Study on the Fitting Equations of Discharge Coefficient for Entrance Flowmeters

Wang Bonian
(University of Shanghai for Science and Technology, Shanghai 200093, China)
Wang Dongwei
（National Institute of Metrology, Beijing 100013, China）
Lu Deyue
(Ningbo Trunsun Instrument Co.,LTD, Ningbo 315204, China)
Zhu Jiashun
(Yuyao Yinhuan Flowmeter Instrument Co.,LTD, Yuyao 315400, China)

Abstract A general form of discharge coefficient for entrance flowmeters, $c = c_0 - c_1 \mathrm{Re}^{c_2}$ is suggested and used to fit the calibrating data successfully. The main problem concerns to solve a set of nonlinear sequations, which is solved by a series of solutions of linear equations. This fitting equations are valuable and suitable for some other flowmeters.

Keywords: Entrance folwmeter; discharge coefficient; Fitting equations; Principle of least squares

1. Introduction

When air is the main flued, or one of the main fluids in a machine or in a equipment, the best approach to measure the air flowrate is applying the entrance flowmeters. It is the special case of pressure differential devices (throttling flowmeters) for $\beta = d/D = 0$, but the discharge coefficient can not be calculated by extensions of the formulate for general throttling flowmeters with $\beta_{\min} \neq 0$, The main problem for developing entrance flowmeters is to set up the fitting equations of discharge coefficient for their designs and applications.

Based on some assumptions and theoretical analysis, the following equation concerning discharge coefficient c, Reynolds number Re, and constant c_1 was obtained[1]-[3]

$$c = 1 - c_1 / \sqrt{\mathrm{Re}} \tag{1}$$

and used commonly[4]-[7]. But from the viewpoint of least squares,equation (1) is not the best one.

In this paper, the general form of fitting equations of discharge coefficient for entrance flowmeters is suggested and analysied in detail and systematically, the applications of this fitting equation explain its excellent characters.

2. Main problem for developing entrance flowmeters

For the general pressure differential devices, the fundamental formula is[8]:

$$q_m = 0.25 c \varepsilon d^2 \sqrt{2 \rho_1 \Delta p} / \sqrt{1 - \beta^4} \tag{2}$$

Where q_m is the mass flowrate, c is the discharge coefficient, ε is the expansion factor, d is the diameter of throat, ρ_1 is the density of upstream fluid, Δp is the pressur-difference (differential pressure) between the upstream and downstream pressure taps, $\beta = d / D$ is the ratio of throat diameter and upstream pipe diameter.

For entrance flowmeters, $\beta = 0$, and $\rho_1 = \rho_0$ is the atmosphere density at the front of inlet, Δp is the pressure-difference between the atmosphere and the fluid at downstream pressure tap. Thus, for entrance floemeter

$$q_m = 0.25 c \varepsilon d^2 \sqrt{2 \rho_1 \Delta p} \qquad (3)$$

For flowmeters with smooth profiles, ε can be calculated by the following[2]

$$\varepsilon = \left\{ [k/(k-1)] \left(p_r^{2/k} \right) \left[\left(1 - p_r^{(k-1)/k} \right) / (1 - p_r) \right] \right\}^{0.5} \qquad (4)$$

Where p_r is the pressure ratio of downstream and upstream, $p_r = p_2 / p_0 = 1 - \Delta p / p_0$. There is no problem to calculate ε. It is seen that the main problem for developing entrance flowmeters is to set up suitable fitting formula of c for their designs and applications.

3. Gerneral form and its special cases of fitting equation

3.1 Gerneral form

On the one hared, (1) has some theoretical bases,and on the other hand, it was not devived from perfect and complete analysis, it is not the final and best form of discharge coefficient. In (1) , if '1' changes to c_0, and '-0.5' changes to c_2, the following equation

$$c = c_0 - c_1 \mathrm{Re}^{c_2} \qquad (5)$$

with three constants c_0, c_1, and c_2, it can be used to fit calibrating data.

3.2 Special case: $c = 1 - c_1 \mathrm{Re}^{-0.5}$ with one unknow c_1

In (5), if $c_0 = 1$, $c_2 = -0.5$, (1) is obtained, and there is only one constant c_1 to be determined.

For n given data points (c_i, Re_i, $i = 1,2,\cdots n$), for each of Re_i, (1)gives

$$c_{fi} = 1 - c_1 \mathrm{Re}_i^{-0.5} \qquad (6)$$

Where c_{fi} is the discharge coefficient from fitting equation.

The deviation δ_i for each Re_i is

$$\delta_i = c_i - c_{fi} = c_i - 1 + c_1 \mathrm{Re}^{-0.5}$$

The sum of squares of the deviations defines the function $s(c_1)$:

$$s(c_1) = \sum_{i=1}^{n} \left(c_i - 1 + c_1 \mathrm{Re}^{-0.5} \right)^2$$

According to the principle of least squares, $s(c_1)$ should be minimum, namely $ds / dc_1 = 0$. Thus,

$$ds / dc_1 = \sum_{i=1}^{n} 2 \left(c_i - 1 + c_1 \mathrm{Re}^{-0.5} \right) \mathrm{Re}^{-0.5} = 0$$

Finally,yields

$$c_1 = (s_r - s_{cr}) / s_{r2} \qquad (7)$$

where $s_r = \sum_{i=1}^{n} \mathrm{Re}_i^{-0.5}$, $s_r = \sum_{i=1}^{n} c_i \mathrm{Re}_i^{-0.5}$,

$$s_{r2} = \sum_{i=1}^{n} \mathrm{Re}_i^{-1.0}$$

3.3 Special case: $c = c_0 - c_1 \mathrm{Re}^{-0.5}$ with two unknows c_0, and c_1

In (5), if $c_2 = -0.5$, it changes to

$$c = c_0 - c_1 \mathrm{Re}^{-0.5} \qquad (8)$$

Similar to above case, the sum of squares of the deviations defines the function $s = s(c_0, c_1)$. Due to $\partial s / \partial c_0 = 0$, and $\frac{\partial s}{\partial c_1} = 0$, yields

$$c_0 = (s_r s_{cr} - s_c s_{r2}) / (s_r^2 - n s_{r2}) \qquad (9)$$

$$c_1 = (n s_{cr} - s_c s_r) / (s_r^2 - n s_{r2}) \qquad (10)$$

where $s_c = \sum_{i=1}^{n} c_i$, other symbols are same as in 3.2.

