Electromagnetic Inductive Flow Pattern Reconstruction

by Means of Spectrum Expanding

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Abstract: In the recent years, the theory of electromagnetic flow meters is being developed to describe more complicated problems in the measurement such as multiphase flow and flow reconstruction. This paper focuses on a method for the reconstruction of flow patterns in two dimension domain by electromagnetic flow induction.

Theoretical analysis and numerical simulation are completed based on the idea of flow pattern reconstruction by means of electromagnetic flow induction. Main concepts of the theory and reconstruction methods are summarized. A spectrum expanding method is used for flow pattern reconstruction in two dimensional domain. General expressions for the weight function W and flow velocity distribution V are obtained by series expansion. Examples of the weight function in the case of 4 pairs of electrodes and 2 pairs of magnetic coils, and some flow velocity distributions are studied. Matrix A and signal vector S are worked out. Some cases of symmetrical and asymmetrical flows are reconstructed using the software of MatLab. The results are showed and discussed.

Keywords：Electromagnetic-induction，weight function, flow pattern reconstruction, spectrum expanding, numerical simulation

INTRODUCTION

Electromagnetic flow meter has some obvious advantages such as fast signal response and non-obstruct to flow. Recent years, the idea of using the principle of electromagnetic flow meter for flow reconstruction has been brought up. Works have been tried. Theoretical consideration and simple examples have shown that it is possible to transfer the theory of electromagnetic flow measurement to the reconstruction of flow field.

Several authors have published their works on this topic, among them are Honda and Tomita [1], Trachtler and Wernsdorfer[2], Zhang [3]. Different reconstruction methods were introduced, however the results were not very good.

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In the author's previous work [4], two methods were bought up: geometry discretization and spectrum expanding. The former one was studied in [5]. This work reports the study on the latter one. The work includes theoretical analysis and numerical simulation..

THEORY FOR ELECTROMAGNETIC FLOW METER

From the theory of electromagnetic flow meter, we have:

$$U_2 - U_1 = \int_a \vec{W} \times \vec{V} dA \qquad (1)$$

where S is the potential difference, $W(r,\theta)$ the weight function which is only determined by electromagnetic boundary conditions, $V(r,\theta)$ the flow velocity field, *A* the cross section of the pipe.

$$\vec{W} = \vec{B} \times \vec{j} \qquad (2)$$

where $\vec{j}$ is called as virtual current, which is determined by electrical boundary condition only. For normal fluid like water, $\vec{j}$ and $\vec{B}$ can be further expressed by potentials as:

$$\vec{j} = -\nabla G \qquad \vec{B} = -\nabla F \qquad (3)$$

G and *F* satisfy Laplacian

$$\nabla^2 G = 0 \qquad \nabla^2 F = 0 \qquad (4)$$

For two dimensional problem, we rewrite U_2-U_1 as *S* and express equation (1) in cylindrical coordinators

$$S = \int_A W(r,\theta) V(r,\theta) dA \qquad (5)$$

where $$W = -1/r(\partial G/\partial r \cdot \partial F/\partial \theta - \partial G/\partial \theta \cdot \partial F/\partial r) \qquad (6)$$

G and F can be obtain by solving Laplacian of (4) with specified boundary conditions.

The reconstruction method by means of spectrum expanding works as bellow:

Expanding *W* and *V* in perpendicular series:

$$W = \sum_{i=1}^{M}\sum_{j=1}^{M} a_{ij} R_{ij}(r) H_i(\theta) \tag{7}$$

$$V = \sum_{i=1}^{M}\sum_{j=1}^{M} c_{ij} R_{ij}(r) H_i(\theta) \tag{8}$$

We have

$$S = \sum_{i=1}^{M}\sum_{j=1}^{M} c_{ij} a_{ij} \int_0^R \left(R_{ij}(r)\right)^2 rdr \int_0^{2\pi} \left(H_i(\theta)\right)^2 d\theta \tag{9}$$

For N times of measurement

$$\left[S_l\right]_{N\times 1} = \left[a_{il} R_{ij}^* H_i^*\right]_{N\times(M\times M)} \left[C_{ij}\right]_{(M\times M)\times 1} \tag{10}$$

Where

$$R_{ij}^* = \int_0^R \left(R_{ij}(r)\right)^2 rdr \qquad H_i^* = \int_0^{2\pi} \left(H_i(\theta)\right)^2 d\theta \tag{11}$$

For simplicity equation (10) is written as $AX=S$ (12)

Where $A=\left[a_{il} R_{ij}^* H_i^*\right]_{N\times(M\times M)}$; $X=\left[C_{ij}\right]_{(M\times M)\times 1}$

By obtaining the general inverse of A, we can have

$(A^T A)X = A^T S$

Then $X = (A^T A)^{-1} A^T S$ (13)

which is the coefficients vector for unknown velocity V.

