Characterization of Oil/Water Two Phase Flow Based on Complexity Theory

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Abstract The flow patterns of oil/water two-phase flow in vertical upward pipe have been characterized by the analysis of LZ Complexity, Fluctuation Complexity and Spectrum Entropy measures using the conductance time series fluctuating signals. It is shown that all the three complexity measures have little changes with oil-in-water flow pattern variations for water cut ranging from 61% to 91% and show irregular sudden changes to transitional flow pattern variations. We conclude that the three complexity measures are potential sensitive 'indicators' of flow pattern transition.

Keywords: Oil/water two-phase flow; Flow pattern; LZ Complexity; Fluctuation Complexity; Spectrum Entropy

1. Introduction

Oil/water two phase flow phenomena are very common in petroleum exploration and development, the pressure drop calculation and interpretation of production logs in oil wells require knowledge of flow pattern behavior. At present, it is fairly difficult to predict the flow patterns that will occur for a given set of flow rates and fluid properties. In early studies, Govier *et al.* [1] studied the oil/water two phase flow patterns in vertical upward pipe with inner diameter of 1.04 in. and described four flow patterns (bubble, slug, froth, mist). Vigneaux *et al.*[2] observed that the transitional flow pattern of oil/water two phase flow in vertical upward pipe with inner diameter of 20 cm occurs in water hold-up of 0.2-0.3 and no slug or froth flow patterns appear, which is similar to Zavarch *et al*'s work[3]. More recently, Flores *et al.*[4] presented a comprehensive study of oil/water two phase flow patterns in vertical and deviated pipes, a mechanism model of flow patterns transition was proposed. Even so, it is still difficult to identify the flow patterns theoretically since the mechanism of the droplets breakage-coalescence phenomena is not thoroughly clear.

Complex phenomenon and complex systems are widely existed in nature. "Complexity" is a general concept and doesn't have a uniform definition hitherto. Kolmogorov[5] proposed the "algorithmic complexity" to describe the complexity of symbol sequences. Since the Kolmogorov Complexity is difficult to calculate, some other complexity measures based on entropy were proposed from the point of view of application[11,12]. Complexity theory has been widely used in the analysis of complex process such as earthquake, finance, physiological signals etc. In China, Jinghua Xu [6,7] *et al.* used complexity measures to study the EEG time series signals. In multiphase flow study, Huang Chunyan *et al.* [8] adopted complexity measures to analysis pressure fluctuation signals from a gas-solid fluidized bed, the complexity parameters were believed to be a new technique for flow regime identification.

In ours previous work [9], the flow patterns of oil/water two phase flow in vertical upward pipe have been characterized by the analysis of Fractal, Chaos and Kolmogorov Entropy using the conductance time series fluctuating signals. Some fairly good conclusions have been derived. In this paper, complexity theory, as another important nonlinear time series analysis method, is used to characterize the oil/water two-phase flow patterns. It is shown that the new tool is a potential sensitive 'indicator' of flow pattern transition.

2. Data acquisition

The test was carried out on the flow loop of Daqing Production Logging Institute. The scheme of the data acquisition system is shown in Fig. 1. The system [10] mainly comprised an inflatable packer, emitting and measuring electrodes, and data processing unit. In operation, the inflatable packer was filled with wellbore fluids by an electromagnetic vibration pump, and the fluids (oil/water mixture) were forced into the port below the packer and passed through the test section with an inner diameter of 18 mm. The test section is equipped with four ring-type stainless steel measuring electrodes mounted flush on the inside wall of the insulated pipe through which the oil/water mixture flows. The top and the bottom electrodes emit constant amplitude of alternating current. From the two pairs of measuring electrodes the fluctuating signals are extracted whose amplitudes are relative to the conductivity of the oil/water mixture. A total of 16 conductance fluctuating signals were acquired under the following flow conditions (Fig. 2):

