

# Simulation Modeling of Electromagnetic Flowmeters at Flows of Intricate Shapes

I.D. Velt, Ju.V.Mikhailova , Russia  
Sun Yan Zuo , China

The problem of a flow measurement of electroconductive mediums with complicit kinematic flow structure is exclusive is relevant, as for the purposes of the count (in a municipal services, for the solution of ecological problems etc.), and for the technological purposes (in chemical, iron and steel industry, by effecting microelectronics etc.). Development and improvement of electrical power and heat power objects, evaluation of ecological problems demands more and more precise and reliable means of flow measurement of a water and heat carriers on the basis of water in pressure pipings .

The signal, excited between electrodes of an electromagnetic flowmeter by a fluid flow , is determined by distribution of flow velocity on cross section of a channel, average with so-called volumetric weighting function dependent only on geometry of a channel of a flowmeter. This property is not characteristic for other converters of the consumption, in which one the usually local flow velocity is measured. In a channel of the device there are not any configuration items, that are handicapping a flow. The indications of electromagnetic flowmeters practically do not depend on change of physical characteristics of measured medium: of the density, viscosity, electrical conductivity, they depend on distribution of flow velocity in a channel very little.

Therefore, when there is an actual problem of increase of accuracy and reliability of a flow measurement, the its solution first of all is binding with an electromagnetic method of measurement.

The conditions of measurement in tubes of small (with diameters 20-80mm) and large diameters are those, that the kinematic flow structure in worked volume of a flowmeter is various.

The flowmeters of small diameter almost always can be established enough far from a pipeline control valves. On this cause it is possible to consider a flow structure in tubes of

small diameter with a high accuracy as axisymmetrical. However requirement of transition to a volume range 1:1000 results foregone in rearrangement of a velocity profile inside a permissible range of the flows.

Flows in tubes of large diameter always turbulent. Such flowmeters often are required to be established near to a pipeline control valves. A velocity profile much asymmetrical. Such flows are characteristic for main, distributive and technological pipe lines delivering water from pump stations to a energyl systems or assigning from they. The flows, as a rule, flow past at the temperature of from +5 up to +35°C, however in systems теплоснабжения temperature of a heat-transfer fluid can reach (150-180)°0C. In cold water probably minor quantity of sand, ooze, algae and even of a small-sized fish. As a rule, for a decrease of quantity of impurity(additives), on a sucking part of the pompe the filters are put. The velocity of a liquid does not exceed 1.5-3.5 m\s, and at the pipe of the large diameter the velocity of a flow, as rule, is lower. The pipe lines of small and angle diameters (from 400 up to 1200 mms) are produced usually from steel, and large diameters - from a reinforced concrete. They are made under ground, on depth of 2-3 m from its surface. The access for the installation of flowmeters is usually possible only in the place, most inconvenient for measurements: at bending of the pipe line near to the pompe, near intake to any aggregate, sump, tank or inside the well, where the stop gate valve, disk shutter, reverse valve or leading-out flanged tee is arranged. As is known, the distribution of flow velocity often has essentially asymmetrical diagram. Besides in these places the access to the pipe line is limited, as he usually directly lies on concrete foundation or close flanks to the aggregate.

The electromagnetic flowmeters allow to measure the flow at composite flow structures: at inexact filling of the channel by measured medium; at nonuniform allocation of a phase

checking of electromagnetic flowmeters in such conditions is necessary the specialized metrological equipment. The flowing plants reproducing indispensable flow pattern are extremely complicated and are expensive. The calibration of electromagnetic flowmeters at composite structures of flow can be yielded by a method of a simulation modeling.

The signal of an electromagnetic flowmeter  $U$  is a functional of distributions of a magnetic field and of flow velocity. In view of a smallness of magnetohydrodynamic interplay the distribution of velocity is determined only by hydrodynamics and for a considered problem is considered as given.

