# Optimization of geometric parameters of the rotor in the turbine flowmeter

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**Abstract:** A new method is presented in this paper for improving the linearity of the turbine flowmeter by optimizing the geometric parameters of the rotor. An objective function is defined to improve the linearity. Four characteristic parameters: the ratio of the blade tip space to pipeline radius, the ratio of the rotor hub radius to the blade tip radius, the fitted angle of the blade at the mean square root radius and the cascade solidity of the blade tip, are used to optimize the rotor design according to the n dimension extremum complex configuration optimization method and the turbine flowmeter's mathematic model. The newly designed optimal rotors of 50mm and 25mm bore turbine flowmeter were manufactured and evaluated on a water flow calibration facility. The linearity of the optimized flowmeters is found to be 0.3657% and 0.283% respectively.

**Keywords:** Turbine flowmeter, Linearity, Optimization, Rotor

**Nomenclature**

$\tau$ Ratio of the blade tip space to pipeline radius
$\theta$ Ratio of the rotor hub radius to the blade radius
$\bar{\beta}$ Blade fitted angle at the position of the mean square root radius
$\sigma$ Cascade solidity of tip blade
$N$ Number of blades
$R_t$ Tip radius of the blade
$R_h$ Rotor hub radius
$L$ Lead of the helical blade
$t_b$ Thickness of the blade
$\gamma$ Bevel edge angle of the blade
$L_h$ Rotor hub length in axial direction
$R_0$ Meter bore radius
$\beta$ Blade fitted angle
$\bar{r}$ Mean square root radius of the blade
$T_d$ Rotor driving torque
$T_b$ Journal bearing retarding torque
$T_h$ Rotor hub retarding torque
$T_m$ Magnetoelectricity detector retarding torque
$T_t$ Blade tip retarding torque
$T_w$ Both hub disks retarding torque
$\rho$ Fluid density
$V_z$ Axial component of absolute velocity at position of $r$
$r$ Radius of the blade element

$c$ Chord of blade at position of $r$
$\beta_\infty$ Angle between average flow direction and meter axis
$C_L$ Lift coefficient
$C_{DS}$ Drag coefficient
$C_D$ Local drag coefficient of a blade
$s$ Rotor blade spacing
$\theta_{TS}$ Momentum thickness of boundary layer at suction side of blade
$\theta_{TP}$ Momentum thickness of boundary layer at pressure side of blade
$\delta^*_{TS}$ Displacement thickness of boundary layer at suction side of blade
$\delta^*_{TP}$ Displacement thickness of boundary layer at pressure side of blade
$U_{\infty C}$ Mean flow velocity relative to blade calculated by airfoil theory ignoring the effect of the boundary layer
$\beta_{\infty C}$ Angle between the mean flow velocity direction and the meter axis ignoring the effect of the boundary layer
$\nu$ Fluid kinematic viscosity
$\beta_1$ Angle made by inlet velocity with the meter axis
$\beta_2$ Angle made by the exit velocity with the meter axis considering the effect of the boundary layer
$\omega$ Rotor rotational speed
$R$ Position of the sources and sinks in conformal mapping
$\alpha$ Angle in potential flow solution
$AR$ Blade aspect ratio
$c_h$ Chord at the root of blade
$c_t$ Chord at the tip of blade
$R_1$ Rotor axis radius
$R_2$ Journal bearing inner radius
$L_b$ Length of friction part between rotor axis and journal bearing
$V_{zh}$ Axial component of absolute flow velocity at hub
$t_{bh}$ Thickness of blade at rotor hub
$\beta_{\infty h}$ Angle between average flow velocity direction and meter axis at rotor hub
$t_{bt}$ Thickness of blade tip
$Q$ Volume flow rate
$K$ Meter factor of turbine flow meter, 1/L
$J$ Objective function
$\delta_1$ Linearity
$n$ Number of measurement times at one test point
$K_i$ Meter factor at one test point, 1/L
$K_i'$ Meter factor of a single measurement at one test point
$K_{imax}$ Maximum of meter factors among all test points
$K_{imin}$ Minimum of meter factors among all test points

## 1. Introduction

The rotor is an essential component of the turbine flowmeter. The structure and geometric parameters of the rotor have direct effects on the performance of the flowmeter. The linearity of the turbine flowmeter is one of significant performance parameters [1], which affects the measurement accuracy of the meter. The optimization of geometric parameters of the rotor is a key approach to improving the performance of the turbine flowmeter. There have been reported activities in the field to optimize the turbine flowmeter. Blows [2] reported that the optimized design of the turbine flowmeter body extended significantly the linear working range of the meter. Zhao [3] used the critical Reynolds number to optimize the characteristic curves of the turbine flowmeter to increase the ratio of the measurement range. Wu [4] introduced a parameter called velocity difference factor which was used to analyzed and optimized the turbine flowmeter. Salami [5] proposed a turbine flowmeter that could be easily manufactured once the key geometric specifications were given; it was claimed that the accuracy can be guaranteed in any flow situation. However, all the above studies have only resulted in some qualitative conclusions and a quantitative method for optimizing the rotor geometric structure has never been reported to date. This paper presents a novel method of improving the linearity of the turbine flowmeter by optimizing the geometric parameters of the rotor.