Water cut: 51%~91%
Total flow rate: 10~60 (m^3/day)
Oil density: 0.82 (g/cm^3)
Water density: 1.00 (g/cm^3)
Oil viscosity: 3.26 (centipoise)

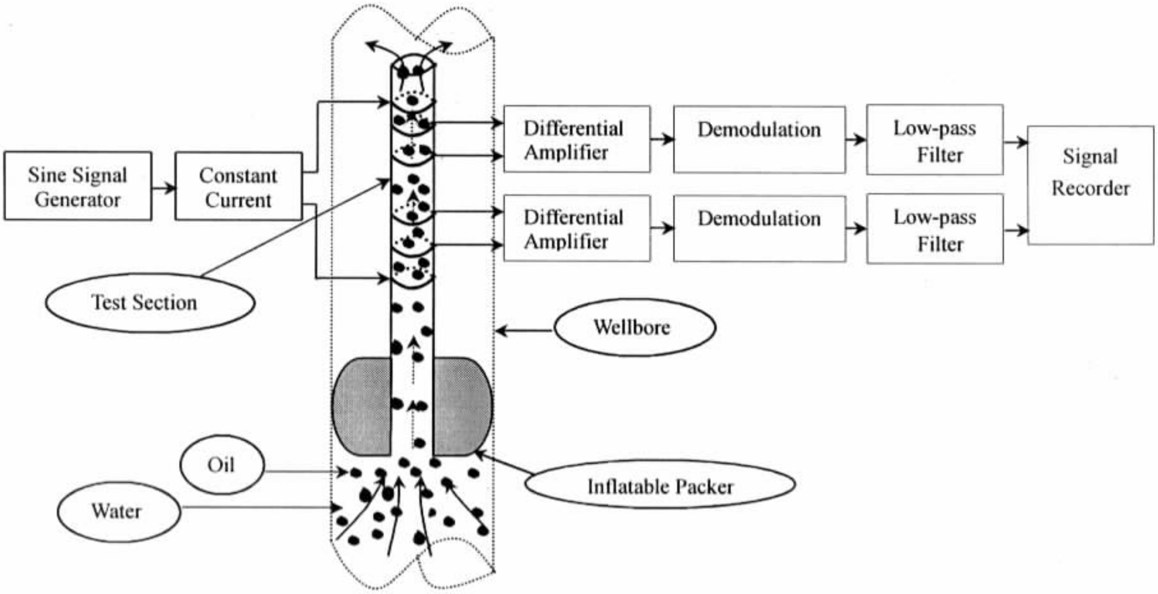

Fig.1 Data acquisition system

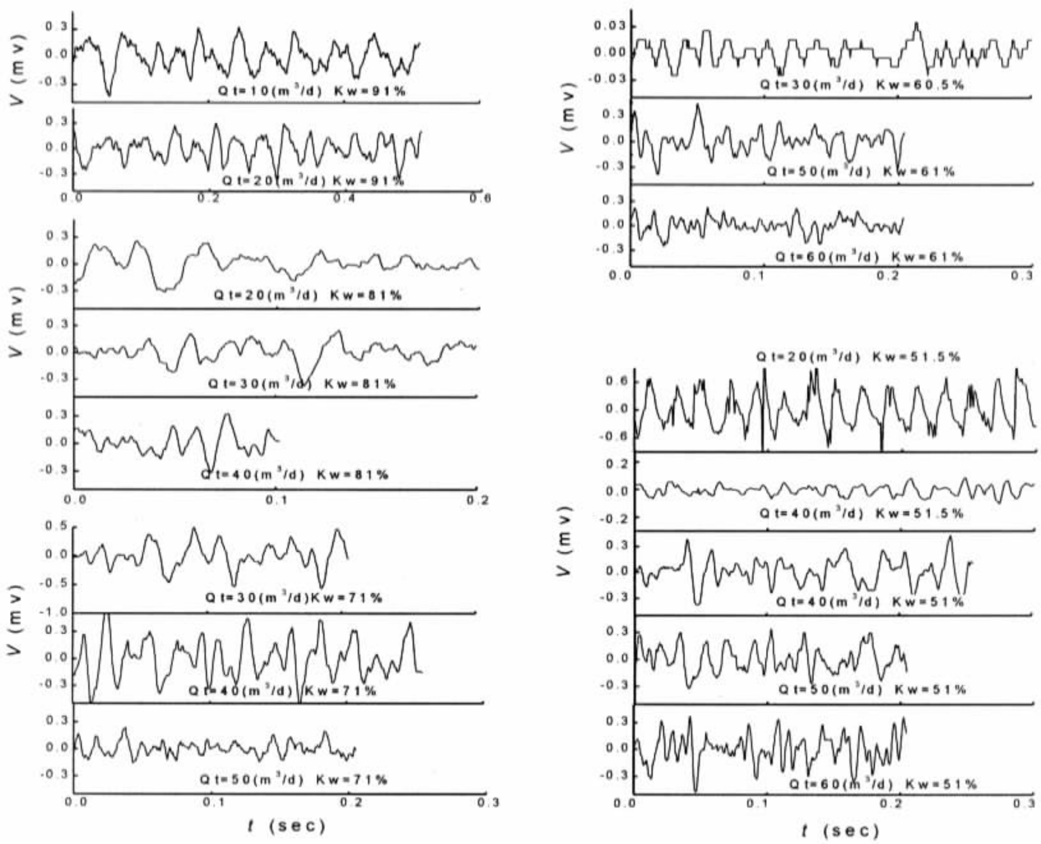

Fig.2 Conductance fluctuating signals of oil/water two phase flow at different flowing conditions

3. Complexity Measures

Complexity is the impersonal measurement of the complex degree of an object. The concept of "Complexity" usually denotes the complex degree of a dynamical system. But the complexity of single sequences that is to say the description of complex degree to a time series is studied relatively much and it indirectly express the complex degree of the dynamical system it comes from. Measures of complexity attempt to capture some features of data sets that make them more 'irregular' or 'random' than others. Kolmogrorov[5] laid one of the cornerstones of complexity theory when he proposed the algorithmic complexity of a binary string to be the shortest "program" that could elicit the string.

3.1. LZ Complexity

Since the Kolmogorov complexity is not computable, Lempel and Ziv [11] suggested a computable measure of Kolmogorov complexity $c(n)$ for sequences of finite length allows only two operations: to copy and to insert. The *LZ* complexity links the complexity of a specific sequence to the gradual buildup of new patterns along the given sequence. It turns out to be a measure of randomness .

