An Experimental Study on the Characteristics of Oval Gear Flowmeters

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Abstract An experimental investigation was conducted into the characteristics of an oval gear flowmeter (OGF) using various fluids with different viscosities such as liquid butane (0.17 cP), gasoline (0.40 cP), spindle oil (12.0 cP), etc. Three OGFs were adopted that have a 50 mm inner diameter and $\pm 0.2\,\%$ repeatability in the range of 3 and 17 m^3/h. The top and side clearances were measured to see their effects on its overall characteristics. The leakage flow through the clearance between its rotor and casing body was determined by liquid viscosity and the size of the clearance. The amount of the leakage flow has been both measured experimentally and estimated theoretically with the calculated velocity profile in the clearance. In the experiment, the piston prover, the gravimetric, and the volumetric calibrators have been employed. Then, the uncertainty of each calibrator was evaluated as being within $\pm 0.1\,\%$ in flow quantity determination using an evaluation procedure that follows ISO recommendation. It was found that the characteristics of OGFs depend on the viscosity of fluid and the magnitude of clearance. Also, the characteristics, when measured with other liquids than water, can be predicted based on the water measurement results. As the results of this study, a correction equation of $\delta Q = 0.272 \ln \mu - 0.0113$ is proposed to predict the flow measurements of other liquids by utilizing a correction method developed in this study.

Keywords: Oval gear flowmeter; calibration; viscosity effect, flowmeter characteristics, Couette flow

Nomenclature

- a : Length of major axis of ellipse [m]
- b : Length of minor axis of ellipse [m]
- C : Clearance [m]
- D : Inner diameter of a pipe [m]
- L : Length of an oval gear rotor [m]
- n : Normal direction to the surface
- p : Pressure [N/m^2]
- q : Leakage flow rate [m^3/s]
- Re : Reynolds number
- u : Velocity in the x direction [m/s]
- v : Velocity in the n direction [m/s]
- V : Velocity at an oval gear rotor [m/s]
- x, y, z : Cartesian coordinates

Subscripts

- C : Based on the clearance
- D : Based on the inner diameter of pipe
- e : Ellipse
- r : Circumferential
- s : Side
- t : Top

1. Introduction

New types of instruments for flow measurements have been developed with increasingly sophisticated technologies and deployed into real applications of experimental fluid mechanics. However, traditional positive displacement (PD) flowmeters, including oval gear flowmeters (OGF), are still widely used, mainly due to their high reliability in quantity measurements. They are used in numerous engineering and scientific applications including metering expensive liquids such as fuels and oils for the purpose of sales.

Basically, an OGF consists of a pair of oval gear rotors that revolve inside the casing, driven by the pressure difference over itself, as shown in Fig. 1. Thus, the flowing stream of a liquid in the OGF is separated by the rotors and the casing into a series of discrete 'pockets' whose volume is fixed, and the flow quantity can be determined by counting the number of pockets. Its principle is very straightforward and it can be accurate.

However, there exist clearances between the rotor and the casing at the top and at the side of the casing that aggravate the accuracy of flow measurements. Particularly, when the fluid to be measured is different from the viscosity of water, the OGF measurement of the fluid flow may be considerably inaccurate, since the OGF is usually calibrated with water for safety.

Most liquid fuels have a slightly lower viscosity than water. This may cause unavoidable measurement errors, leading frequently to arguments between buyers and suppliers of the liquid fuels in commercial transactions.

In this study, the feasibility of a correction method for OGFs has been investigated, when used for flow measurements of liquids other than water, especially those having various viscosities. Also, the effects of geometrical configurations of OGF components on its performance have been studied.

2. Analytic Consideration of the Leakage

As mentioned, the performance of an OGF is primarily dependant on the amount of leakage flow through the clearances between the rotor and the top of the casing and between the rotor and the side of the casing (see Fig. 2). The leakage flow is driven by the pressure difference the upstream and the downstream of the OGF.

