonic wave propagation trough the fluid flow can be written as:

$$\Delta u - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad (1)$$

Ultrasonic wave propagates only  $y$  direction (fig.1.) so relation (1) becomes:

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2} = 0$$

Solution of this equation is harmonic function that can be written:

$$u(y,t) = U(y) \cdot \cos(\omega_0 t - k_y + \varphi_0) \quad \text{where are:} \quad (2)$$

$\omega_0$  - ultrasonic frequency ,  $\omega_0 = 2\pi f_0$

$k$  - wave number,  $|k| = \frac{2\pi}{\lambda} = \frac{2\pi f_0}{c}$ ,

$\varphi_0$  - phase in the beginning of ultrasonic wave,

Amplitude of the ultrasonic wave  $U(y)$  is a function of distance from ultrasonic transmitter to ultrasonic receiver

$$U(y) = U_0 \cdot e^{-my} \quad \text{where:}$$

$$m \sim \frac{\omega_0^2 \xi}{2c^3}, \quad \text{absorption coefficient in liquid.} \quad (3)$$

Ultrasonic signal on the transmitting side ( $y = 0$ ) is:

$$U(t) = U_0 \cdot \cos(\omega_0 t + \varphi_0), \quad \text{and on the receiving side } (y = D) \text{ (fig.1.)}$$

$$U(t) = U_0 \cdot e^{-mD} \cos(\omega_0 t - k_D + \varphi_0)$$

When ultrasonic wave passes through the vortices (tangential velocity of vortex is  $V_\theta$ ), relation (2) could be written as:

$$u(y,t) = U_0 \cdot e^{-m(c \pm V_\theta)t} \operatorname{Re}[e^{j\phi(t)}]$$

$$\phi(t) = \omega \cdot t - ky + \varphi_0$$

On the receiving side this equation could be written:

$$u(y,t) = U_0 \cdot e^{-mD} \cdot e^{\pm \frac{mV_\theta \cdot D}{c}} \operatorname{Re}[e^{j\phi(t)}] \quad (4)$$

Expression  $U_1 = U_0 \cdot e^{-mD}$  is result of absorption of ultrasonic wave on the receiving side when the vortices are not present.

Relation (3) becomes:

$$u(y,t) = U_1 \cdot e^{\pm \frac{mV_\theta \cdot D}{c}} \cdot \operatorname{Re}[e^{j\phi(t)}] \quad (5)$$

factor  $e^{\pm \frac{mV_\theta \cdot D}{c}}$  shows effect of influence of vortices to the ultrasonic wave amplitude. It is result of absorption of ultrasonic wave in the vortices and their tangential velocity.

If we expand  $e^{\pm \frac{mV_\theta \cdot D}{c}}$  in the Taylor series and take first two terms we will get:

$$u(y,t) = U_1 \cdot \left( 1 \pm \frac{m \cdot D}{c} \cdot V_{\theta y} \cdot \cos \omega t \right) \operatorname{Re}[e^{j\phi(t)}] \quad (6), \text{ where: } V_{\theta y} = V_\theta \cdot \cos \omega t$$

Sign  $\pm$  indicates opposite direction of velocity circulation of generated vortices.

Denoting  $m_a = \pm \frac{m \cdot D \cdot V_\theta}{c}$  expression (5) becomes:

$$u(y,t) = U_1 \cdot (1 \pm m_a \cdot \cos \omega t) \cdot \operatorname{Re}[e^{j\phi(t)}] \quad (7)$$

Equation (6) presents well known amplitude modulated signal where  $m_a$  is coefficient of modulation.

Both, phase and amplitude of the ultrasonic signal are modulated. Equation on the receiving side has the form [6]:

$$u(t) = [U_0 \cdot (1 + m_a \cdot \cos \omega t)] \cdot \cos(\omega_0 t + m_p \cos \omega t) \text{ where:}$$

$m_p$  – coefficient of phase modulation, but in this case we have inters only for amplitude modulation.

We are going to estimate coefficient of amplitude modulation  $m_a$ :

$$\text{According to relation (2) } m_a \sim \frac{\omega_0^2 D^2 \cdot V_\theta}{2c^4} \quad (8)$$

This expression is very important for selection of working frequency for a given value of the diameter  $D$ .

For prototype vortex flow meter PVMP100, DN 100, working frequency is:  $f=2$  MHz, velocity flow from  $0,2$  m/s to  $4$  m/s, and water as working fluid. Coefficient of amplitude modulation is about 1% to 20% of the amplitude of ultrasonic wave on the receiving side. Coefficient of amplitude modulation is less than one and amplitude modulation will be correct for the full range of velocity. It should be noticed that amplitude modulation is not only caused by absorption and tangential velocity of vortices but also by other parameters such as: ultrasonic dispersion, pressure and coefficient of compression, especially if the working fluid is gas [2].

Results of analysis agrees well with simulation and experimental results as fig.2 shows.

## Experimental results

Prototype vortex meter PVMP 100 has been developed on this phenomenon. Calibration of prototype was carried out in Laboratory for calibration of flowmeters "Delta Petrol" Bosnia and Herzegovina.

Petroleum was working fluid, and PD volume meter F4.S1 Smith Meter GmbH, range from 250 [l / min] up to 2500 [l / min], and linearity  $\pm 0,15\%$  was the laboratory standard meter.

Testing procedure consist of measurement of volume passed trough the laboratory standard meter and Vortex prototype during a fixed time period (three minutes) and counting a number of pulses (shedding frequency) on Contrec integrator (202).

Reference meters used in calibration was described in procedures above belong to A1 group (ISO/FDIS 11631: 1998 [E]).