The calculation of $c(n)$ proceeds as follows:

For a given time series, symbolize it first over a finite alphabet A to acquire a string denoted by $s_1 s_2 \cdots s_n$. In this paper, the alphabet of the string are 2 symbols (0,1) and 4 symbols (0,1,2,3) that are derived by dividing original data evenly. We adopted 4 symbols partition for the sake of avoiding excessive coarse may be occurred in only 2 symbols partition.

Scanning the known string from left to right. The string up to s_r will de denoted by $S = s_1 s_2 \cdots s_r \bullet$, where the dot indicates that s_r is newly inserted (it was not obtained by simply copying it from $s_1 s_2 \cdots s_{r-1}$). Taking $Q = s_{r+1}$ and judge whether Q is one of the substrings of $SQ\pi$ where $SQ\pi$ denotes the string which is composed of S and Q, and π means that the last digit has to be deleted. The string Q can contain two ($Q = s_{r+1} s_{r+2}$) or more elements. If s_{r+1} can be copied from the vocabulary of S, then next judging whether $Q = s_{r+1} s_{r+2}$ is contained in the vocabulary of $SQ\pi$ and so on until Q becomes so large that it can no longer be obtained by copying a word from $SQ\pi$ and one has to insert a new digit and add a 'dot' after it. Repeat the above process until the last digit of the given string. The *LZ* complexity $c(n)$ is defined as the number of 'dot' (the number of newly inserted digits).