NUMERICAL SIMULATION RESULTS

The construction of the measurement configuration for numerical examples consists of two pair of magnet coil and eight point electrodes, as is shown in Fig. 1. Two dimensional assumption is considered.

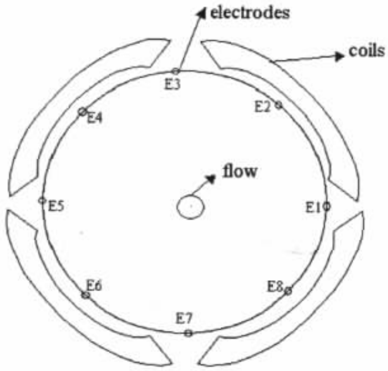

Fig. 1 configuration of the magnet coils and electrodes

The coils are so built that 'ideal magnet' model can be applied. The eight electrodes are at the pipe wall with equal interval circumferentially.

By solving equation (4) with specified boundary conditions we obtain

$$G=\sum_{n=1}^{\infty} r^n[(\cos n\theta_1-\cos n\theta_2)\cos n\theta+(\sin n\theta_1-\sin n\theta_2)\sin n\theta]/n\pi \quad (14)$$

$$F=\sum_{k=1}^{\infty}[(\cos k\theta_3-\cos k\theta_4)\sin k\theta+(\sin k\theta_4-\sin k\theta_3)\cos k\theta]/k\pi \quad (15)$$

where G is the virtual current potential for any two electrodes, F is the potential for the magnetic flux of any coil.

W is worked out from equation (6) as

$$W=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} r^{(n+k-2)}/\pi^2 \begin{Bmatrix} [(\cos n\theta_2-\cos n\theta_1)\sin n\theta+(\sin n\theta_1-\sin n\theta_2)\cos n\theta]\cdot \\ [(\sin k\theta_4-\sin k\theta_3)\cos k\theta+(\cos k\theta_3-\cos k\theta_4)\sin k\theta]- \\ [(\cos n\theta_1-\cos n\theta_2)\cos n\theta+(\sin n\theta_1-\sin n\theta_2)\sin n\theta]\cdot \\ [(\sin k\theta_3-\sin k\theta_4)\sin k\theta+(\cos k\theta_3-\cos k\theta_4)\cos k\theta] \end{Bmatrix} \quad (16)$$

Velocity field is unknown, but it can be expended in series as

$$V=\sum_{n=0}^{N}\sum_{m=1}^{M} J_0(x_m r)(A_{mn}\cos n\theta+B_{mn}\sin n\theta) \quad (17)$$

Distributions of typical weight functions calculated are shown in Fig. 2.

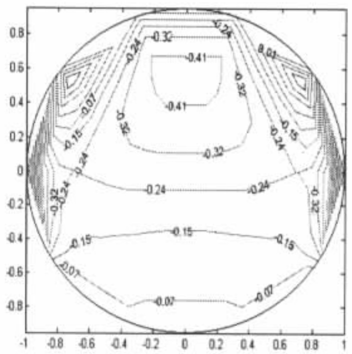

Fig.2(a) weight function distribution I

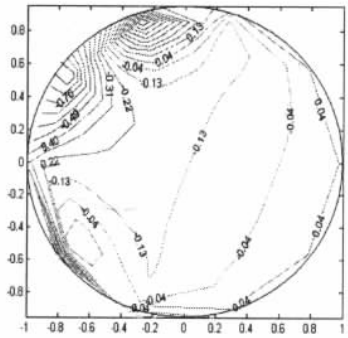

Fig2(b) weight function distribution II

Equations (16) and (17) are used to work out matrix A in equation (12).

Dimension of A depends on the up limits of the series of W and V in equations (15) and (16).

Here we choose the 112 terms for W and 60 terms for V, so we have 112×60 matrix of A.

Owing to the serious illness of the matrix A, the observation of the general inverse A^T is very poor. We think this is because of the high divergence of the weight functions from the electrode ('soft field' property). To overcome the weakness in a certain degree, we do the sharpening of the weight function by replacing any value of the weight functions less than 0.2 by 0.

Three velocity patterns to be reconstructed are shown in Fig.3

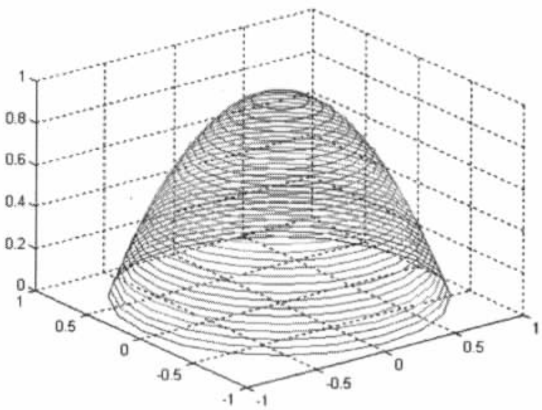

Fig4(a) flow pattern I (parabola profile)

