Iterative Computations of Unkowns for pressure Differential Devices by Means of Secant Method

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Abstract The secant method for solving nonlinear equation is used to calculate the unknown parameters of pressure differential devices. It is shown that the important advantages are needing only unique iterative equation without considering different unknows, easily to obtain convergent solutions without to calculate the derivatives which can not be finished for complex nonlinear equation, and yielding high precision solutions. The formulae of initial values and the example for showing the results are presented, the results are excellent.

Key words: Pressure differential devices, Iterative equation, Secant method, Initial values of unknown

1. Introduction

For pressure differential devices (throttling devices), the problems of designs and applications concern to solve nonlinear equation, or a set of nonlinear equations. In the new edition of ISO 5167-1 2003[1] and its old editions, iterative computations are given to slve these problems. But there are some problems of such iterative computations :

(1) The unknown is one of q_m, β, D, and Δp, the corresponding iterative equations are different, namely the iterative equation is not unique.

(2) The first guess value is not same as the unkown; and when β (or d) is the unknown, two first guesses c, and ε need to offer;

Simply speaking, the iterative computation in [1] is not uniform and perfect.

Eight years ago, the first author of this paper published a paper to solve this problem[2]. The final form of nonlinear equation is the basic equation of mass flowrate itself , but the unknown Re is substituted for q_m . Now, the original form of mass fowrate equation is used directly to give the complete solutions, and the initial values of unknown are given by definit formulae.

2. Nonlinear and fundamental equation

The more complex case of orifices is discussed in following.

The mass flowrate q_m is shown by (1):[3], [4]

$$q_m = 0.25\pi d^2 c\varepsilon \sqrt{2\rho_1 \Delta p} \Big/ \sqrt{1-\beta^4} \qquad (1)$$

Where d is the diameter of orifice, c is the discharge coefficient, ε is the expansion factor, ρ_1 is the density of upstream fluid, Δp is the pressure difference between the upstream and downstream pressure taps, β is d/D, the ratio of diameter of orifice d to the diameter of upstream pipe D.

For orifices, the Reynolds number Re_D is expressed by

$$\mathrm{Re}_D = \frac{4q_m}{\pi D \mu} \qquad (2)$$

Thence

It is suitable to express (1) with the form including D, such as

$$\begin{aligned} q_m &= 0.25\pi D^2 c\varepsilon\sqrt{2\rho_1 \Delta p}\,\beta^2 \big/ \sqrt{1-\beta^4} \\ &= f_q(D,\beta,c,\varepsilon,\Delta p) \end{aligned} \qquad (3)$$

The discharge coefficient c is given by [4]:

$$\begin{aligned} c &= 0.5961 + 0.0261\beta^2 - 0.261\beta^8 + 0.000521 \\ &\quad (10^6\beta/\mathrm{Re}_D)^{0.7} + (0.0188 + 0.0063A)\beta^{3.5} \\ &\quad (10^6/\mathrm{Re}_D)^{0.3} + (0.043 + 0.080e^{-10L_1} - 0.123 \\ &\quad e^{-7L_1})(1-0.11A)\beta^4/(1-\beta^4) - 0.031(M'_2 - \\ &\quad 0.8M'^{1.1}_2)\beta^{1.3} + c_{sp}\cdot 0.011(0.75-\beta)(2.8 - \\ &\quad D/25.4) \\ &= f_C(\beta,\mathrm{Re}_D,A,L_1,M'_2,C_{sp},D) \end{aligned} \qquad (4)$$

Where

$$A = (19000\beta/\mathrm{Re}_D)^{0.8} = f_A(\mathrm{Re},\beta) \qquad (5)$$

$L_1 = L'_2 = 0$ (for cormer tappings) (6)

$L_1 = 1,\ L'_2 = 0.47$

(for D and $D/2$ tappings) (7)

$L_1 = L'_2 = 25.4/D = f_L(D)$

(for flange tappings) (8)

$$M_2 = 2L_2'/(1-\beta) = f_M(\beta), or = f_M(D,\beta) \qquad (9)$$

$C_{sp} = 1.0$ (for $D < 71.12mm$) (10)

$C_{sp} = 0.0$ (for $D > 71.12mm$) (11)

(5) to (11) are substituted for the corresponding parameters in (4), it can be expressed by

$$\begin{aligned} c &= \varphi_c(\beta,\mathrm{Re}_D,D,\Delta p) \\ &= \psi_c(q_m,\beta,D,\Delta p) \end{aligned} \qquad (12)$$

Even above equation is very complex, c can be calculated when q_m, β, Re_D, D, and Δp are given.