It has been demonstrate that along with $n \to \infty$ the complexity $c(n)$ trends to the same value:

$$\lim_{n\to\infty} c(n) = b(n) = \frac{n}{\log_2 n} \tag{1}$$

So define the asymptotic complexity:

$$C(n) = \frac{c(n)}{b(n)} = \frac{c(n)}{n/\log_2 n} \tag{2}$$

The function above is usually adopted to express the complexity's changes of finite sequences. From it we can see that for a completely random sequence the value of $C(n)$ trends to "1" and it trends to "0" for an orderly period one. In order to acquire stable result, the length of the sequence must at least be 10^3~10^4.

The calculation method of LZ complexity based on 4 symbols is similar to that of based on 2 symbols. Its asymptotic complexity is defined as:

$$C(n) = \frac{c(n)}{b(n)} = \frac{c(n)\log_2 n}{n \log_2 N_1^G} \tag{3}$$

where N_1^G denotes the base of alphabet which create the symbol sequence "S" from original data.

3.2. Fluctuation Complexity

Bates and Shepard [12] introduced information fluctuation as a measure of complexity. They related complexity to the computational power of a dynamical system. As a new information measure of complexity, it reflects the fact that genuinely interesting complex behavior lies between the extremes of order and disorder, where both influences combine to permit higher levels of computation.

Let p_i denote the probability that state i is the present-state and p_{ij} the probability that a transition from state i to state j occurs. The forward conditional transition probability $p_{i\to j}$ means that if i is the present-state, j will be the next-state with probability $p_{i\to j}$. Similarly the reverse conditional transition probability $p_{i\leftarrow j}$ means that if state j is the present-state, the state i was the prior-state with probability $p_{i\leftarrow j}$. Then

$$p_{ij} = p_i p_{i\to j} = p_{i\leftarrow j} p_j . \tag{4}$$

The information gain G_{ij} associated with a transition from state i to state j is defined as:

$$G_{ij} = \log(1/p_{i\to j}). \quad (5)$$

The information loss L_{ij} is similarly:

$$L_{ij} = \log(1/p_{i\leftarrow j}). \quad (6)$$

The net information gain Γ_{ij} is just:

$$\begin{aligned}\Gamma_{ij} &= G_{ij} - L_{ij} = \log(p_{j\to i}/p_{i\to j}) \\ &= \log(p_i/p_j) = I_j - I_i\end{aligned} \quad (7)$$

where I is the Shannon information.

Then the mean of the net information gain over all transitions is:

$$\langle\Gamma\rangle = \sum_{ij} p_{ij}\Gamma_{ij}. \quad (8)$$

Because of $\sum_j p_{i\to j} = 1$, it can be derived that $\langle\Gamma\rangle = 0$. So the mean square deviation of Γ,

$$\sigma_\Gamma^2 = \langle(\Gamma - \langle\Gamma\rangle)^2\rangle = \langle\Gamma^2\rangle - \langle\Gamma\rangle^2 = \langle\Gamma^2\rangle \quad (9)$$

need not be zero. σ_Γ captures the degree of variability and hence the fluctuation in the relative dominance of chaos or order as the system evolves in time. Consequently this variable can be defined as a measure of complexity:

$$C_f = \sigma_\Gamma^2 = \langle\Gamma^2\rangle = \sum_{i,j=1}^{N} p_{ij}(\lg\frac{p_i}{p_j})^2 \quad (10)$$

Calculating method of fluctuation complexity :

From equation (10), we can see that C_f can be computed from state probabilities and the transition probabilities alone. In this paper, for the sake of calculating the probability of each state, we adopt the equidistant cells method to divide the original data.

1. Compute the maximum value x_{max} and the minimum value x_{min} of the time series $X = \{x_1, x_2, ..., x_N\}$, then divide X into n equidistant cells. The length of each cell is $L = (x_{max} - x_{min})/n$.