The manifestative relation of a signal  $U$  from the indicated distributions can be written as:

$$U = \int_{\tau} d\tau \bar{v} \left[ \frac{\partial g}{\partial \bar{r}} \bar{B} \right] \quad (1)$$

where the function  $g(\bar{r})$  is meaningful to volumetric weighting function and in cylindrical axials have the form:

$$g(z, \rho, \theta) = \int_0^{\infty} dk \cos kz \sum_{\substack{m=0 \\ n=2m+1}}^{\infty} (-1)^m \frac{I_n(k\rho)}{I_n'(kr)} \sin n\theta. \quad (2)$$

Here  $I_n(k\rho)$  is modified function of a Bessel,  $I_n'(kr) = \frac{dI_n(kr)}{dr}$ . It is supposed, that the

$$A(z, \rho, \theta) = \int_{-\infty}^{+\infty} dk \left[ e^{ikz} \sum_{n=0}^{\infty} [a_n(k) \cos n\theta + b_n(k) \sin n\theta] \frac{I_n(k\rho)}{I_n'(kr)} \right]. \quad (4)$$

We use that circumstance, that the signal of an electromagnetic flowmeter is determined by that component of a magnetic field, which one is determined only by factors  $a_n(k)$ . Therefore to present a signal through a normal component of a magnetic field  $H(z, y)$  to a plane  $S(z, y)$ , passing

$$H(z, y) = \frac{\partial A}{\partial x} \Big|_{x=0} = \int_{-\infty}^{+\infty} dk \left[ e^{ikz} \sum_{\substack{m=0 \\ n=2m+1}}^{\infty} (-1)^m n a_n(k) \frac{I_n(ky)}{y} + \sum_{\substack{m=0 \\ n=2m}}^{\infty} n b_n(k) (-1)^m \frac{I_n(ky)}{y} \right] \quad (6)$$

and we shall enter notation

electrodes are in points with coordinates  $\rho = r, z = 0, \theta = \pm\pi/2$ .

Let's remark that from expression (1) follows(outflows), that the volumetric weighting function characterizes the contribution to a signal of a flowmeter an electromotive force, induced magnetic field of the inductor in different points of a channel.

The integral on  $k$  in (2) can be presented as the sum of deductions of the integrand on poles

being zero points  $I_n'(kr) = \frac{dI_n(kr)}{dr}$ . Limiting by deductions in zero points with minimum values  $|k|$ , we shall receive approximated expression for  $g(z, \rho, \theta)$ :

$$g(z, \rho, \theta) = \frac{\sin \theta e^{-|z|/r} \left( 1 - \frac{\rho^2}{r^2} e^{-2|z|/r} \right)}{1 + 2 \frac{\rho^2}{r^2} \cos 2\theta e^{-2|z|/r} + \frac{\rho^4}{r^4} e^{-4|z|/r}}. \quad (3)$$

Allowing, that the scalar magnetic potential  $A$  ( $\bar{B} = \text{grad } A$ ) satisfies an Poisson equation, the most common expression for a scalar magnetic potential in a channel of a flowmeter looks like:

through electrodes and axis of a tube, it is enough to receive a ratio:

$$a_n = f_n(H(z, y)), \quad (5)$$

where  $f_n$  is the functional from  $H(z, y)$ .

Let's remark, that normal to a plane  $S(z, y)$  component of a magnetic field is:

$$\Phi(y) = \sum_{\substack{m=0 \\ n=2m+1}}^{\infty} (-1)^m n a_n(k) \frac{I_n(ky)}{y}. \quad (7)$$

As  $\Phi(y)$  - even function for,  $\Phi(y)$  is possible to decompose in series on  $\cos(\pi my/r)$ :

$$\Phi(y) = \sum_{m=0}^{\infty} \alpha_m \cos \pi my/r, \quad (8)$$

$$\sin t \cos(z \cos t) = \frac{2}{z} \sum_{p=0}^{\infty} (-1)^p (2p+1) J_{2p+1}(z) \sin(2p+1)t. \quad (10)$$

Noting  $z=iky$ ,  $\cos t = -i\pi m/kz$ , we have

$$\cos \pi my/r = \frac{1}{\sqrt{1 + (\pi m/rk)^2}} \sum_{\substack{p=0 \\ n=2p+1}}^{\infty} (2p+1) \frac{I_n(ky)(-1)^p}{ky} \times \left[ \frac{\pi m}{kr} - \sqrt{1 + \left(\frac{\pi m}{kr}\right)^2} \right]^{2p+1}. \quad (11)$$

Substituting expression (11) in the formula (8) and comparing with (7), we shall receive

$$a_n(k) = \sum_{m=0}^{\infty} \frac{\alpha_m}{\sqrt{1 + (\pi m/rk)^2}} \left[ \frac{\pi m}{kr} - \sqrt{1 + \left(\frac{\pi m}{kr}\right)^2} \right]^{\frac{n-1}{2}}. \quad (11)$$

