

What if the Calculated Uncertainty of a Calibrated Instrument is Lower than the Standard Instrument Uncertainty?

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Abstract In the uncertainty of measurement evaluation it is essential a solid grasp of the physical and mathematical theory underlying the measurement model to make a sensible analysis of the experimental data. There may occur particular situations, which lead to results in apparent contradiction to our usual understanding of error analysis. This paper presents a detailed case of such situations arising in a flow meter calibration model.

Keywords: Uncertainty of measurement; flow meter error.

1. Introduction

In the evaluation of the uncertainty of a measurement a solid knowledge of the theory underlying the measurement model is necessary to perform adequate analysis. This requires special attention since there are some particular situations in which unusual results may be obtained. In these situations a careful interpretation of the mathematical and physical aspects of the measurement model is required.

For instance, there are situations in which the uncertainty of the quantity, which is been calibrated, may be lower than the uncertainty of the standard that was used in its calibration, although the relative uncertainty of the standard must always be, by definition the lowest. In chemistry, for example, the concentration uncertainty of a diluted solution - prepared using a reference material - can be lower than the concentration uncertainty of the reference material, but again, this does not occur if one uses the relative uncertainties of the concentrations.

To better illustrate some of these unusual situations one may find in evaluating measurement uncertainties, we present in this paper a case study of a flow meter calibration model which may lead to a relative error uncertainty of a calibrated meter lower than its

corresponding standard meter. We show that a careful analysis is needed in such cases to avoid a wrong interpretation.

2. Unusual situations

$$C_d = \frac{M_N}{M_d} = C_p \cdot \frac{M_c}{M_d},$$

We present here as a first example a situation occurring in chemistry in which the uncertainty of a calibrated concentration may be lower than the uncertainty of its reference concentration.

Let us consider the preparation of a NaOH solution in water from a standard solution with known concentration and uncertainty. We collect 10.00 g of this standard solution, diluting it with water to obtain a 10 times diluted solution. The uncertainty in the measurement of the solution mass is 0.02 g. The evaporation, water impurities and other contamination in the production process of the solution are negligible. As for the concentrations, we have

where C_d is the diluted solution concentration (4.00 g of NaOH per kg of solution); M_N is the mass of NaOH contained in 10.00 g of the standard solution (4.00 g);

M_d is the mass of the prepared diluted solution (1,000.00 g); C_p is the standard solution concentration (40.00 g of NaOH per kg of solution) and M_c is the standard solution mass collected to prepare the diluted solution (10.00 g).

Finally, for the uncertainties we have the expanded uncertainty for the standard solution concentration (0.005 g of NaOH per kg of solution); the expanded uncertainty of the diluted standard solution (g of NaOH per kg of solution) and the expanded uncertainty of the solution mass (0.002 g). All these uncertainties have coverage factor of $k = 2$ with coverage probability of approximately 95%.

The expanded uncertainty of the diluted solution is 0.0011 g of NaOH per kg of solution, more than 4.5 times smaller than that of the standard solution. However, the relative uncertainties for the standard and diluted solution are 0.0125% and 0.0257%, respectively. The uncertainty in the diluted solution is small due to its very low concentration.

A similar situation may occur in the calibration of viscometers by the comparison method. The calibrated viscometer constant uncertainty may be lower than that of the standard viscometer, if the constant of the calibrated viscometer is lower than the constant of the standard viscometer. However, when one compares the relative uncertainties, the uncertainty of the calibrated viscometer is greater than that of the standard one.

Next we proceed to describe a detailed case in which a flow turbine is calibrated by a standard turbine using the total volume comparison method. We assume that the two turbines are installed in series in a pipe and that the measurements begin only when the flow is stabilized. We then let a fluid volume pass through the turbines. The total volume is evaluated by multiplying the number of pulses sent by the turbines by their constant K .

The calibration consists in the determination of the relative error ε_i of the turbine, which is been calibrated.

To simplify the problem, we assume that the uncertainties due to repeatability/reproducibility, pulse loss and volume variation due to changes in temperature and pressure are negligible. We also assume that only the relative error uncertainty ε_s of the standard turbine is relevant.

By definition, the relative error ε_i is given by

$$\varepsilon_i = 100\% \cdot \frac{V_i - V_{sc}}{V_{sc}}$$

Here, V_{sc} is the (corrected) totalized volume of the standard turbine, and V_i is the (uncorrected) totalized volume of the calibrating turbine. These quantities can be written as $V_{sc} = N_s K_s / (1 + \varepsilon_s/100\%)$ and $V_i = N_i K_i$, where N is the number of pulses sent by its respective turbine.