2. Count the number of x_k in each cell and then divide it by n as the evaluation of p_i , the probability of each state. The transition probability p_{ij} can be derived as follows: count the number event that x_k is in the i th cell and simultaneously x_{k+1} in the j th cell, the divide the number of such event by n as the evaluation of transition probability p_{ij}.

3. Compute C_f according to equation (10) and the calculated probabilities.

3.3 Spectral Entropy [13]

The concept of entropy originated from statistical thermodynamics. Entropy expresses the inordinate degree of a system. Shannon entropy uses the form of entropy to denote the even uncertainty of the information source. A variety of spectral transforms exist but Fourier transform is probably the most well known. Application of Shannon's entropy gives an estimate of the spectral entropy of processes.

For a given time series $\{x_i\}$, its Fast Fourier Transform (FFT) is $X(\omega)$, So its power spectrum is:

$$S(\omega) = \frac{1}{2\pi N}|X(\omega)|^2 \quad (11)$$

The Spectral Entropy is defined as:

$$H_f = -\sum_{i=1}^{N} p_i \log p_i \quad (12)$$

where p_i represents the probability at frequency f. the entropy has been interpreted as a measure of uncertainty about the event at f. Spectral entropy computes the uncertainty quantitatively of a system in frequency domain. Thus it may be used as a measure of system complexity. High uncertainty (entropy) is due to a large number of processes, whereas low entropy is due to a small number of dominating processes that make up the time series.

3.4 An example for complexity computation

We use three common signals (sine signal, Lorenz signal and white noise) to compare the three complexity parameters. The Lorenz equation to acquire the chaotic time series signal is:

$$\begin{cases}\frac{dx}{dt} = -10(x-y) \\ \frac{dy}{dt} = -y + 28x - xz \\ \frac{dz}{dt} = xy - \frac{8}{3}z\end{cases} \quad (13)$$

The initial value is (2,2,20). We adopted x series to compute, the length N of the x series is 10000.

From table 1, it is shown that the complexity measures of white noise have high complexity values. In contrast, the sine signal gives low values in complexity measures. The chaotic Lorenz signal is between that of sine signal and white noise. This shows that all the three complexity measures consider that periodic signal is the simplest whereas completely random signal is the most complex.

To Lempel and Ziv complexity, the values of sine signal trend to 0 whereas the values of white noise which are completely random trend to 1. This can validate the theory of Lempel and Ziv.

Table 1 Three complexity measures comparison for different type signals

		Sine signal	Lorenz signal	White noise
LZ complexity $C(n)$	Based on 2 symbols	0.01063	0.067767	1.02714
	Based on 4 symbols	0.009966	0.114939	0.975318
Fluctuation Complexity C_f		0.005054	0.040924	1.01763
Spectral Entropy H_f		2.04944	6.41416	9.27623

4. Results

The calculating results to the conductance time series fluctuating signals of 16 different flow conditions using LZ complexity and fluctuation complexity and spectral entropy measures are shown on Fig. 3, 4, 5, 6.

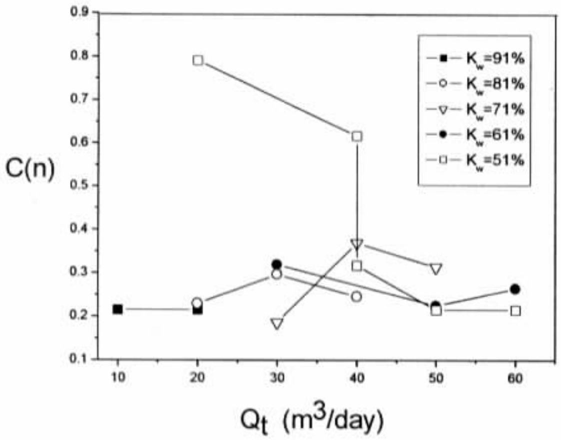

Fig. 3 *LZ* complexity based on 2 symbols *versus* total flowrate *Qt* and water cut *Kw*