Therefore, the potential of a magnetic field looks like

$$A(z, \rho, \theta) = \int_{S(z, y)} dS(z, y) H(z, y) W(z, -z, y, \rho, \theta), \quad (12)$$

where

$$W(z - \tilde{z}, \rho, \theta) = \int_{-\infty}^{+\infty} dk e^{ik(z - \tilde{z})} \sum_{\substack{p=0 \\ n=2p+1}}^{\infty} \frac{I_n(k\rho)}{I_n(kr)} \cos n\theta \times \sum_{m=0}^{\infty} \frac{\cos \pi my/r}{\sqrt{1 + (\pi m/rk)^2}} \left[ \frac{\pi m}{kr} - \sqrt{1 + \left(\frac{\pi m}{kr}\right)^2} \right]^p. \quad (13)$$

Allowing, that  $\vec{B} = \text{grad } A$ , and substituting received expression  $\vec{B}$  in the formula (1), for a signal of a flowmeter  $U$  we shall find:

$$U = \int_{S(z, y)} dS(z, y) B_n(z, y) G(z, y), \quad (14)$$

where  $G(z, y)$  is Surface weighting function for cross-section  $S(z, y)$ :

$$G(z, y) = \int_{\tau} d\tau \vec{v} \left[ \frac{\partial g}{\partial \vec{r}} \frac{\partial W}{\partial \vec{r}} (z - \tilde{z}, y, \tilde{\rho}, \tilde{\theta}) \right]. \quad (15)$$

The surface weighting function  $G(z, y)$  by the formula (15) is shown analytically depending on design data of a channel and flow pattern.

and the factors  $\bar{\alpha}_m$  are determined through  $H(z, y)$  as follows:

$$\alpha_m = \frac{1}{2\pi^2 r} \int_{-\infty}^{+\infty} dz e^{ikz} \int_{-r}^r dy \frac{H_n(z, y) \cos(\pi my/r)}{1 + \delta_{m,0}}. \quad (9)$$

It is known, that

Therefore, the surface weighting function also is notably distinct from zero point only near electrodes. Therefore greatest influencing on surface weighting function render the shape deteriorations of velocity near to electrodes.

The expression (15) demonstrates, that the change of velocity distribution can sharply change a picture of level lines of surface weighting function. As an example we shall consider a axisymmetrical flow. Two most typical modes are advanced turbulent, realised at large Reynold's numbers, and laminar, answering to small hydraulic Reynold's numbers.

At a turbulent mode it is possible to consider distribution of velocity homogeneous, such, that  $\vec{v} = \vec{e}_z v_0$  and  $v_0 = \text{const}$ .

For homogeneous distribution of velocity we have

$$G(z, y) = \int_{-\infty}^{\infty} dk \cos kz \sum_{\substack{p=0 \\ n=2p+1}}^{\infty} \frac{I_n(kr)}{I_n'(kr)} \cos n\theta \times \sum_{m=0}^{\infty} \frac{\cos \pi my/r}{\sqrt{1 + (\pi m / kr)^2}} \left[ \frac{\pi m}{kr} - \sqrt{1 + \left(\frac{\pi m}{kr}\right)^2} \right]^p \approx \quad (16)$$

$$\cong \frac{e^{-|z|/r}}{1 - (y/r)^2} \frac{1}{\exp(-2|z|/r) r}$$

The surface weighting function is figured in a fig. 1, the level lines of surface weighting

function are figured in a fig. 2.

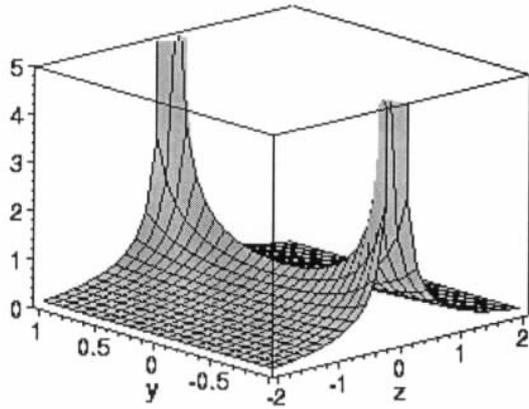


Fig.1.

