Study on the Mathematical Model of the Turbine Flowmeters

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Abstract In order to accurately predict the variation of the performance curve of the turbine flowmeters according to the change of the viscosity of the flow, several previous mathematical models are compared and analyzed. On the base of the airfoil theory, considering the effect of the finite wing spread and the interference between the blades among the cascade, viscous boundary layer theory is applied to the calculation of the lift coefficient C_L and the drag coefficient C_D at the surface of the blades in this paper. The drag induced by the effects of the leakage flow at the blade tip clearance and the loss of the secondary fluid is calculated also. The new mathematical model of the turbine meter is better suitable for measuring the viscous flow than before. The validity of the model is verified preliminarily, for the performance curves predicted by it are fitted well with those plotted by the experimental data got from the water flow facility.

Keywords: Turbine flowmeters; mathematical model; performance curve; boundary layer theory; viscosity

1. Introduction

The turbine flowmeter is widely applied to many fields, as its unique virtue. However the performance of the turbine flowmeter is affected by several parameters of the flow such as the viscosity, the density as so on. There is great difference of performance between the turbine flowmeters with different design. In order to predict the variation of performance according to the change of the characteristics of the flow, and optimize its design, many mathematical models of the turbine flowmeter have been constructed based on different theory since the last few years of 1950's to now.

The comparison of several traditional mathematical models of the turbine flowmeter is as following. Hochreiter[1] made a relationship between the meter factor and the design parameters of it. The relationship was constructed by dimensional reasoning and one-dimensional calculation based on the assumption of the velocity profile just ahead the flow enter the rotor blades is uniform, the flow regime is fully turbulent, the rotor blade is one-dimensional and there is no nonfluid power input or output. Though the accuracy of the relationship for calculation the meter factor is better than 10%, the variation of the meter factor according to the change of the flow rate, the viscosity, the density and the mechanical drag can not be described. Adopting the method of physics analysis and experiment test, Lee[2] presented a mathematical model based on momentum change theory and the assumption of the velocity profile is uniform, the rotor blade is one-dimensional and there is no nonfluid force. In the model, fluid drag consists of resisting torque due to skin friction drag of blades and it due to the sum of all secondary fluid drag. The model regards the drag coefficient for secondary fluid as a constant, ignores the effect of boundary layer on the momentum change and the effect of other drag on the performance of the meter. And the calculation of the skin friction drag of blades is dependent on experimental data. Zhao[3] showed a detailed analysis of the fluid drag of the blades due to the viscosity of the fluid and got a formula for calculation the resisting torque of blades based on boundary layer theory. However the effect of the drags except for the bearing drag and the flow leakage drag in the blade tip clearance on the performance is neglect, the effect of boundary layer on the momentum change is ignored also. Tsukamoto[4] considered the effect of the boundary layer on the momentum change and computed many drag items. But the skin friction drag of blades and the secondary fluid drag were neglect. Thompson[5] applied airfoil theory to the calculation of the driving torque of the rotor and put forward a mathematical model of turbine flow meter based on turbulent flow in an annulus. This model ignores the effect of the boundary layer on the calculation of the drag coefficient and the lift coefficient of the blade, and the effect of the secondary fluid drag on the performance of the meter. Weng [6] introduced a factor to the model made by Thompson for the calculation of the lift coefficient, applied boundary layer theory to the calculation of drag coefficient and constructed a mathematical model of the meter.

A new mathematical model of the turbine flowmeter is constructed in this paper, based on airfoil theory, considering the effect of finite wing spread, the interference between the blades among the cascade, the drag due to leakage flow in the clearance of the blade tip and the secondary fluid loss, applying viscous boundary layer theory to the calculation of the lift coefficient C_L and the drag coefficient C_D of the blades. It should be better suitable for the turbine meter measuring the viscous flow. The validity of the model is verified preliminarily, for the performance curves predicted by it are fitted well with those plotted by the experimental data got from the water flow facility.

2. Construction of the model

2.1. Torques equation of the turbine flowmeter

The equation for the meter performance is developed from a torque balance on the rotor as below:

$$T_d - T_b - T_h - T_m - T_w = 0 \qquad (1)$$

where T_d is the rotor driving torque, T_b is journal bearing retarding torque, T_h is rotor hub retarding torque due to fluid drag, T_m is retarding torque due to mechanical friction in journal bearing and attractive force of magnetoelectricity detector, T_w is both hub disks retarding torque due to fluid drag.