Contact lengths of the rotor with the casing at the top and at the side are 1 mm and 3 mm, respectively, and their clearances are about 70 μm and 80 μm, respectively, which are relatively small compared to the corresponding contact lengths. Also, the length of the rotor, 80 mm, is sufficiently larger than the clearances. Thus, they could be modeled as the Couette flow between two parallel plates.

The Reynolds number, based on the clearance Re_C of the leakage flow, is less than 1500, leading to the assumption that it is laminar flow. Thus, the following is a governing equation to be dealt with in this study:[1-2]

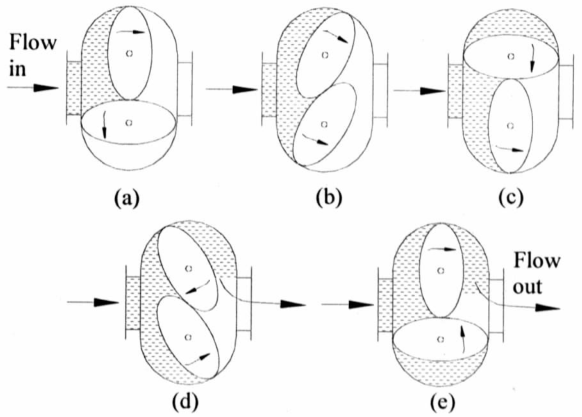


Fig. 1 Operational principle of an OGF

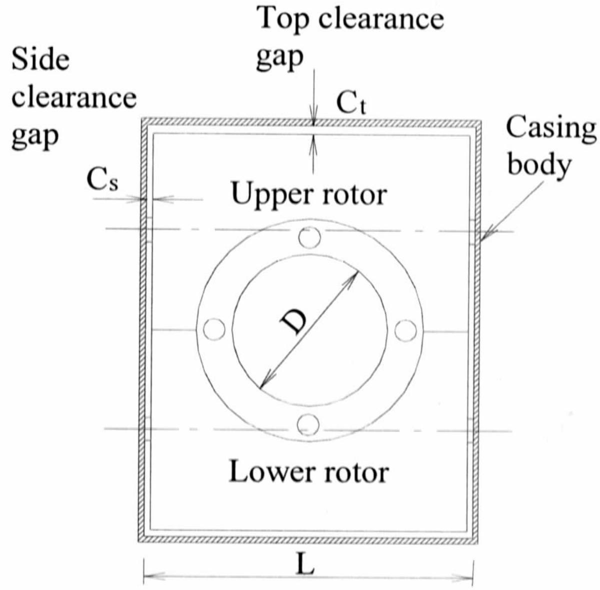


Fig. 2 Side view of an OGF

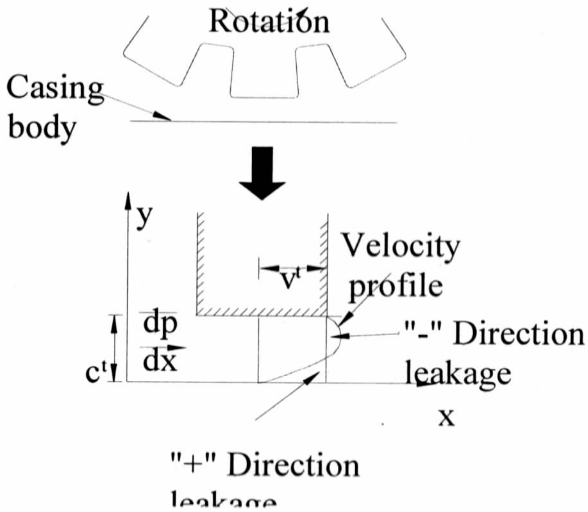


Fig. 3 Velocity profile of the leakage flow in the top clearance

$$-\frac{dp}{dx}+\mu\frac{d^2u}{dy^2}=0 \quad (1)$$

2.1. Leakage through the top clearance

Leakage flow through the top clearance can be sketched, as shown in Fig. 3, after putting it upside down for convenience. The rotor end moves steadily at a velocity V_t relative to the casing at rest.

