

Assessment of Reproducibility and Linearity of the NMIA Bell Prover using a High Flowrate Sonic Nozzle Array

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Abstract: Critical flow Venturi, or sonic, nozzles are widely used in gas flow metrology as stable and convenient transfer standards. Once the nozzle coefficient has been determined by calibration against a primary standard, the mass flowrate can be determined by knowledge of the upstream gas composition and its pressure and temperature. As these can be determined with great precision, the mass flow achieved can be more reproducible than the nozzle calibration. At NMIA, sonic nozzles used to cover the range 1 to 25 m³/h are calibrated against a bell prover, with an uncertainty of 0.13% ($k=2$). Calibration of the nozzles in additive combination, using a sonic-nozzle array developed at NMIA, allows estimation of the non-linearity of the bell prover primary standard at its maximum flow. In this paper, extensive measurements of the reproducibility of the bell prover calibration of sonic-nozzles are used to identify correlated uncertainty terms in the nozzle calibration, which inturn used to assess the non-linearity as 0.07% for the highest flows, with a reduced standard uncertainty of 0.015%.

Keywords: Gas Flow, Bell Prover, Uncertainty Analysis, Sonic Nozzle Array, Linearity, Reproducibility

1. Introduction

Sonic nozzles operate by allowing a stream of gas to flow through a small orifice and expand rapidly into a region of much lower pressure. At a sufficiently high pressure drop, the increase in gas velocity as it passes through a gradually constricting throat region of the orifice leads to the formation of a shock-wave when the gas speed is equal to the speed of sound. This decouples the flow in the region downstream of the nozzle from the region upstream, and the flowrate is fully determined only by the gas composition and its inlet temperature and pressure. Sonic nozzles are often characterised by a quantity called the nozzle coefficient, defined [1] as:

$$N(p_N) = \frac{Q_m}{\sqrt{p_N \rho_N}} \quad \text{and} \quad Q_m = Q_V \rho_{Bell} \quad (1)$$

where

Q_m : Calculated mass flowrate from the bell prover

Q_V : Measured volume flow from the bell prover

ρ_{Bell} : Density of air inside the bell prover calculated from measurements of the bell's pressure, temperature and relative humidity

p_N : Upstream pressure of the sonic nozzle

ρ_N : Density of air calculated from measurements of the nozzle's upstream pressure, temperature and relative humidity

The nozzle coefficient N has the dimensions of area, and in the limiting case of a large nozzle for which boundary layer effects are negligible, is exactly the area of the most constricted region of the nozzle. Ideally, the nozzle coefficient is independent of the upstream gas composition, pressure and temperature, although boundary layer effects, turbulence and non-ideal-gas effects make it a weak function of these parameters. It is common practice to combine these into a single number characterising the flow, the Reynolds number, and to make the nozzle coefficient a function of this parameter. Nozzles are then calibrated at a number of upstream pressures, corresponding to a range of Reynolds numbers, allowing a user to accurately determine the mass flowrate under a range of upstream conditions. For example, for a toroidal-throat type nozzle, a rise of 1.8% in the nozzle coefficient is expected for an increase in the Reynolds number from 2.1×10^4 to 3.2×10^7 [2].

2. The Bell Prover at NMIA

At NMIA, a commercial SINGER 300 L bell prover is used as the primary standard in the measurement of the gas flow, up to 24 m³/h using dry air at atmospheric pressures and room temperatures. The commercial system was extensively modified [3,4] to improve its performance. Some of these modifications were:

- Numerous calibrated PRT probes, providing point measurement of the air temperature with uncertainties less than 0.02 °C within the bell volume and in the gas stream adjacent to the sonic nozzles.
- A laser interferometer to monitor the displacement of the bell.
- Precision pressure transducers (Paroscientific model 745-16B) providing less than 2 Pa pressure uncertainty in the upstream sonic-nozzle gas pressure.
- A thorough survey of the axial deviations of the bell cross sectional area [3].

Traditionally, bell provers can provide a measurement uncertainty of around 0.5%; however, the modifications discussed above reduce this to 0.13% ($k=2$). NMIA's participation in the CCM.FF-K6 key comparison [5] showed agreement of NMIA's results with the key comparison reference value to around 0.05%.