Based on airfoil theory and the application of boundary layer theory in turbomachinery [7] T_d can be presented as follow:

$$T_d = \tfrac{1}{2}\rho N \int_{R_h}^{R_t} \frac{r V_z^2 c}{\cos\beta_\infty}(C_L - tg\beta_\infty C_{DS})dr \qquad (2)$$

where ρ is fluid density, N is number of rotor blades, R_h is turbine rotor hub radius, R_t is turbine rotor tip radius, r is radius to differential blade element, V_z is axial component of absolute velocity at radius of r, c is chord of blade at radius of r, β_∞ is angle between the mean flow velocity direction and the meter axis, C_L is theoretical lift coefficient of a blade, C_{DS} is sum of drag coefficient.

According to the application of boundary layer theory in turbomachinery[7] C_L and C_D can be calculated by:

$$C_L = \frac{s}{c}(2\delta_u \cos\beta_\infty + \xi_V \cos\beta_\infty \sin\beta_\infty)$$

$$C_D = \frac{s}{c}\xi_V \cos^3\beta_\infty$$

$$C_{DS} = C_D + C_{Di}$$

where C_D is local drag coefficient of a blade, s is rotor blade spacing, C_{Di} is drag coefficient due to finite wing spread, secondary fluid loss and blade tip clearance drag.

Based on airfoil theory and its application in turbomachinery[8] C_{Di} is:

$$C_{Di} = \frac{C_L^2}{\pi(AR)} + 0.04C_L^2\sigma\frac{s}{R_t - R_h} + \frac{1}{4}C_L^2\sigma\frac{R_0 - R_t}{R_t - R_h}\frac{1}{\cos\beta_2}$$

where first item is the effect of finite wing spread, second item is the effect of secondary fluid loss, the last item is the effect of blade tip clearance drag.

Where

$$\xi_V = \frac{2\theta}{\cos^2\beta_{2corr}}$$

$$\delta_u = (1 + \Delta^* - \theta)tg\beta_{2corr} - tg\beta_1$$

$$\Delta^* = \frac{\delta_{TS}^* + \delta_{TP}^*}{s\cos\beta_{2corr}} \qquad \theta = \frac{\theta_{TS} + \theta_{TP}}{s\cos\beta_{2corr}}$$

where β_{2corr} is angle made by the exit velocity with the meter axis calculated by airfoil theory ignoring the effect of the boundary layer, θ_{TS} is momentum thickness of boundary layer at suction side of blade, θ_{TP} is momentum thickness of boundary layer at pressure side of blade, δ^*_{TS} is the displacement thickness of boundary layer at suction side of blade, δ^*_{TP} is the displacement thickness of boundary layer at pressure side of blade.

Where

$$\begin{cases} \theta_{TS}/c = \theta_{TP}/c = 0.664R_c^{-\frac{1}{2}} \\ \delta^*_{TS}/c = \delta^*_{TP}/c = 1.721R_c^{-\frac{1}{2}} \end{cases} \quad \left(R_c < 2.5\times10^5\right)$$

$$\begin{cases} \theta_{TS}/c = \theta_{TP}/c = 0.0463R_c^{-0.2} \\ \delta^*_{TS}/c = \delta^*_{TP}/c = 0.036R_c^{-0.2} \end{cases} \quad \left(R_c \ge 2.5\times10^5\right)$$

$$R_c = \frac{U_{\infty c}c}{\nu}$$

$$U_{\infty c} = \frac{V_z}{\cos\beta_{\infty c}\left(1 - t_b/(s\cos\beta_{\infty c})\right)}$$

$$tg\beta_{\infty c} = \frac{tg\beta_1 + tg\beta_{2corr}}{2}$$

where $U_{\infty c}$ the mean flow velocity relative to blade calculated by airfoil theory ignoring the effect of the boundary layer, $\beta_{\infty c}$ is angle between the mean flow velocity direction and the meter axis ignoring the effect of the boundary layer, t_b is thickness of the blade, ν is fluid kinematic viscosity, β_1 is angle made by the inlet velocity with the meter axis.