The velocity distribution can be found by solving Eq. (1) with boundary conditions of $u(0)=0$ and $u(C_t)=V_t$ which is,

$$u(y)=V_t\frac{y}{C_t}-\frac{1}{2}\frac{C_t^2}{\mu}\frac{dp}{dx}\frac{y}{C_t}(1-\frac{y}{C_t}) \quad (2)$$

where dp/dx the pressure gradient over the clearance. The velocity profile is a function of dp/dx , μ, V_t and C_t

The amount of leakage flow, which results from the relative velocity of flow to the circumferential velocity of the rotor end, could be estimated by integrating the relative velocity over the cross -sectional area of the clearance measured perpendicular to u, that is,

$$q_t=\int_0^{C_t}(u(y)-V_t)dy\cdot L \quad (3)$$

where L is the width of the oval gear rotor. Based on the magnitudes of u and V_t, the direction of leakage flow can be changed either from the upstream to the downstream or from the downstream to the upstream of the main flow, leading to both negative and positive errors, respectively.

2.2. Leakage through the side clearance

Figure 4 is a series of diagram indicating the leakage flow through the side clearance as a function of rotation angle of the two oval gear rotors. As shown in the figure, the direction of the leakage flow can be assumed to be normal to the circumference of the oval rotors.

The rotors are of an elliptical shape having the length

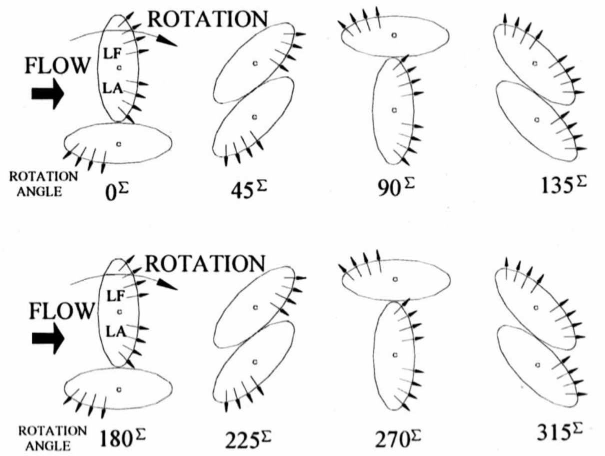


Fig. 4 Leakage flow through the side clearance with difference angles of the oval gear rotors

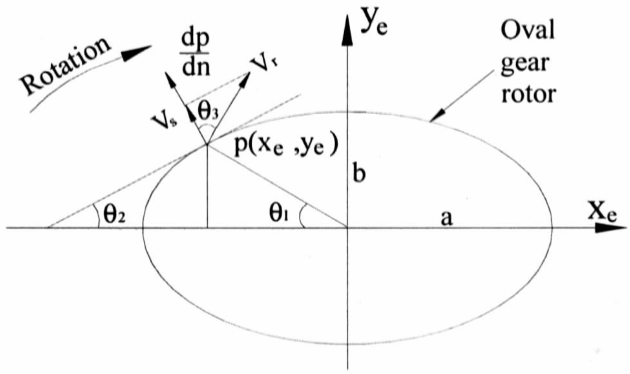


Fig. 5 Geometry of an oval gear rotor for the leakage flow in the side clearance

of the major axis of 2a and that of the minor axis of 2b, as shown in Fig. 5. Then, the velocity V_s at an arbitrary point $p(x_e, y_e)$ on the ellipse is the normal component of the circumferential velocity V_r caused by the rotation of the rotor, and can be expressed as follows:

$$V_s=V_r\cdot\cos\theta_3 \quad (4)$$

$$\theta_3=\theta_1+\theta_2=\arctan\left(b\frac{\sqrt{1-x_e^2/a^2}}{x_e}\right)+\arctan\left(\frac{-b}{a^2}\frac{x_e}{b\sqrt{1-x_e^2/a^2}}\right) \quad (5)$$

The driving force resulting in the leakage flow may be the pressure gradient across the side clearance, which is combined with other minor sources of leakage flow such as centrifugal force, etc. However, the minor forces are neglected in the analysis for simplicity.