## 2. Rotor characteristic parameters

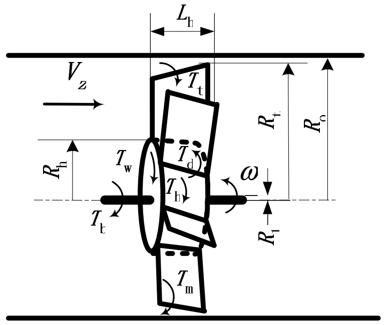


*Figure 1. Main parameters of a rotor and associated torques*

Rotor structure can be described by a number of geometric parameters. For a helical rotor blade, the main geometric parameters are the number of blades($N$), tip radius of blade($R_t$), rotor hub radius($R_h$), lead of helical blade($L$), thickness of blade($t_b$), bevel edge angle of blade($\gamma$) and rotor hub length($L_h$). The performance curves of the turbine flowmeter vary with the changes of rotors' geometric parameters [6-9]. Figure 1 illustrates the dimensional parameters of the rotor (the number of blades $N$ is excluded). In order to increase the versatility of the method, dimensionless parameters were introduced and applied to analyze the influence of the rotor's geometric parameters on meter performance [10, 11].

### 2.1. The ratio $\tau$ of the blade tip space to the pipeline radius

For a rotor as show in Figure 1, if $R_0$ is fixed, the blade tip space is determined by $R_t$. The blade tip space decides the amount of fluid flowing through the gap (which puts influence on the fluid component of acting on the blade) and the retarding torque between the blade tip and the internal surface of meter bore and the turn speed of the rotor. Naturally, the meter performance will depend on $\tau$. Some researchers reported that suitable blade tip space could reduce turbine flowmeter's sensitivity to viscosity variation [12-14]. The ratio $\tau$ of blade tip space to pipeline radius is defined to describe the blade tip space as follows:

$$\tau = \frac{R_0 - R_t}{R_0} \tag{1}$$

### 2.2. The ratio $\theta$ of the rotor hub radius to the blade radius

If $R_t$ is fixed, the value of $\theta$ affects a number of parameters including the height of blade in radial direction, fluid velocity through the surface of the blade and the retarding torque and driving torque on the blade. Ultimately, the value of $\theta$ will affect the meter performance. Some recent work has shown that a suitable ratio of the rotor hub radius to the blade radius can reduce turbine flowmeter's sensitivity to viscosity change [5, 14, 15]. $\theta$ is therefore selected as one of the characteristic parameters and is defined as follows:

$$\theta = \frac{R_\mathrm{h}}{R_\mathrm{t}} \tag{2}$$

### 2.3. The blade fixed angle $\overline{\beta}$ at the mean square root radius

For a helical rotor blade, the blade fitted angle $\beta$ at different radial positions depends on the lead of helical blade $L$, which puts influence on the value of lift force and drag force that the fluid acts on the blade, the force's axial component and tangential component on the blade's surface and the turn speed of the rotor. Because of the change of $\beta$ at different radial position, the mean value of $\beta$ ( $\overline{\beta}$ ) is selected as a characteristic parameter of the rotor, which can be presented as:

$$\overline{\beta} = tg^{-1}\left(\frac{2\pi\overline{r}}{L}\right) \tag{3}$$

$$\overline{r} = \sqrt{(R_\mathrm{t}^2 + R_\mathrm{h}^2)/2}$$

Where $\overline{r}$ is mean square root radius of the blade.

### 2.4. The cascade solidity $\sigma$ at blade tip

The overlap degree of the rotor blade is related to the number of blades $N$, rotor hub length in axial direction $L_\mathrm{h}$ and blade fitted angle $\beta$. Normally, the rotor blade is expanded into cascade. Cascade solidity [11] is often used to describe the overlap degree of rotor blade. However, the cascade solidity is changing along the radius of the rotor blade. Thus, $\sigma$ is selected as a characteristic parameter, which is defined as:

$$\sigma = \left(\frac{c}{s}\right)_\mathrm{t} = \frac{NL_\mathrm{h}}{D_\mathrm{t}}\left(\frac{1}{\pi\cos\beta_\mathrm{t}}\right) \tag{4}$$

Where $NL_h / D_t$ is the cascade solidity parameter, $D_\mathrm{t}$ is the diameter at the blade tip, and $\beta_\mathrm{t}$ is the fixed angle of the blade tip.

## 3. Constraints of characteristic parameters

The typical values of the parameters are listed in Table 1, but Table 1 has the values used by others.