At NMIA the usual procedure for the calibration of a sonic nozzle is to perform 10–15 descents of the bell. When sonic nozzles are used to calibrate rotary flow meters at high flowrates, small pressure drops can occur across the meter, which is usually upstream of the nozzle. At NMIA, nozzles are calibrated at a number of pressures around atmospheric pressure, by slightly throttling the flow between the nozzle and the bell to generate pressure differences up to 10 kPa.

The uncertainty analysis for the mass flowrate from the bell prover has been given in [3,4] and is reproduced here in Table 1.

Table 1 Standard uncertainty components associated with measuring mass flowrate using the bell prover at NMIA

Standard Uncertainty Components	Flow Uncertainty
Bell Pressure Transducer Calibration ($< \pm 1.0$ Pa at 101 kPa)	0.001%
Bell Pressure Variation (± 1.2 Pa at 101 kPa)	0.001%
Bell Temperature Sensor Calibration (± 0.005 °C)	$< 0.002\%$
Bell Temperature Uniformity (± 0.11 °C)	0.037%
Bell Relative Humidity ($\pm 0.5\%$ at 2–3%RH air)	0.004%
Bell Volume	0.050%
Time (sampling rate limit better than 100kHz)	$< 0.003\%$
Storage Effect	0.010%
Nozzle Pressure Transducer Calibration (± 1 Pa), p_N	0.001%
Nozzle Pressure Variation (~ 10 Pa), p_N	0.005%
Nozzle Temperature Sensor Calibration (± 0.005 °C)	0.01%
Nozzle Temperature Uniformity ($< \pm 0.01$ °C)	$< 0.02\%$
Nozzle Relative Humidity ($\pm 0.5\%$ at 2–3%RH air)	0.002%
Total Standard Uncertainty	0.065%

The two major contributions to the uncertainty in mass flowrate at the sonic nozzle are the bell's volume and its temperature uniformity. However, it is important to note that if the same system is used to (a) calibrate nozzles repeatedly or (b) calibrate these nozzles in combination, then many of these components will be fully or partially correlated, and the uncertainty in the difference between two calibrations will be substantially lower than simply estimated by combining two sets of data each with an uncertainty of 0.065%.

3. Repeatability Testing

Eight sonic nozzles, constructed from 316 grade stainless steel, have been calibrated using NMIA's standard calibration methods against the bell prover (Table 2). The calibration of the set of nozzles was repeated several times; each nozzle received four calibrations spaced over a year. Figure 1 shows the relative deviation of these four calibrations from the mean value, together with the standard error of the mean of the 10–15 descents of the bell prover contributing to each calibration (or data point in Figure 1). In this figure, the nozzle coefficients have been corrected back to a common standard pressure using the measured pressure dependence of the nozzle coefficient for each nozzle. This correction varied from 0.02% to 0.06% per 1 kPa for the different nozzles, and is a necessary correction, as atmospheric air pressure varies by approximately ± 1 kPa at the Lindfield laboratory.

It is evident that the four calibrations for each nozzle vary (a) much less than the estimated calibration standard uncertainty of 0.065% and (b) more than the ESDM (typically 0.002%) of each data point. It is interesting to note that similar variability results are obtained for all eight nozzles, covering a range from 2 to 9 m³/h.

Table. 2 Sonic nozzles used to test the reproducibility and linearity of the NMIA bell prover.

Nozzle Serial No.	Nozzle Coefficients, N (m^2)	Approximate Volumetric Flowrate for dry air at 101 kPa and 21 °C (m^3/h)
B2	2.492×10^{-6}	2.60
B4	2.236×10^{-6}	2.34
B5	2.207×10^{-6}	2.31
C3	4.411×10^{-6}	4.61
C4	4.349×10^{-6}	4.35
C5	4.425×10^{-6}	4.61
D3	8.233×10^{-6}	8.60
D4	8.501×10^{-6}	8.91