The pressure gradient occurs in the normal direction to the circumference of the rotors, as does the velocity of the leakage flow. Similar to the leakage flow through the top clearance, the side of the rotor moves at a velocity V_s relative to the casing at rest, which can lead to the assumption of being the Couette flow. The velocity profile across the side clearance can be obtained by solving Eq. (1) with boundary conditions of $v(0) = 0$ and $v(C_s) = V_s$, that is,

$$v(z) = V_s \frac{z}{C_s} - \frac{1}{2} \frac{C_s^2}{\mu} \frac{dp}{dn} \frac{z}{C_s} (1 - \frac{z}{C_s}) \tag{6}$$

Where z is the coordinate that is normal to the rotor surface.

Similarly, the amount of the leakage flow through the side clearance can be estimated by integrating the velocity over the cross-sectional area of the clearance enclosed by both the side clearance C_s and the length of the major axis of the ellipse 2a as follows:

$$q_s = \int_{-a}^{0} \int_{0}^{C_s} (v(z) - V_s) \, dz \cdot \frac{dx_e}{\sin \theta_1} + \int_{a}^{0} \int_{0}^{C_s} (v(z) + V_s) \, dz \cdot \frac{dx_e}{\sin \theta_1} \tag{7}$$

Alternating the sign of the term V_s reflects the change in the direction of the velocity, depending on the coordinate x_e.

The total amount of leakage flow across the rotor can be estimated by adding each of the two calculated Eqs., Eqs. (3) and (7), respectively, that is,

$$q = q_t + q_s \tag{8}$$

Exact calculation of the total leakage amount requires the data for the liquid viscosity, each of the two clearances, the pressure gradient, etc. The first two quantities are measurable without difficulty. The

Table 1. Measured top and side clearance gaps of oval gear flowmeters.

OGF	C_t (mm)	C_s (mm)	Test liquids
A	0.069	0.086	Water, liquid Butane, spindle oil
B	0.071	0.087	Water, liquid butane, spindle oil
C	0.073	0.089	Water, liquid butane
D	0.060	0.083	Water, gasoline