$$tg\beta_{\infty} = \frac{tg\beta_1 + tg\beta_2}{2}$$

$$tg\beta_2 = (1 + \Delta^* - \theta)tg\beta_{2corr}$$

$$tg\beta_1 = \frac{r\omega}{V_z}$$

$$tg\beta_{2corr} - tg\beta_1 = \frac{2q}{1+q}\left(\frac{2\pi r}{L} - \frac{r\omega}{V_z}\right)$$

$$q = \frac{2R}{R^2+1}\cos\alpha$$

$$\begin{cases} tg\alpha = (tg\beta)\dfrac{R^2-1}{R^2+1} \\ \dfrac{s}{c} = \dfrac{1}{\pi}\left\{\cos\beta\ln\left(\dfrac{R^2+2R\cos\alpha+1}{R^2-2R\cos\alpha+1}\right) + 2\sin\beta\left(tg^{-1}\dfrac{2R\sin\alpha}{R^2-1}\right)\right\} \end{cases}$$

$$tg\beta = \frac{2\pi r}{L}$$

$$c = \frac{L_h}{\cos\beta} - (r - R_h)tg\left(\frac{\pi}{2} - \gamma\right)$$

$$AR = \frac{R_t - R_h}{c_a} \qquad c_a = \frac{c_t + c_h}{2}$$

$$\sigma = \frac{c}{s} \qquad s = \frac{2\pi r - Nt_b}{N}$$

where β_2 is angle made by the exit velocity with the meter axis considering the effect of the boundary layer, ω is rotor speed of a real meter, L is lead of helical blade, R is position of the sources and sinks in conformal mapping, α is angle in potential flow solution, β is blade stagger angle, AR is blade aspect ratio, L_h is length of rotor hub, γ is bevel edge angle of blade, c_a is average chord of blade, c_h is chord at the root of blade, c_t is chord at the tip of blade, σ is solidity ratio.

According to the accurate solution of Navier-Stokes equation to the problem of steady flow in two turning coaxial cylinder[9], T_b can be presented as:

$$T_b = \frac{4\pi R_1^2 R_2^2}{R_2^2 - R_1^2} L_b \rho\nu\omega \qquad (3)$$

where R_1 is radius of rotor axis, R_2 is inner radius of journal bearing, L_b is length of friction part between rotor axis and journal bearing.

Based on researches of fluid drag due to skin friction on a flat plate in a same direction as a flow, T_h is expressed as:

$$T_h = \frac{1}{2} V_{zh}^2 A_h R_h C_h \frac{tg\beta_{\infty h}}{\cos\beta_{\infty h}} \quad (4)$$

where
$$C_h = 1.328 R_{eh}^{-\frac{1}{2}} \quad (R_{eh} < 2.5\times10^5)$$
$$C_h = 0.074 R_{eh}^{-0.2} \quad (R_{eh} > 2.5\times10^5)$$

$$R_{eh} = \frac{U_{\infty h} c_h}{\nu}$$

$$A_h = 2\pi R_h L_h - N t_{bh} c_h$$

V_{zh} is axial component of absolute flow velocity at rotor hub, it can be regarded as V_z, $\beta_{\infty h}$ is angle between the mean flow velocity direction and the meter axis at rotor hub, $U_{\infty h}$ is mean flow velocity relative to blade calculated by airfoil theory considering the effect of the boundary layer at rotor hub, t_{bh} is thickness of the blade at rotor hub.

The value of T_m is dependent on the parameters of the magnetoelectricity detector and the effect of journal bearing lubricating. For a certain design of turbine flowmeter T_m can be regarded as a constant:

$$T_m = Const \quad (5)$$

According to the accurate solution of Navier-Stokes equation to the problem of retarding torque due to skin friction of turning round disk in a flow [9], T_w is:

$$T_w = \tfrac{1}{2} \rho \omega^2 R_h^5 C_M \quad (6)$$

where
$$C_M = 3.87 R_w^{-\frac{1}{2}} \quad R_w < 3\times10^5$$
$$C_M = 0.146 R_w^{-0.2} \quad R_w > 3\times10^5$$

$$R_w = \frac{R_h^2 \omega}{\nu}$$

The formula (1)~(6) form the mathematical model of the turbine flowmeter. In the model, there is not any parameters need to be regulated by person or dependent on the experimental data of the meter.