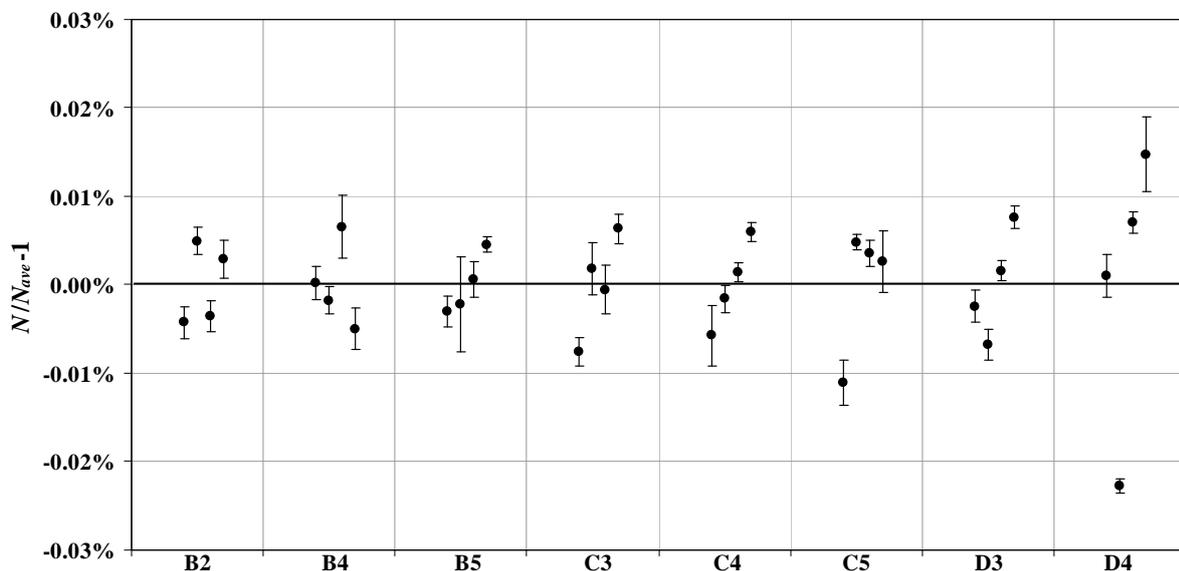


Fig. 1 Plots of the four calibrations of the set of eight sonic nozzles, compared to the average for each nozzle. The error bars represent relative standard deviations obtained from the 10–15 bell descents contributing to each calibration.

We can conclude that some of the uncertainty components estimated in Table 1 are likely to be highly correlated between measurements made by the NMIA nozzle calibration facility. Clearly any error in the bell volume is expected to be fully correlated between calibrations. The uncertainties related to the calibration of, and measurements from, the pressure, temperature and humidity sensors will be partially correlated; however, their contribution is negligible.

The other major term, the temperature uniformity of the gas in the bell, is also partially correlated. It arises mainly from the spatial variations in room temperature, which varies by no more than 0.4 °C throughout the laboratory. As the laboratory is temperature controlled with a stability of better than ± 0.1 °C, 24 hours a day, and no significant changes to the heat-load in the laboratory occurred over the 1 year measurement period, it is expected that any such gradients will be fairly constant between the 32 calibrations here (i.e. a systematic but repeatable error).

However, as it is difficult to determine the exact degree of correlation, it was decided to use the actual dataset to estimate the magnitude of the uncorrelated calibration uncertainty. The Birge ratio, given by [6]:

$$R_B = \sqrt{\sum_{i=1}^n \left[\frac{(x_i - x_w)^2}{u^2(x_i)} \times \frac{1}{(n-1)} \right]} \quad (2)$$

where

n : Number of tests conducted

x_i : Results of the measurements conducted here (for $n=1$ to 4)

x_w : Weighted mean

$u(x_i)$: Repeatability of each test, or the standard deviation of each set of measurements

provides a measure of the consistency of a set of data x_i with individual uncertainty estimates $u(x_i)$ of each data point. If the observed spread of the measured data x_i is consistent with the estimated uncertainties, $u(x_i)$, then a value of $R_B \leq 1$ is expected. If the Birge ratio is significantly less than 1, then the uncertainties have been overestimated. Figure 2 shows the Birge ratio calculated for each of the eight nozzles, as a function of an additional uncertainty added in quadrature to the measured standard error of the mean (ESDM) for each measurement. Using the uncertainty estimate in Table 1, the Birge ratio is typically 0.1, and has a maximum of 0.24. From Figure 1, if we assume an uncorrelated uncertainty of 0.015% together with the measured ESDM, the data will be consistent with the calculated total uncertainties. This value of 0.015% represents both the long term stability of the nozzles and bell system and the calibration reproducibility. The quadrature sum uncertainty may be taken as an experimentally derived estimate of the contributions of the uncorrelated components in Table 1.