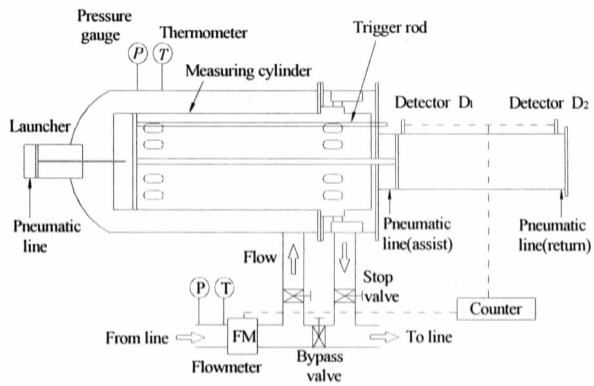


Fig. 6 Piston prover

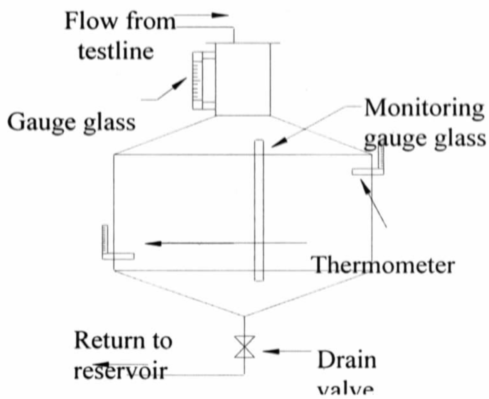


Fig. 7 Volumetric calibration system

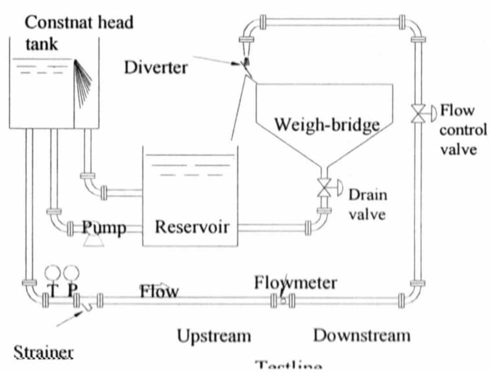


Fig. 8 Gravimetric calibration system

last one, however, is not easily measured, since this should be determined across the narrow clearances, not between the far upstream and the far downstream the OGF.

In order to investigate the characteristics of the OGFs more preciously, therefore, experiments have been conducted by varying parameters of interest.

3. Experiment

Four OGFs, which were picked up randomly from the production line, have been used for tests. Their top and side clearances, which satisfy the manufacturer's tolerance, have been measured and listed in Table 1. The OGFs are for pipes 50 mm in diameter. Repeatability of the OGFs in volume measurements claimed by the manufacturer is ±0.2%.

Figures 6-8 show the calibration systems used in this study, namely, the piston prover, the volumetric calibrator, and the gravimetric calibrator, respectively. All of them have traceability of national flow standard system maintained by KRISS (Korea Research Institute of Standards and Science). The uncertainty[3-6] of each calibrator was evaluated in accordance with the procedure recommended by the ISO (International Organization for Standardization), and turned out to be less than ±0.1% in flow quantity determination.

During the tests, the flow rate has been maintained in the range from 3 to 15 m^3/h. This means that the Reynolds number based on the pipe diameter Re_D varies from 20,000 to 400,000. Test fluids include the liquid butane, the spindle oil and the gasoline. All or some of these liquids have been used to characterize each of the test OGFs, as listed in Table 1.

Figure 9 shows the viscosities of the test liquids as a function of the temperature. The viscosity of water, gasoline, and spindle oil were measured and that of butane was taken from the table.[7] As shown in the figure, the range of the viscosities of the test liquids is very wide, from less than 1 cP to greater than 14 cP. All the tests have been carried out in open air conditions.

Other test conditions including the fluid temperature and pressure for a test liquid were differed. To allow

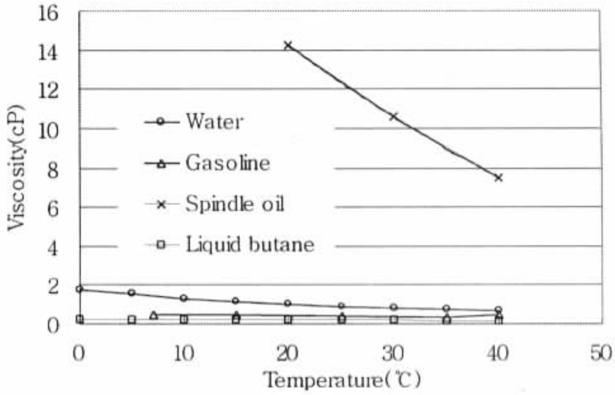


Fig. 9 Viscosity of test fluids

comparison, the obtained data have been converted to the values in the standard condition, that is, 15°C with atmospheric pressure.