2.2. Velocity profile at rotor blades inlet

The velocity profile at rotor blades inlet is significant for the calculation of the meter factor of the turbine flowmeter. In order to determine the velocity profile just ahead of flow enter the rotor blades, the flow fields inner DN50mm and DN15mm turbine meter are calculated by commercial CFD software FLUENT. The conditions of calculation are as following: the fluid is water liquid, the flow rate for DN50mm is 21.55 m^3/h and DN15mm is 3.0 m^3/h. The axial velocity profiles at rotor blades inlet are shown in Fig.1and Fig. 2.

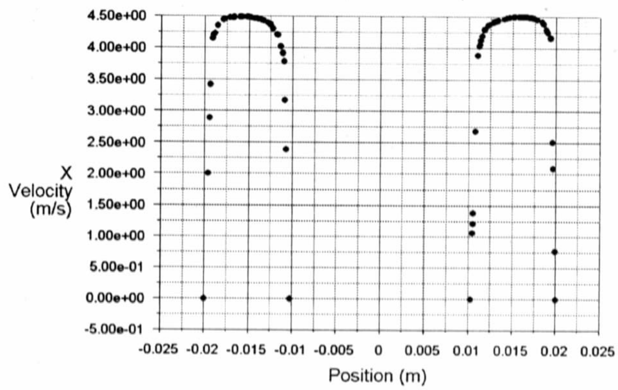

Fig. 1 DN50mm turbine flowmeter velocity profile

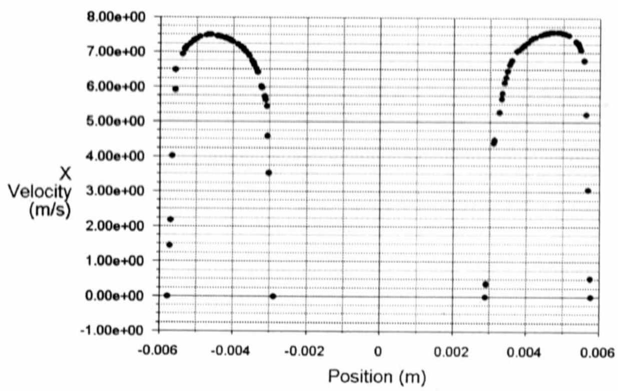

Fig. 2 DN15mm turbine flowmeter velocity profile

Taking the region of 0.01028 m~0.01581 for DN50mm and 0.00289 m~0.00483 m for DN15mm, is that a quarter of cross-line of the annulus between rotor hub and meter shell, the velocity profiles in fig.1 and Fig. 2 can be made to be dimensionless as square point plot in Fig. 3 and Fig. 4. The dimensionless velocity profiles can be fitted by exponent law, as the thin lines shown in Fig. 3 and Fig.4. The exponential number n for DN50mm is 6.7186, and DN15mm is 4.6121.