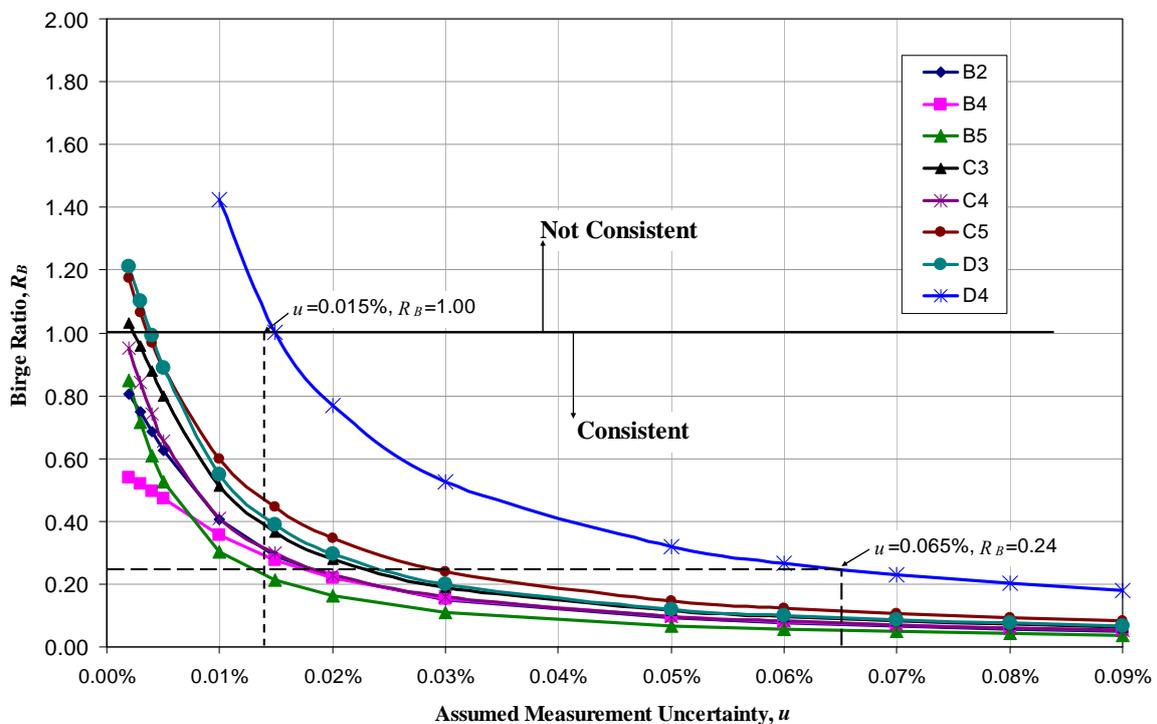


Fig. 2 Plot of the Birge Ratio, R_B , for the four calibrations of each nozzle as a function of the assumed uncertainty arising from the reproducibility of the measurement system.

4. Linearity Testing

As bell provers provide only transient and intermittent sources of known flows which are unsuitable for the calibration of mass-flowmeters and rotary flowmeters, at NMIA such devices are calibrated against a “sonic nozzle array” (Figure 3). This array consists of two chambers between which a number of calibrated nozzles can be activated under computer control. The chambers ensure that the upstream pressure and temperature is the same for all of the nozzles in the array, and that the downstream pressure is below 30 kPa (absolute) downstream of all nozzles, even when nozzles are switched in combination.