For a laminar flow the distribution of speed looks like

$$v = v_0 \left(1 - \rho^2 / r^2\right). \quad (17)$$

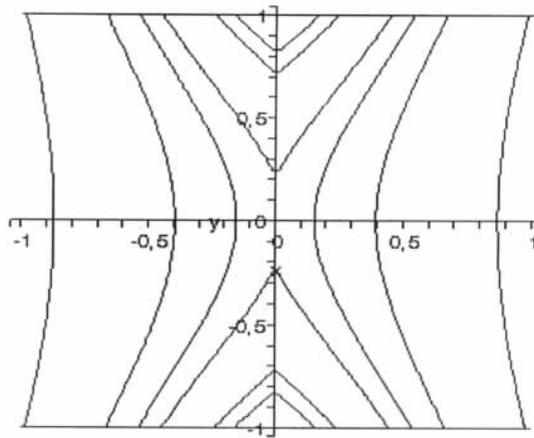


Fig.2.

Substituting expression (17) in (15), we shall receive, that for a laminar mode the surface weighting function is :

$$G(z, y) = \int_{-\infty}^{\infty} dk \cos kz \sum_{\substack{p=0 \\ n=2p+1}}^{\infty} (-1)^p \frac{I_n(kr)}{I_n'(kr)} \left[ 1 + \frac{n^2}{k^2 r^2} - \left( \frac{I_n'(kr)}{I_n(kr)} \right)^2 \right] \cos n\theta \times \sum_{m=0}^{\infty} \frac{\cos \pi my/r}{\sqrt{1 + (\pi m / kr)^2}} \left[ \frac{\pi m}{kr} - \sqrt{1 + \left(\frac{\pi m}{kr}\right)^2} \right]^p \quad (18)$$

The surface weighting function for a laminar mode is figured in a fig. 3, the level lines

of surface weighting function are figured in a fig. 4.

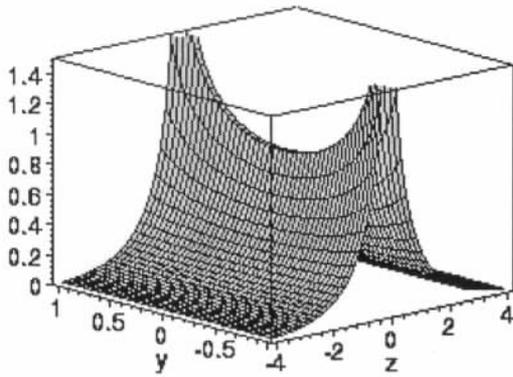


Fig.3.

Thus, the analysis of behavior of surface weighting function demonstrates, that, basically, for any function  $v_z(r,\theta)$  it is possible to receive the surface weighting function, conforming to her, and this function will descend always at  $z \rightarrow \infty$ , and will increase always up to maximum value at an approaching to electrodes.

For physical basis of the simulation model can be an induction coil allocated on an interior canal surface of a flowmeter, dismantled from the flowmeter pipe. If the coils of induction coil correspond to lines of a surface weight function ( $W_n$ ), a voltage, induced by the magnetic field of flowmeter, will proportionally to voltage  $U$ , incipient between electrodes at a motion of a flow with the relevant allocation of velocity and of flow structure. Thus, the simulation model with the transformer of a magnetic field as an

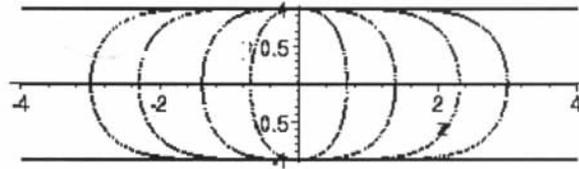


Fig..4.

induction coil fulfilled according to the surface weight function, allows to simulate devices of a different construction and at different magnetohydrodynamic modes and at flow structures. As the surface weight function depends on geometry of the channel, of kinematic structure of flow, of allocation of a phase composition of measured medium in the channel, of level of filling by a fluid of the channel at a unpressure flow, it is possible by simulation method to explore the metrological performances of devices under change of each of the above numbered factors separately or all of them together. It is enough to apply for this purpose the coil fulfilled according to that surface weight function, which one reflects any explored factor or their plurality.