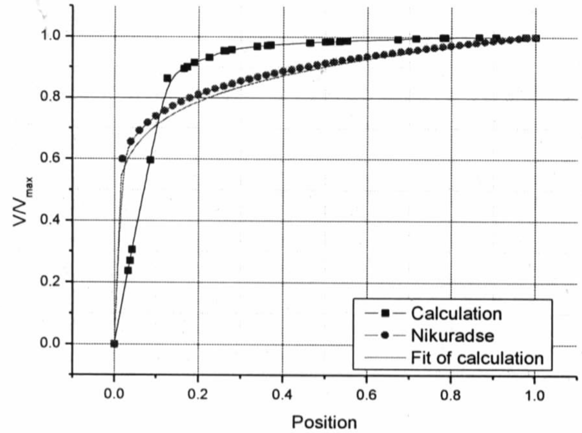

Fig. 3 DN50 mm turbine meter dimensionless velocity profile

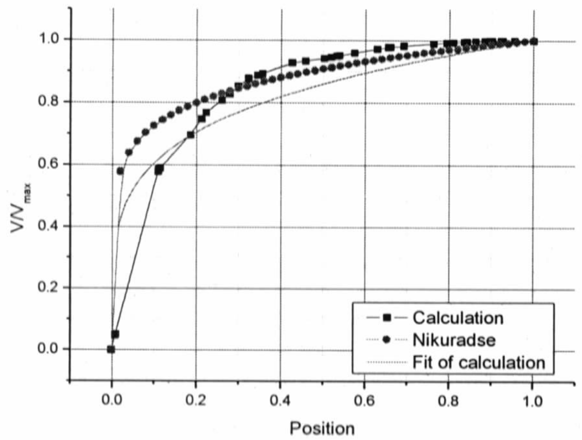

Fig. 4 DN15 mm turbine meter dimensionless velocity profile

According to the relationship between pipe Reynold number and the exponential number n of exponent law velocity profile[10], drawn from experimental data made by Nikuradse, the exponent law velocity profile can be gotten at the same conditions as FLUENT calculation, as the round dot plots shown in Fig. 3 and Fig. 4. The exponential number n for DN50mm is 7.6235 and DN15mm is 7.1684.

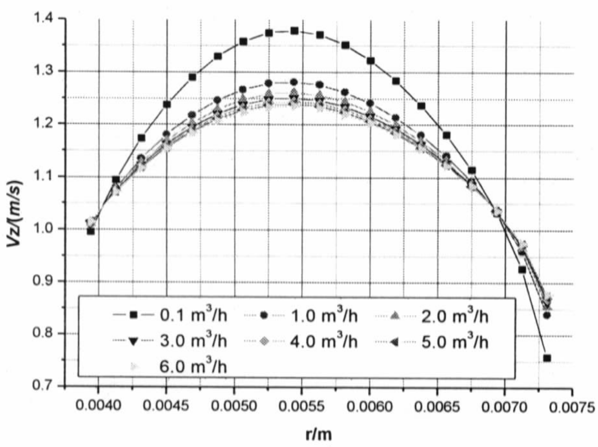

Fig. 5 DN15mm turbine flowmeter velocity profile

Based on the results of research on developed flow in annulus[11~13], the velocity profile at rotor blades inlet for DN15mm turbine meter can be calculated as the lines shown in Fig. 5.

As shown in Fig. 1 and Fig. 2, the flow velocity at outside of annulus is larger than that of inside. However, as the velocity profile shown in Fig. 5, the flow velocity at inside of annulus is larger than that of outside. The reason for this is that, according to the formula for calculating the length of the developing area[14], the flow regime is far from fully developed, is that it is developing still, at the inlet of rotor blade. The flow velocity is larger than actual one at outside of annulus if the uniform velocity profile is adopted to calculate it. The difference is like that between the dimensionless profiles drawn from FLUENT calculation and the cross lines of unit one. Salami[15] indicated that at the root and the middle part of the blade, the lift coefficient is positive, so the acts of the rotor like the turbo the fluid driving the rotor, however, at the top part the lift coefficient is negative and the rotor act as the pump, the fluid blocking the turn of the rotor. In this way, the meter factor is always larger than the real one if the calculation is based on annulus profile, and it is reversed if based on uniform profile. For measuring water liquid choose an exponent law velocity profile with suitable exponential number n is good for performance predict.

3. Validity of the model

3.1. Meter factor predict at one flow rate

The main parameters of DN50mm and DN15mm turbine flowmeters are shown in Tab. 1. Where R_o is meter bore radius.

Adopting different velocity profile, the meter factors of DN50mm and DN15mm turbine meter are calculated by the model when water liquid flow rate is 21.55 m^3/h and 3.0 m3/h separately. The results are shown in Tab.2. The experimental meter factor obtained from the water flow facility and the error of model calculation relative to the experimental data are shown in Tab.2 also. During the calculation of the meter factors, T_m is set as zero.

Table 1 Parameters of turbine flowmeter

	DN50 mm	DN15 mm
c_h	0.009	0.0075
L	0.157	0.0414
L_b	0.0045	0.006
L_h	0.008	0.007
N	6	4
R_1	0.0015	0.001
R_2	0.00151	0.00101
R_h	0.01275	0.00375
R_o	0.025	0.0075
R_t	0.024	0.0073
t_b	0.0008	0.0005
t_{bh}	0.0008	0.0005
γ	$\pi/2$	0.925

Table 2 Experimental and calculated meter factors

	Experimental /(1/L)	n	Calculated /(1/L)	Error /%
DN15	776.842	7.1684	804.142	3.5142
		4.6121	838.228	7.9020
		Uniform	737.234	-5.0986
		Annulus	827.830	6.5635
DN50	27.416	7.6235	29.124	6.2299
		6.7186	29.480	7.5285
		Uniform	26.479	-3.4177
		Annulus	30.049	9.6039

It can be known from the Tab. 2 that the error is relative small when the Nikuradse experimental velocity profile or the uniform profile is adopted. However, compared with experimental data, the meter factor calculated is small when uniform profile is adopted, and it is reversed if based on Nikuradse experimental profile. So, to choose a suitable exponential number n, larger than that of Nikuradse experiments, maybe a good predict accuracy can be reach for water measuring.