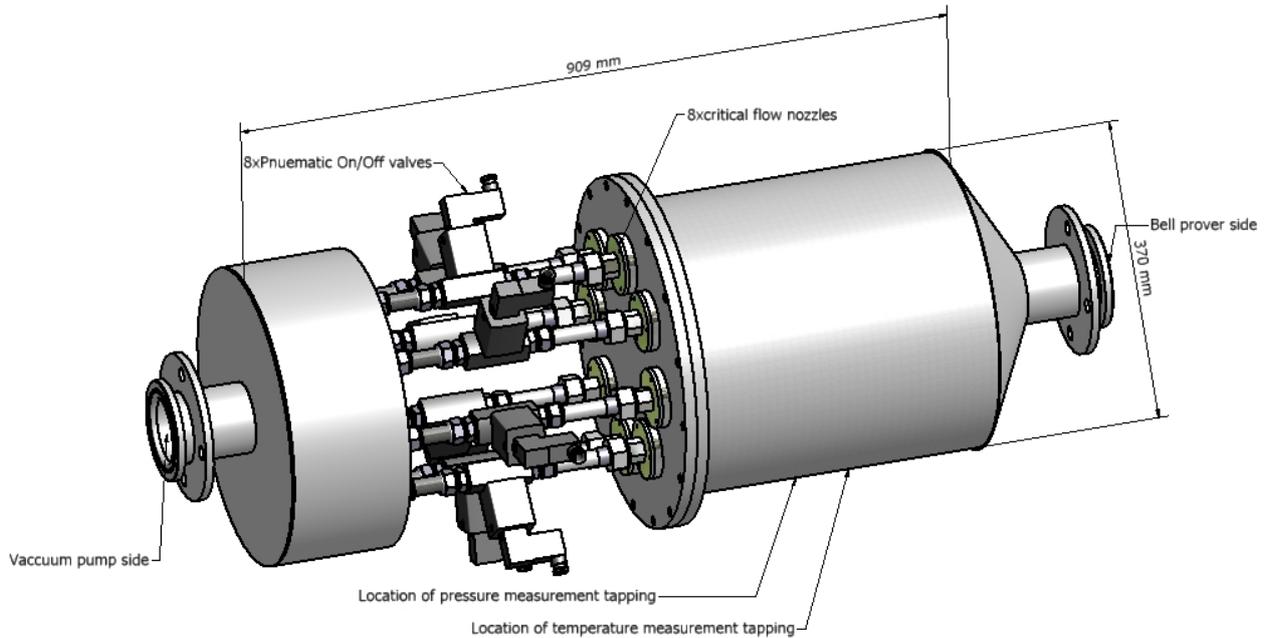


Fig. 3 The nozzle array used in the linearity testing of the bell prover at NMIA.

As the bell prover is a mechanical device, characterised under stationary conditions, some level of non-linearity may be expected if it is used with larger flowrates. In particular, issues such as non-equilibrium of the temperature within the bell, or inadequate or incomplete wetting of the surface of the descending bell may give rise to flowrate errors not fully accounted for in the calibration uncertainty analysis. The sonic-nozzle array used for flowmeter calibration has been used to assess any such non-linearity experimentally. For example, two nozzles with a nominal flowrate of 2 m³/h can be individually calibrated against the bell and the sum of these calibrations compared to the calibration of their parallel combination i.e. 2+2=4. More generally, we define

$$D_Q = \frac{\sum Q_k - Q_{sum}}{Q_{sum}} \quad (3)$$

where

Q_k : Mass flowrate calculated from the individual calibration of a single nozzle

Q_{sum} : Mass flowrate, measured by the bell prover, of two or more nozzles in parallel

Values of D_Q greater than zero imply that the bell is delivering a lower mass flowrate than that calculated from its displacement, pressure and temperature.