The expansion factor ε is given by :[4]

$$\begin{aligned} \varepsilon &= 1 - (0.351 + 0.256\beta^4 + 0.93\beta^8) \\ &\quad \left[1-(p_2/p_1)^{1/k}\right] \\ &= 1 - (0.351 + 0.256\beta^4 + 0.93\beta^8) \\ &\quad \left[1-\left(1-\Delta p/p_1\right)^{1/k}\right] \\ &= f_e(\beta,\Delta p) \end{aligned} \qquad (13)$$

(12) and (13) are substituted for c, and ε in (3), yields

$$q_m = \varphi_q(q_m,\beta,D,\Delta p)$$

Finally,

$$f(q_m,\beta,D,\Delta p) = q_m - \varphi_q(q_m,\beta,D,\Delta p) = 0 \qquad (14)$$

(14) is strongly nonlinear equation. In principle, it can be solved to get one unknown by iterative methods.

3. Secant method for solving nonlinear equations

3.1 Newton's method for solving nonlinear equation

Newton's method is a effective method for solving simple nonlinear equation, $f(x)=0$, the basic idea of Newton's method is discussed firstly . In Fig.1, assuming x_1 is guess value closed to the root x_s, from x_1

determing $y_1 = f(x_1)$ and devivative $y_1' = f'(x_1)$. The tangent passing through point 1 intersects x coordinate at x_2. Obiously,

$$y' = \frac{0 - y_1}{x_2 - x_1}$$

$$x_2 = x_1 - y_1 / y_1' \qquad (15)$$

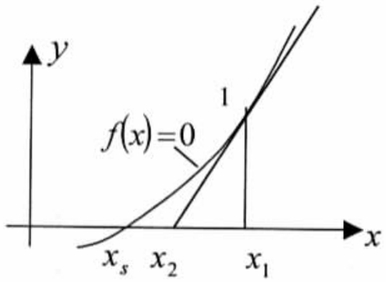

Fig.1 Newton's method

Then, point $(0, x_2)$ can be a new point to repect above procedure until $|f(x_n)| \le \delta_e$, (δ_e is given error, such as $1 \cdot 10^{-6}$), the root is $x_s = x_n$.

For simple nonlinear equation, Newton's method is convergent rapidly, but for complex nonlinear cases, devivative y' can not be calculated, this method is noneffective.

3.2 Secant method for solving equations $f(x) = 0$

For Secant method, it needs to assume two initial values of x, x_1, and x_2, then calculating $f_1 = f(x_1)$, and $f_2 = f(x_2)$ the slopes of the lines passint through points 1 and 2, and points 2 and 3 are

$$\text{slope} = \frac{f_2 - f_1}{x_2 - x_1} = \frac{0 - f_2}{x_3 - x_2}$$

$$x_3 = x_2 - f_2(x_2 - x_1)/(f_2 - f_1) \qquad (16)$$

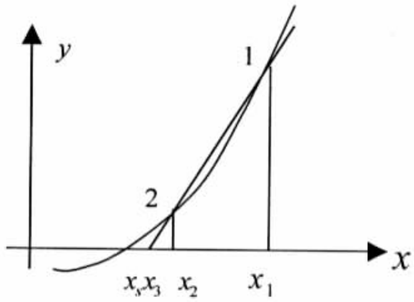

Fig 2 Secant method

above equation is used to calculate a new point $(0, x_3)$, the x coordinate of secant line at $f(x_3) = 0$.

Then, assigning: $x_2 \to x_1, x_3 \to x_1, f_2 \to f_1$, a new value of f_2 can be calculated, (16) can be repeated until $|f| < \delta_e$, or $|(x_n - x_{n-1})/x_n| < \delta_e$, the convergent solution is obtained.

The important advantage of secant method is easy to calculate all the values in (16). For a set of nonlinear equations, even the final form of nonlinear equation can not be obtained, (16) can still be completed to get the solution.

4. Calculating the initial values of unknown

For pressure differential devices, the final form of basic equation is (14), only one unknown can be solved from it, such as q_m, or β (or d), or D, or Δp.

If the two initial values of unknown are too far from the root, convergent result may be difficult

to get .But for the present case, it is easy to calculate the initial values, because the values of c and ε can be guessed approximately errors.