4. Results and discussion

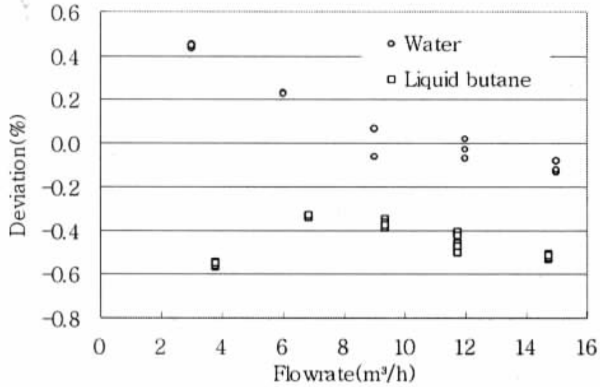
Due to the difficulty of dealing with hazardous liquids, the experiments have taken quite a long time. Thus, collected data may be less systematic, but useful.

The experimental results to be discussed hereafter are a percentile deviation of the flow quantity between test OGFs and calibrator measurements as a function of the flowrate. Then, the negative deviation means that the OGF measurement is smaller than the calibrator measurement and vice versa.

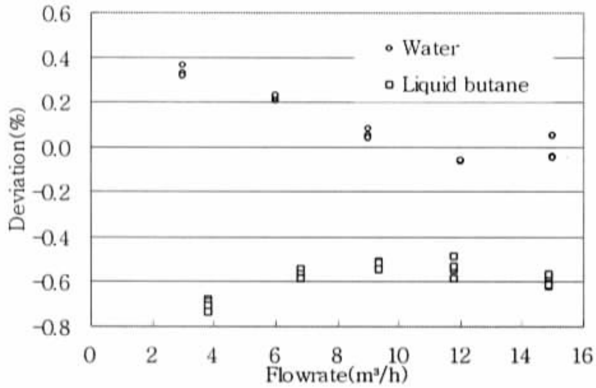
First of all, three of the test OGFs have been tested with both liquid butane and water by utilizing the piston prover, as shown in Fig. 10. For all three OGFs, the deviation against the calibrator measurements decreases as the flowrate is increased in the case of the water measurements. However, that for the liquid butane varies quadratically, that is, the minimum deviation occurs in the middle of the flowrates investigated.

As for the influence of the magnitude of the clearances, it is clear that the deviation gets smaller, meaning that its absolute value gets larger, as the magnitudes of the clearances are increased regardless of the type of liquids.

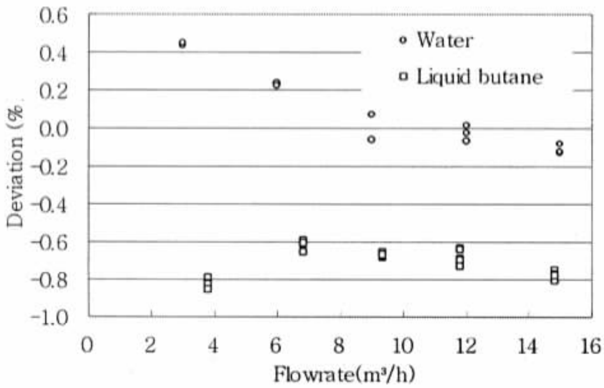
The difference of the clearances between OGF A and OGF C, listed in Table 1, results in 0.3% difference in



(a) OGF A



(b) OGF B



(c) OGF C

Fig. 10 Test results of OGFs with liquid butane and water

flow measurements. This result manifests the theoretical consideration, based on Eqs. (2), (3), (6), and (7), that the amount of the leakage flow is strongly dependent on the magnitude of the clearance. Average deviation over the flowrate investigated due to the difference of the viscosity between the water