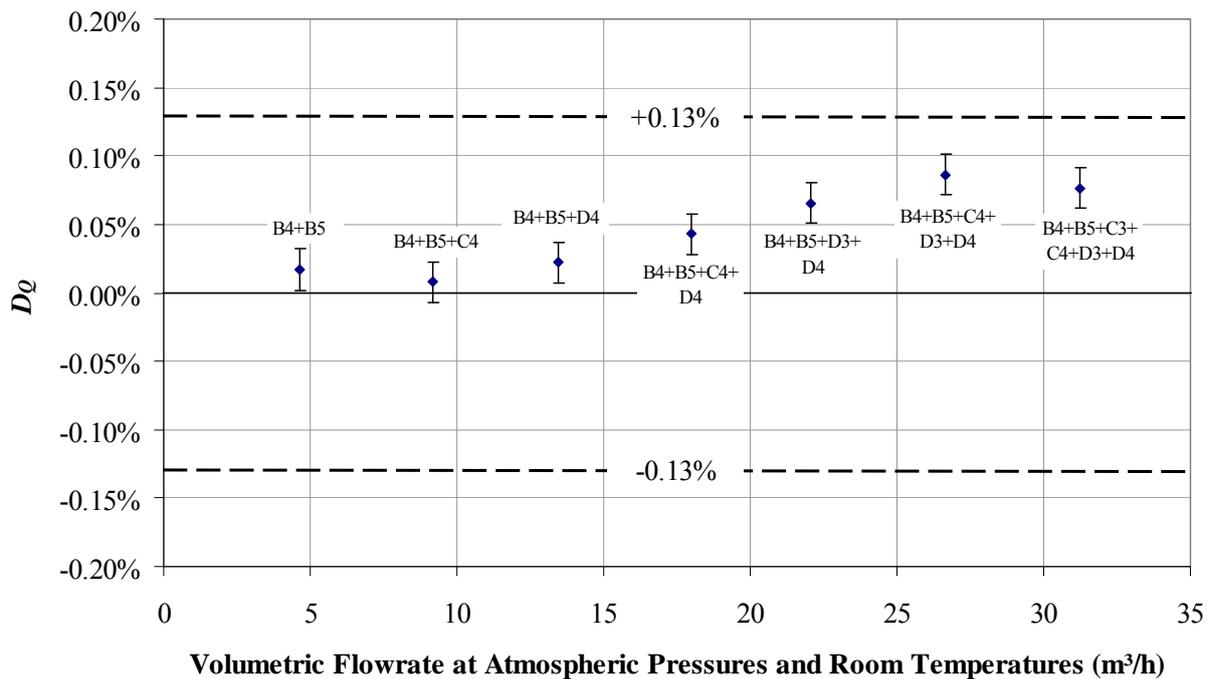


Fig. 4 Plot of the deviation from linearity for the NMLA bell prover as a function of the volumetric flowrate. Note the error bars represent the repeatability of the calibrations (not including the systematic contributions from the bell volume or temperature uniformity). The calibration uncertainty of 0.13% (at $k=2$) is also shown (dotted lines).

Figure 4 shows the value of D_Q for a number of nozzle combinations up to 32 m³/h, together with the uncertainties resulting from the combined ESDM and the 0.015% calibration reproducibility. At flowrates below about 10 m³/h no nonlinearity is evident, indicating that the calibrations of the “B” (2 m³/h), “C” (4 m³/h) and “D” (9 m³/h) type nozzles are unaffected by any nonlinearities present in the system. However in the region above 15 m³/h, the bell appears to deliver slightly less, about 0.07%, than calculated. This non-linearity is statistically significant compared to the calculated uncertainty. At the maximum flowrate of 32 m³/h, the measured pressure drop between the nozzles and the bell is 40 Pa, which corresponds to a temperature drop due to adiabatic expansion of approximately 0.12 °C. However, it should be noted that as the temperature and pressure are measured both in the bell and immediately upstream of the sonic nozzle, and appropriate corrections applied, this should not contribute to any measured flow discrepancy.

5. Discussion

As the bell is used in descending mode, oil film effects on the inner bell surface can be neglected, and it is unlikely that inadequate wetting of the descending bell could explain the 0.07% (i.e. nearly 0.2 mm). If the measured flow discrepancy is assumed to arise solely from errors in the measurement of the temperature of the gas within the bell, this would imply that the true average temperature of the gas dispensed by the prover was either (a) 0.2 °C *higher* than indicated by the PRT probes at the higher flowrates or (b) 0.2 °C *lower* than indicated by the PRT probes at the lower flowrates. This is consistent with the value of u_i of 0.11 °C ($k=1$) taken in Table 1 obtained from extensive measurements of the temperature uniformity of the air volume within the prover under static conditions. It would appear that at the higher flowrates the temperature variations in

the air within the bell are mixed differently compared to that at lower flowrates, although no direct experimental evidence for this hypothesis is available. It should be noted that both the bell and laboratory gradients are transverse as well as vertical, so the effects are not solely due to simple stratification within the prover volume.

It is interesting to note that despite the relatively rapid air circulation of 2–3 m³/s within the laboratory and the large mass of the prover, the temperature gradients in the prover volume (as inferred from the measured nonlinearity) are similar to those of the laboratory (i.e. they are *not* significantly smaller).

5. Conclusions

A thorough assessment of the repeatability and reproducibility of sonic nozzle calibrations by the NMIA bell prover has been performed, giving a surprisingly low value of 0.015% ($k=1$) for this quantity. Together with NMIA's sonic nozzle array, this reproducibility has allowed the estimation of the non-linearity of the bell prover at high flowrates. The maximum measured non-linearity of 0.07% is consistent with the temperature variation within the bell prover, though further investigations are required to understand the flowrate dependence of this systematic error.

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