To give a numerical example, suppose the turbine constants K_i and K_s are equal to $0.1 \text{ m}^3/\text{pulse}$. If it sends $N_i = 9,800$ and $N_s = 10,201$ pulses, its measured volumes are $V_i = 980 \text{ m}^3$ and $V_{sc} = 1,010 \text{ m}^3$, considering $\varepsilon_s = 1.0\%$. Taking these values in the above equation, we get $\varepsilon_i = -2.97\%$. Let us now consider the expanded uncertainties for both ε_i and ε_s :

$$U(\varepsilon_i) = U(\varepsilon_s) \cdot \frac{\left(1 + \frac{\varepsilon_i}{100\%}\right)}{\left(1 + \frac{\varepsilon_s}{100\%}\right)}$$

The crucial observation here is that the fraction on the right hand side of the above equation is less than 1. In fact, the use of the above values leads to

$$U(\varepsilon_i) = 0.9607 U(\varepsilon_s)$$

In other words, the expanded uncertainty of the calibrated turbine is less than the expanded uncertainty of the standard turbine. This analysis seems to suggest

that as long as we are able to get a bigger ε_s and a smaller ε_t , the better we are, since in this way we can proportionately reduce the expanded uncertainty of the calibrated turbine relative to the expanded uncertainty of the standard turbine. In the example presented before, we obtained $u(\varepsilon_t) < u(\varepsilon_s)$, since $\varepsilon_t > \varepsilon_s$. Naturally, if $\varepsilon_t = \varepsilon_s$ we would have $u(\varepsilon_t) = u(\varepsilon_s)$, and if $\varepsilon_t < \varepsilon_s$, the result would be $u(\varepsilon_t) > u(\varepsilon_s)$.

In conclusion, it seems to be a good procedure to calibrate a turbine with a high relative error so as to obtain a smaller uncertainty in the instrument. Taking this logic to an extreme situation, where $\varepsilon_t = -100\%$, we see that the expanded uncertainty on ε_t is zero. Thus the best possible scenario is to calibrate an instrument whose relative error is minus one hundred percent, so that the uncertainty on its calibration is zero. In the next section we will explain the reasons for this apparently paradoxical result.

3. Solving the puzzle

According to the ISO-GUM [1] (in annex E) the standard uncertainty formula for a measurand $Y=f(X_1, X_2, \dots, X_N)$ is correct only in the first order Taylor expansion for the entrance quantities X_k .

In the extreme situation where $\varepsilon_t = -100\%$ discussed above the sensitivity coefficient is zero. There is an inflexion which fails to attend the requirements described in the annex E of ISO-GUM, so that the sensitivity coefficient cannot be done through partial derivatives of the measurand Y . If $\varepsilon_t = -100\%$ the product $N_t K_t$ is zero, i.e., the instrument to be calibrated does not record the volume that passes through it. Therefore, in this case our model breaks down.

The fact that the uncertainty of the relative error ε_t of the calibrated instrument is lower than the relative error ε_s of the standard instrument does not imply that the former will measure the totalized volume with lower uncertainty than the latter, since this uncertainty depends on both the relative error uncertainty and

value.

Finally, let us conclude the example of the two turbines to see how the measure of physical quantities are not affected by the apparent lack of consistency in the expanded uncertainties of the relative errors. As stated before, the corrected measure of volume that passes through the standard turbine is

$$V_{sc} = \frac{N_s K_s}{\left(1 + \frac{\varepsilon_s}{100\%}\right)}.$$

From this formula, the expanded uncertainty on the volume V is given by

$$U(V_{sc}) = U(\varepsilon_s) \frac{N_s K_s}{(100\% + \varepsilon_s)^2}.$$

In the same manner, the corrected measure of volume that passes through the turbine to be calibrated is

$$V_{tc} = \frac{N_t K_t}{\left(1 + \frac{\varepsilon_t}{100\%}\right)},$$

and its expanded uncertainty is

$$U(V_{tc}) = U(\varepsilon_t) \frac{N_t K_t}{(100\% + \varepsilon_t)^2}.$$

Taking in this expression the values listed in Section 2, we have

$$U(V_{sc}) = 3.00 \text{ m}^3,$$

$$U(V_{tc}) = 3.0186 \text{ m}^3.$$

Therefore, the uncertainties in the physical quantities are what we should expect, even as the uncertainties in the relative errors are not.

4. Conclusions

A solid grasp of the theory underlying the

measurement process is an indispensable requirement for an appropriate mathematical and physical analysis of the results obtained in measurement, calibration or comparison.

The uncertainty in the measurement of a physical quantity depends on the value of the instrument relative error and its uncertainty. As we have seen, the correct way to proceed in the comparison and analysis of uncertainties is through physically meaningful quantities - which reveal the true nature of the measurement - rather than through mathematical constructions like relative errors.

5. References

- [1] *Guide to Expression of Uncertainty in Measurement* - International Organization for Standardization, ISO, 1993.
- [2] Reglamento de Prueba del PTB - Contadores Volumétricos de gas - Physikalisch-Technische Bundesanstalt, Braunschweig y Berlin.