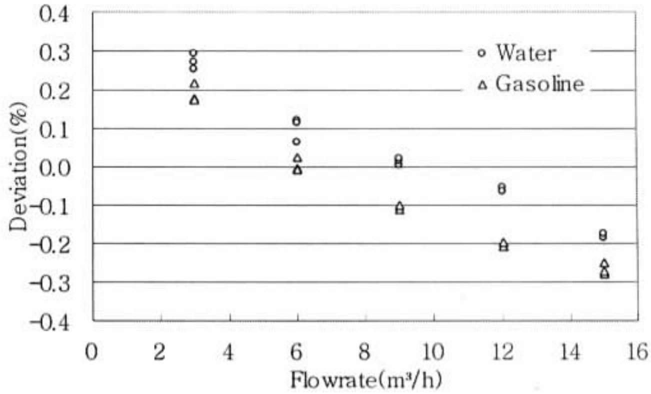


Fig. 11 Test results of OGF D with gasoline and water

and the liquid butane is about 0.6%. As shown in the figure, the effect of viscosity on the deviation in flow measurements is stronger at the lower flowrate.

Figure 11 shows OGF D test results against the volumetric calibrator (see Fig. 7) with gasoline and the water. As shown in Table 1, among the three, the clearances of OGF D are closest to those of OGF A.

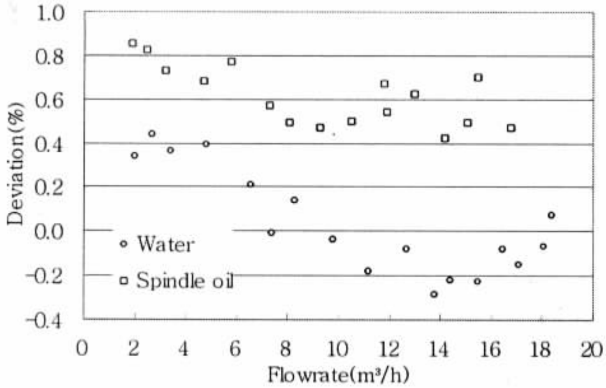
Thus, its deviation characteristics against the calibrator measurements for the water are supposed to be similar to those of OGF A; that is, being linear to the flowrate in the range investigated, and they are.

The difference in deviations between water and gasoline is less than 0.1% in most cases. This may be because the viscosity of the gasoline is very close to that of water. It also manifests the theoretical consideration that the viscosity of the liquids is a single dominant fluid property.

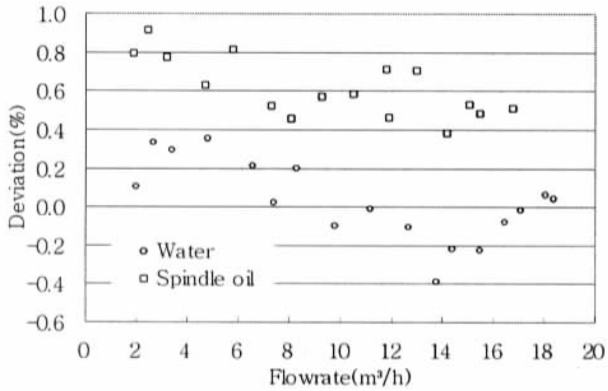
The characteristics of OGFs A and B have been tested against the gravimetric calibrator (see Fig. 8) with the spindle oil and the water, as shown in Fig. 12. The results of both OGFs for the water are quite comparable to those in Fig. 10, that is, linear characteristics of the positive deviation at lower flowrate and the negative deviation at higher flowrate.

The tests with spindle oil indicate much more slowly varying characteristics of deviation with the flowrate. Also, their average deviation, which is always positive in the range investigated, is approximately 0.6% with a repeatability of ±0.2%.

Thus, it is found that the measurement accuracy for highly viscous liquids such as spindle oil is less sensitive to the flowrate of the main flow. It is also found that the magnitudes of the clearances of OGFs