3.2.Performance curve predict

The performance curves of DN50mm and DN15mm turbine flowmeters measuring the water flow are calculated by the mathematical model based on Nikuradse experimental velocity profile and uniform profile. The results are shown in Fig. 6 and Fig. 7. The experimental performance curves drawn from the water flow facility is shown in Fig. 6 and Fig. 7 also.

For DN50mm turbine flowmeter, at the flow range of 4 m^3/h~40 m^3/h, the maximal error is –3.4636% and the minimum error is –2.6962% adopting the uniform velocity profile, the maximal error is 7.8044% and the minimum error is 5.8552% adopting the Nikuradse experimental velocity profile. For DN15mm turbine flowmeter, at the flow range of 0.6 m^3/h~6.0 m^3/h, the maximal error is –5.0146% and the minimum error is –3.9273% adopting the uniform velocity profile, the maximal error is 7.1305% and the minimum error is 3.2112% adopting the Nikuradse experimental velocity profile. Now the predict performance curves fit the experimental curves relatively well. To choose a suitable exponential number n, larger than that of Nikuradse experiments, maybe a better predict accuracy can be reach for water measuring.

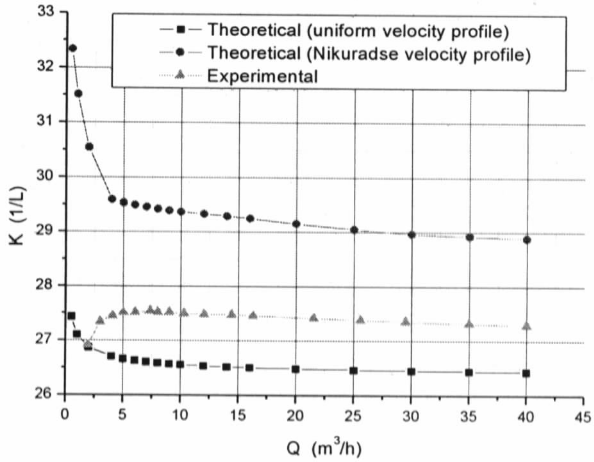

Fig. 6 DN50mm turbine flowmeter performance curves

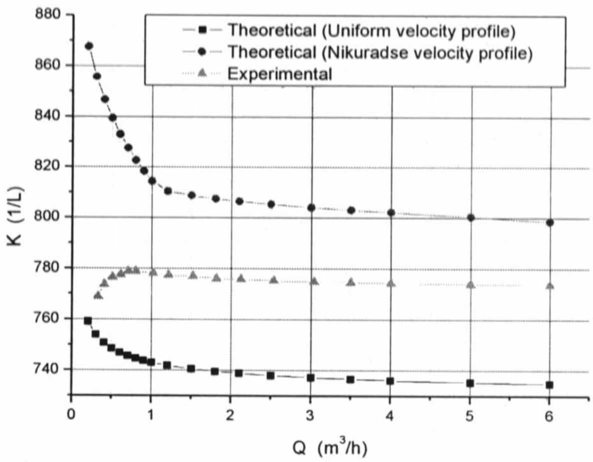

Fig. 7 DN15mm turbine flowmeter performance curves

4. Conclusion

Based on airfoil theory, according to the application of boundary layer theory in turbomachinery a mathematical model of turbine flowmeter is developed. There is not any parameters need to be regulated by person or dependent on the experimental data of the meter in the model. The model is better suitable for measuring viscous flow in theory. The validity of the model is verified preliminarily, for the performance curves predicted by it are fitted well with those plotted by the experimental data got from the water test facility. The validity of the model for predict the performance of turbine flowmeter measuring high viscosity flow will be test in the future.

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