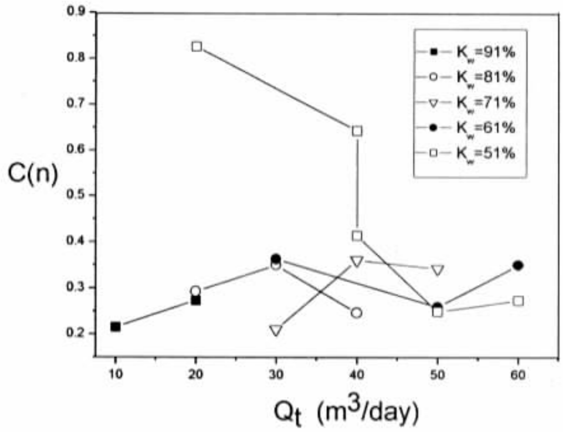

Fig. 4 *LZ* complexity based on 4 symbols *versus* total flowrate *Qt* and water cut *Kw*

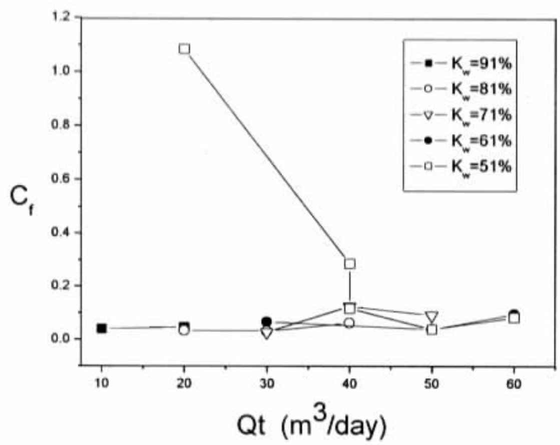

Fig. 5 Fluctuation complexity *versus* total flowrate *Qt* and water cut *Kw*

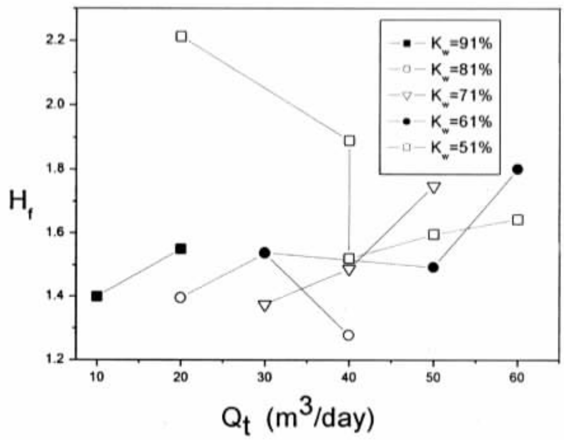

Fig. 6 Spectrum entropy *versus* total flowrate *Qt* and water cut *Kw*

For $61\% \le K_w \le 91\%$, in our previous study[9], we have proved that the flow conditions belong to the oil-in-water flow pattern. From the present calculated three complexity measures, they show little changes with the total flowrate Q_t and are not sensitive to the oil in water flow pattern variations.

For K_w=51%, the three complexity measures show irregular sudden changes with the total flowrate Q_t. we have known that such flow conditions belong to the oil/water transitional flow pattern (oil in water or water in oil) and are strongly supported by the characteristics of complex power spectrum [9]. It is shown that the measures of complexity are sensitive to transitional flow pattern.

We conclude that the three complexity measures are potential sensitive indicators of flow patterns variations.

5. Conclusions

Results from this study, it is shown that the complexity measures analysis can be used to characterize the oil/water two phase flow pattern transition, the transitional flow pattern corresponds to the irregular changes of the LZ Complexity and Fluctuation Complexity and Spectrum Entropy. We conclude that complexity time series analysis can be a useful tool for flow pattern identification in vertical upward oil/water two phase flow pipes.

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63