When β is the unknown in (3), it can be calculated approximately (assuming $c\varepsilon = 0.6$):

$$\beta_0 = \left\{\left[2q_m{}^2 \big/ (D^4 \rho_1 \Delta p)\right] \Big/ \left[1 + 2q_m{}^2 \big/ (D^4 \rho_1 \Delta p)\right]\right\}^{0.25} \tag{17}$$

then two initial values of β are:

$\beta_1 = 0.9\beta_0$, and $\beta_2 = 1.1\beta_0$

Similarly, for q_m, or D, or Δp, yields:

$$q_{m_0} = D^2 \beta^2 \sqrt{\rho_1 \Delta p} \Big/ \sqrt{1-\beta^4} \tag{18}$$

$q_{m_1} = 0.9 q_{m_0}$,and $q_{m_2} = 1.1 q_{m_0}$

$$D_0 = \left[q_m \sqrt{1-\beta^4} \Big/ \left(\beta^2 \sqrt{\rho_1 \Delta p}\right)\right]^{0.5} \tag{19}$$

$D_1 = 0.9 D_0$, and $D_2 = 1.1 D_0$

$$\Delta p_0 = 2\left[q_m \left(1-\beta^4\right) / \left(D^4 \rho_1 \beta^4\right)\right] \tag{20}$$

$\Delta p_1 = 0.9 \Delta p_0$, and $\Delta p_2 = 1.1 \Delta p_0$

5. Schemes of iterative computations by secant method

For given properties μ, k, and ρ_1, the schemes for iterative computations by secant method are shown in Table 1

Table 1 Schemes of iterative computation by scant meth

unknow	β (or d)	q_m	D	Δp
given parameters	$q_m, D, \Delta p$	$\beta, D, \Delta p$	$q_m, \beta, \Delta p$	q_m, β, D
Is Re known?	no	no	no	yes
Is ε known?	no	yes	yes	no
Iterative equation	$f(q_m, \beta, D, \Delta p) = 0$ (it is not expressed to a definite form)			
Initial values	$\beta_1 = 0.9\beta_0$ $\beta_2 = 1.1\beta_0$ β_0, see (17)	$q_{m_1} = 0.99 q_{m_0}$ $q_{m_2} = 1.19 q_{m_0}$ q_{m_0}, see (18)	$D_1 = 0.9 D_0$ $D_2 = 1.1 D_0$ D_0, see (19)	$\Delta p_1 = 0.9 \Delta p_0$ $\Delta p_2 = 1.1 \Delta p_0$ Δp_0, see (20)
Precision criterion	$\lvert f(x_n) \rvert < \delta_e$, or $\lvert (x_n - x_{n-1}) / x_n \rvert < \delta_e$ $\delta_e = 1 \cdot 10^7$			
result	$\beta = \beta_n$, $d = \beta_n D$	$q_m = q_{m,n}$	$D = D_n$	$\Delta p = \Delta p_n$

As an example, the properties of fluid at upstream are: $p_1 = 1578\ kPa$, $\rho_1 = 13.1273\ kg/m^3$, $\mu = 10.96 \cdot 10^{-6}\ Pa \cdot s$, and $k = 1.36$. For each of four cases, the answers are shown in Table 2

Table 2 Example of iterative computations by secant method

unknow	β (or d)	q_m	D	Δp
given parameters	$q_m = 8.1368 \, kg/s$ $D = 259.37mm$ $\Delta p = 24.0kPa$	$\beta = 0.55434$ $D = 259.37mm$ $\Delta p = 24.0kPa$	$q_m = 8.1368 \, kg/s$ $\beta = 0.55434$ $\Delta p = 24.0kPa$	$q_m = 8.1368 \, kg/s$ $\beta = 0.55434$ $D = 259.37mm$
answer	$\beta = 0.55434$ $d = 143.78mm$	$q_m = 8.1346 \, kg/s$	$D = 259.3702mm$	$\Delta p = 24.0006 \, kPa$
precision	$\lvert f \rvert < 4e-7$ $\lvert \delta x_r \rvert \to 0$	$\lvert f \rvert < 1.5e-7$ $\lvert \delta x_r \rvert \to 0$	$\lvert f \rvert < 3.5e-7$ $\lvert \delta x_r \rvert \to 0$	$\lvert f \rvert < 5.8e-8$ $\lvert \delta x_r \rvert \to 0$

6. Conclusions

(1) The methods of iterative computations for pressure differential devices given in [1] and other references can be improved greatly. Secant method is a standard and effective method in numerical analysis and methods. Even the final form of nonlinear equation can not be obtained from a set of nonlinear equations , it is still can be used to get the solutions satisfactoryly.

(2) It is shown that the iterative equation for computing the parameters of pressure differential devices is the mass flowmeter itself. This equation is $f(q_m, \beta, D, \Delta p) = 0$, as shown in (14).

(3) The key problem of secant method is to calculate two initial values of unknown. For orifices some formulae are given in (17) to (20),

(4) From the example, it is shown that the precisions of iterative computations by secant method are excellent.

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