3.4 Special case: $c = c_1 \mathrm{Re}^{c_2}$ with two unknows c_1, and c_2

In (5), if $c_0 = 0$, and changing the negative sign of c_1 to positive, it changes to

$$c = c_1 \mathrm{Re}^{c_2} \qquad (11)$$

where c_1, and c_2 are two unknows to be determined.

Taking logarithmic operation of (11), yields

$$\ln c = \ln c_1 + c_2 \ln \mathrm{Re}$$

considering $\ln c$ and $\ln \mathrm{Re}$ are two new

is same for other linear equations.

variables, above is still a linear equation, it can be determined easily by least squares.

Similar to (8), two unknows in (11) are obtained:

$$c_1 = \ln^{-1}\left[(s_{lc} s_{lr2} - s_r s_{cr}) / (n s_{r2} - s_r^2)\right] \qquad (12)$$

$$c_2 = (n s_{lcr} - s_{lc} s_{lr}) / (n s_{lr2} - s_{lr}^2) \qquad (13)$$

where $s_{lc} = \sum_{i=1}^{n} \ln c_i$, $s_{lr} = \sum_{i=1}^{n} \ln \mathrm{Re}_i^{-0.5}$,

$$s_{lr2} = \sum_{i=1}^{n} (\ln \mathrm{Re}_i)^{-1.0}, \quad s_{r2} = \sum_{i=1}^{n} \mathrm{Re}_i^{-1.0}$$

$$s_{lcr} = \sum_{i=1}^{n} (\ln c_i)(\mathrm{Re}_i^{-0.5}).$$

3.5 Special case: $c = 1 - c_1 \mathrm{Re}^{c_2}$ with two unknows c_1, and c_2

In (5), if $c_0 = 1$, if changes to

$$c = 1 - c_1 \mathrm{Re}^{c_2} \qquad (14)$$

$(1 - c)$ in (14) and c in (11) are correspondent. So (12), and (13) can be used for (14), only c_i in (12) and (13) needs changing to $(1 - c_i)$.

3.6 Nonlinear case: $c = c_0 - c_1 \mathrm{Re}^{c_2}$ with three unknows

For the general case of (5), there are three unknows to be determined, it concerns to solve a set of nonlinear equations. If solving these nonlinear equations directly, getting the correct answer is very difficult[9].

One approach to solve such a problem is to change the nonlinear equations to linear. For this aim, first is to assume a series of c_0 in (5), second is to solve the equations similar to (14) for each c_0 ,third is to calculate the maximum of $\left|\left(c_i - c_{fi}\right)/c_i\right|$, for each of c_0, finally comparing all of $\left|\left(c_i - c_{fi}\right)/c_i\right|_{\max}$ determines the best one with the minimum of $\left|\left(c_i - c_{fi}\right)/c_i\right|_{\max}$ for all c_0. With the help of programming, this problem is solved automatically.

3.7 Example

For the data of c_i , and Re_i concerning entrance flowmeters given in [10], the solutions of all cases, are shown in Table 1 ($5\cdot10^3 \le \mathrm{Re} \le 2\cdot10^5$).

Table 1 Fitting equations and their fitting precisions

Fitting equations	$\left\|\left(c_i-c_{fi}\right)/c_i\right\|_{\max}$	Order of fitting preci-sion
$c = 1 - 5.9426\,\mathrm{Re}^{-0.5}$	0.0059	4
$c = 1.006 - 6.6224\,\mathrm{Re}^{-0.5}$	0.0019	2
$c = 0.7651\,\mathrm{Re}^{0.0223}$	0.0135	5
$c = 1 - 18.3795\,\mathrm{Re}^{-0.6198}$	0.0084	3
$c = 1.01 - 4.8773\,\mathrm{Re}^{-0.4602}$	0.0014	1

4. Conclusion

(1) Although there are some applications of $c = 1 - c_1 \mathrm{Re}^{c_2}$ to fit the calibrating data of entrance flowmeters, or other flowmeters, this equation is not the best one. Comparing to other equations (seeing Table 1), its fitting precision is not satisfactory.

(2) A general form of fitting equation for entrance flowmeter, $c = c_0 - c_1 \mathrm{Re}^{c_2}$ is suggested. There are three unknows c_0, c_1, and c_2 to be determined. For over-come nonlinear difficuts, one approach of changing the nonlinear to linear is suggested and completed perfectly.

(3) One special case of $c = c_0 - c_1 \mathrm{Re}^{c_2}$ is $c = c_0 - c_1 \mathrm{Re}^{-0.5}$ with two unknows c_0, c_1. In such a case, the problem is linear fully. c_0 , and c_1 are very easy to calculate from (9), and (10). The fitting precision of this equation is very good, and very closed to the fitting precision of the best one $c = c_0 - c_1 \mathrm{Re}^{c_2}$ (see Table 1).

(4) Besides $c = c_0 - c_1 \mathrm{Re}^{c_2}$ is suitable for entrance flowmeters, it will be suitable to some others flowmeters with the fitting equations similar to the suggested form.

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