Both amplitude modulation method and phase modulation method of ultrasonic waves were used. Relation between the number of pulses from prototype vortex meter and the volume of fluid registered at laboratory standard flow meter in every point of examination presents calibration constant K written as:

$$K = \frac{N}{\Delta V \cdot K_p} \text{ where are:}$$

$N$  - number of pulses from prototype of vortex meter,

$K_p$  - programmable constant of Contrec 202 integration.

Diagram of linearity and uncertainty of calibration curves, for booth methods: amplitude and phase modulation ultrasonic wave by Von Karman vortex street are shown in fig.3.

Maximal flow that could bee used (900[l/min]) was limited by tank capacity. It doesn't influence our general conclusions, because large flow results in a higher level of modulation making detection of vortices easier.

### Conclusion:

1. Influence of vortices tangential velocity and absorption of ultrasonic wave, caused by that, are enough for using amplitude modulation method for detecting vortices in liquid flow at vortex flow meter.
2. Both methods (amplitude and phase modulation of ultrasonic wave) gave good results.
3. Uncertainty for both curves is better than  $\pm 0,75\%$  in the full calibration range.
4. Minimal rates of flow registered using amplitude modulation method were 130 [l/min] and for phase modulation method 98 [l/min].
5. Minimal registered velocity was 0,28[m/s] and 0,2 [m/s] for amplitude and phase modulation methods respectively.
6. Corresponding Reynolds numbers were:  $1,2 \cdot 10^4$  and  $1 \cdot 10^4$  respectively.
7. Phase modulation method gives better results in low flow region, but amplitude modulation method is simpler for realization.
8. It is possible to improve results for amplitude modulation causing shape of bluff body and well tuned adaptive low phase filter.

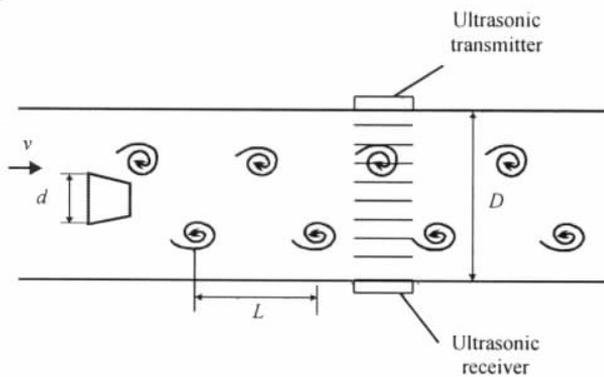


fig.1.

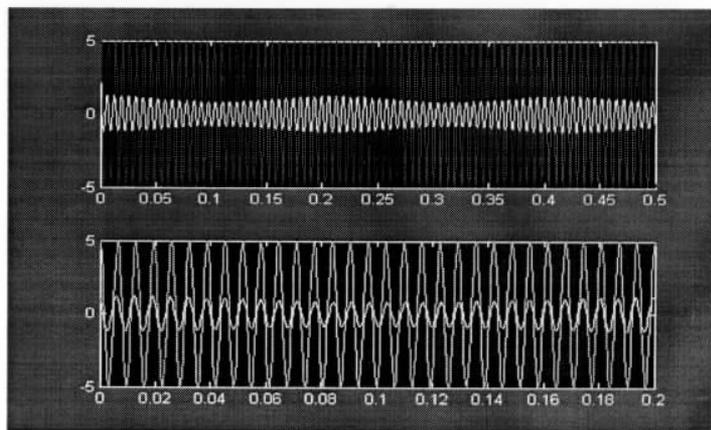


fig.2.

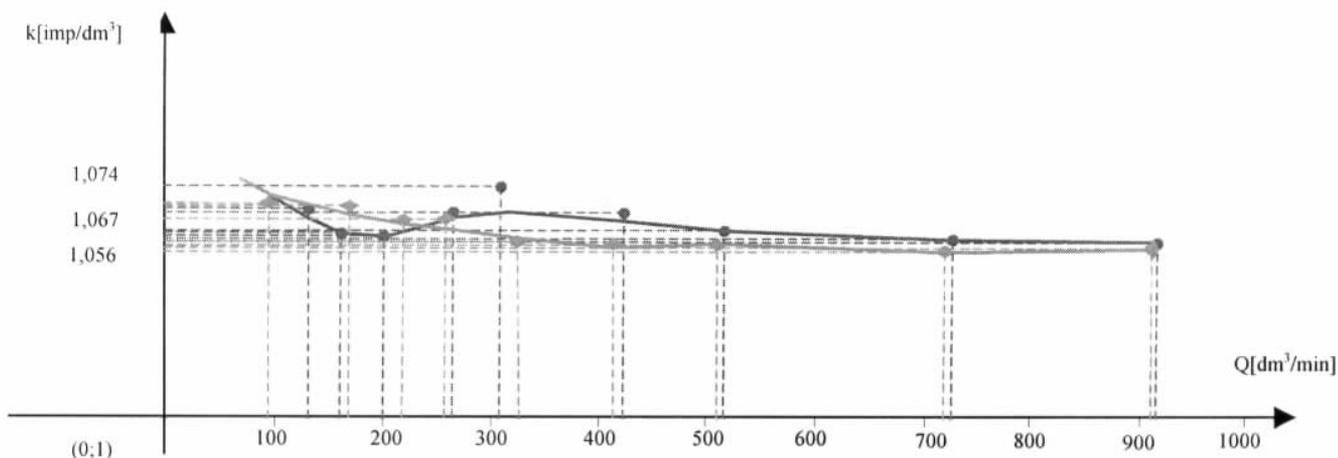


fig.3.

- calibration curve of vortex prototype based on amplitude modulation of ultrasonic wave
- ◆ calibration curve of vortex prototype based on phase modulation of ultrasonic wave

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