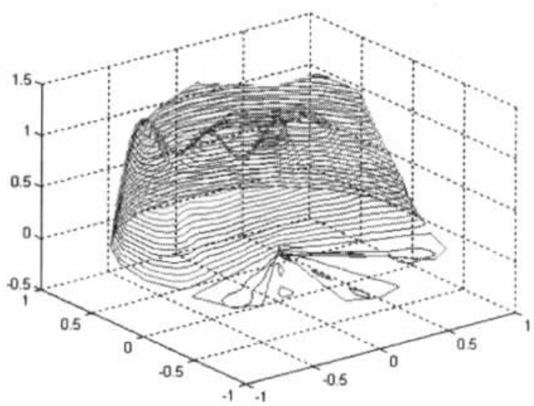

Fig4(a) flow pattern I (flat flow in half pipe)

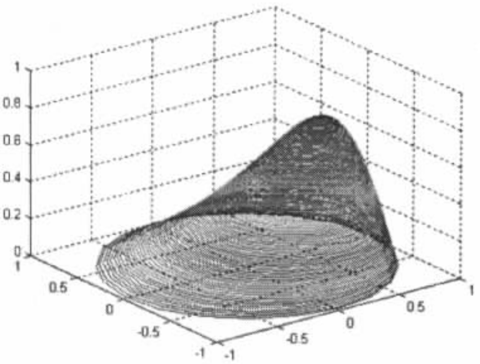

Fig.4(c) flow pattern III (asymmetrical profile)

Numerical calculation is finished in PC using Matlab. It offers good tools for matrix operation , special functions such as Bessel function, and plotting as well.

Figure 4 Reconstructed flow patterns

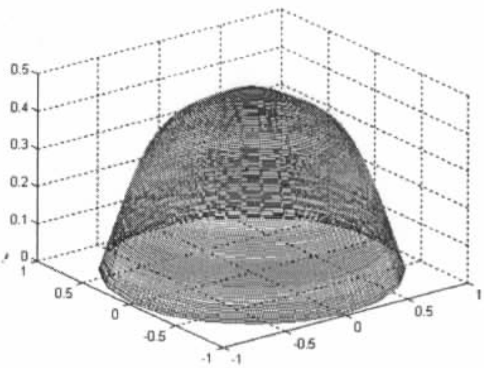

Fig4(a) reconstructed flow pattern I

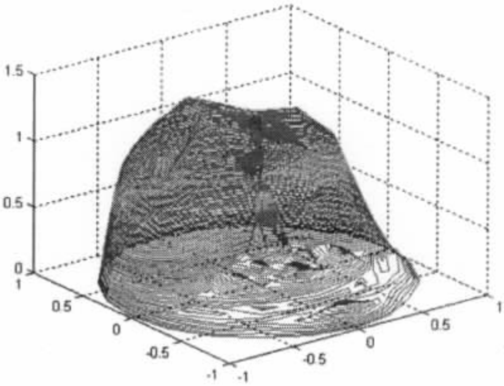

Fig4(a) reconstructed flow pattern II

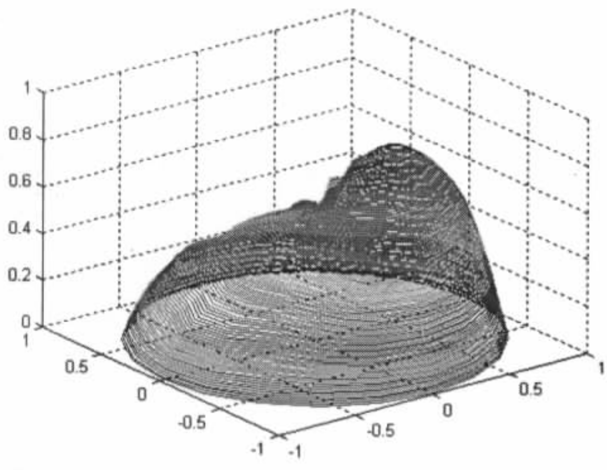

Fig.4(c) reconstructed flow pattern III

From the figures we cab see, the reconstructed pattern can fairly reflect different flow profiles. However ,around the center of the pipe cross section, reconstructed patterns are not quite satisfied. As mentioned above, the 'soft field' effect makes both virtual current and magnetic flux divergent near the center. This makes the sense of the velocity change there to be weak. We have tried some other cutting-off values of the weight functions, but 0.2 seems the better one.

SUMMARY

A new reconstruction algorithm is introduced, which uses spectrum expending. Numerical works are basically reduced to find a revised general inverse of matrix *A*. Three flow profiles are constructed. The results are fairly good except for the parts near the center of the pipe cross section. Increasing the series term of *W* and *V* might help overcome this. Then we are facing much more computation time and other problems.

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