(a) OGF A



(b) OGF B

Fig. 12 Test results of OGFs with spindle oil and water

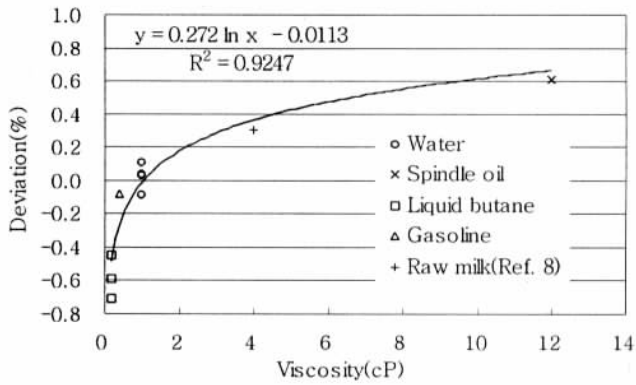


Fig. 13 Prediction of deviations as a function of fluid viscosity

do not affect much on the amount of the leakage flow for those liquids of high viscosity, by comparing Figs. 10, 11, and 12. Those parameters that may affect the characteristics of OGFs have been experimentally investigated and the test results can be, in general, explained well with the theoretical consideration developed in this study.

The magnitude of the clearances affects the amount of the leakage flow noticeably for the liquid butane, but this is hardly noticeable for the spindle oil. The amount of the leakage flow with different liquids varies with the flowrate of the main flow.

This trend is very clear for those liquids having low viscosity including the liquid butane, the gasoline, and the water, but it is less definite for a highly viscous fluid like the spindle oil in this investigation.

However, the repeatability of all the measurements with the four different liquids lies still within the manufacturer's claimed one of ±0.2%, thus the experimental results with the test OGFs are good and reliable.

All the experimental data is combined to see the effect of the viscosity of the test liquids on the deviation more thoroughly, as shown in Fig. 13. The data for water show deviations scattered in the range of ±0.1%, which may indicate the uncertainty of all the measurements of the OGFs in this study; this is supposed to be zero. The data for gasoline deviate by -0.1%, but this is still within the range of uncertainty for overall measurements. However, the data for the liquid butane and the spindle oil deviate by -0.6% and 0.6%, respectively.

All the data in Fig. 13 are curve-fit with a model of $\delta Q = A \cdot \ln \mu + B$, where δQ and μ represent the relative deviation in volumetric flow quantity measurement and the viscosity of the test liquid, respectively. The constants A and B in this study turn out to be 0.272 and - 0.0113, respectively. As shown in the figure, the correlation equation is quite tight.

In order to validate the correlation equation, flowrate data of milk, which is usually measured with a PD meter at a dairy farm, is applied. A technical report[8] regarding the calibration of a PD meter when used for the milk flow quantity measurements suggests that the reading, based on the water, should be corrected by -0.3%. The viscosity of milk collected at a dairy farm is in the range of 3.5 and 4.5 cP at 5°C.[9] This data point can be found in the figure.

Considering the repeatability of a PD meter and the experiments in this study, this is in a good agreement with the curve of the correlation equation. Thus, it turns out that the correlation equation proposed in this study may be useful in correcting the flowrate measurements with OGFs when used for other liquids than water.

5. Conclusions

The characteristics of test oval gear flowmeters (OGF), especially the effect of viscosity of liquids on the deviation of flow quantity measurements, are experimentally investigated. Four OGFs having different magnitudes for the top and side clearances are tested with four different liquids, that is, the liquid butane (0.17 cP), the gasoline (0.40 cP), the water (1 cP), and the spindle oil (12.0 cP).

The following conclusions can be drawn in this study. First, the deviation of flow quantity measurements increases at the magnitude of the clearances gets larger. This trend is distinct for lower viscosity liquids, while it is less sensitive for higher viscosity liquids. Secondly, the viscosity of the liquids obviously influences the deviation of flow quantity measurements of the OGFs, regardless of the magnitude of the clearances of the OGFs. Lastly, a correlation equation of $\delta Q = 0.272 \cdot \ln \mu - 0.0113$ is proposed for correction of flowrate measurements of OGFs when used with liquids other than water. The equation indicates that the correction in flowrate measurements is dominantly a function of the viscosity of the liquid, and it is probably applicable to other types of positive displacement flowmeters.

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