*Table 1. Classical values of the characteristic parameters*

| | Diameter $D$ (mm) | $\delta$ (%) | $\theta$ | $\overline{\beta}$ (°) | $\sigma$ |
|---|---|---|---|---|---|
| Tianjin instruments Group Co.,LTD | 10 | 5 | 0.4421 | 42.71 | 0.3914 |
| | 15 | 2.6667 | 0.5137 | 41.37 | 0.7809 |
| | 50 | 4 | 0.5354 | 37.61 | 0.4414 |
| Xu, Y.[16] | 101.6 | 2.0079 | 0.4950 | 42.38 | 1.2540 |
| Tsukamoto[10] | 36.576 | 1.5311 | 0.5388 | 43.65 | 1.1710 |
| Lee[17] | 422.0 | 2.2569 | 0.6081 | 50 | 1.21 |
| Jepson[18] | 50.8 | 0.5906 | 0.1259 | 35.33 | 0.2961 |
| | 50.8 | 4.4094 | 0.1259 | 34.30 | 0.3021 |

For optimizing the geometric parameters of the rotor, the constraints of the four selected characteristic parameters have to be defined. Nicholl [19] used 30° as the blade fitted angle of a turbine flowmeter. Liang [20] concluded that the blade fitted angle at the mean square root radius should be 10° ~ 15° for a gas turbine flowmeter and 30°~45° for a liquid turbine flowmeter. Salami [5, 14] reported that the commercial turbine flowmeter, with a ratio of the rotor hub radius to the blade radius of 0.5 and a ratio of the blade tip space to the pipeline radius of 10%, was not sensitive to the uniform velocity profile or turbulent velocity profile. Tsukamoto [10] suggested that the blade tip space to pipeline radius should be 3% when $\theta$ =0.5. Liang [20] indicated that: the blade tip space should be 0.05$D$～0.07$D$ when the pipeline diameter $D$ was less than 10 mm, 0.01$D$～0.015$D$ when $D$ was from 10 mm to 80 mm, and 0.01$D$ when $D$ was more than 80 mm. Baker [21] considered that a suitable value of $\theta$ is 0.5. Rubin [11] studied the effect of changing $\theta$ on solidity parameter of blade and concluded that $\theta$ should be between 0.4 and 0.8. Liang [20] also suggested that the range of the overlap degree of rotor blade in axial direction was from 0.9 to 1.2, however, there was no quantitative equation to calculate the overlap degree. Rubin [11] proposed that the cascade solidity of blade should be more than 2.53 if the fluid was wholly led through the meter in the process of calculating torques on rotor with momentum method.

In consideration of the classical values of the four characteristic parameters listed in Table 1 and the views of other researchers as stated above, the upper and lower limits of the constraint conditions of the parameters was equal to 110% of the maximum values and 90% of the minimum values of each group of parameters. These limits are $0.5\%\le\delta\le15.5\%$, $0.1\le\theta\le0.8$, $25°\le\bar{\beta}\le55°$, $0.26\le\sigma\le1.4$. It is believed that the constraints of the four characteristic parameters are wide enough to include the optimal geometric parameters of the rotor.

## 4. Mathematic model of the turbine flowmeter

Referring to Figure 1, the equation for turbine flowmeter performance under stable working conditions is developed by balancing the torques on the rotor:

$$T_{\mathrm{d}} - T_{\mathrm{b}} - T_{\mathrm{h}} - T_{\mathrm{m}} - T_{\mathrm{t}} - T_{\mathrm{w}} = 0 \qquad (5)$$

Where $T_{\mathrm{d}}$ is the rotor driving torque, $T_{\mathrm{b}}$ journal bearing retarding torque, $T_{\mathrm{h}}$ rotor hub retarding torque due to fluid drag, $T_{\mathrm{m}}$ magnetoelectricity detector retarding torque, $T_{\mathrm{t}}$ blade tip retarding torque, and $T_{\mathrm{w}}$ both hub disks retarding torque.

Based on the airfoil theory and the application of boundary layer theory in turbomachinery [22], $T_{\mathrm{d}}$ can be presented as follows:

$$T_{\mathrm{d}} = \tfrac{1}{2}\rho N \int_{R_{\mathrm{h}}}^{R_{\mathrm{t}}} \frac{r V_z^2 c}{\cos\beta_\infty}(C_{\mathrm{L}} - tg\beta_\infty C_{\mathrm{DS}})dr \qquad (6)$$

Where $\rho$ is the fluid density, $N$ number of rotor blades, $R_{\mathrm{t}}$ rotor tip radius, $R_{\mathrm{h}}$ rotor hub radius, $r$ radius at differential blade elements, $V_{\mathrm{z}}$ axial component of absolute velocity at radius of $r$, $c$ chord of blade at radius of $r$, $\beta_\infty$ angle between average flow velocity direction and meter axis, $C_{\mathrm{L}}$ theoretical lift coefficient of a blade, and $C_{\mathrm{DS}}$ the sum of drag coefficient.

According to the application of boundary layer theory in turbomachinery [22] $C_{\mathrm{L}}$ and $C_{\mathrm{D}}$ can be calculated by:

$$C_{\mathrm{L}} = \frac{s}{c}\left(2\delta_u \cos\beta_\infty + \xi_V \cos\beta_\infty \sin\beta_\infty\right),\quad C_{\mathrm{D}} = \frac{s}{c}\xi_V \cos^3\beta_\infty$$

$$C_{\mathrm{DS}} = C_{\mathrm{D}} + C_{\mathrm{Di}}$$

Where $C_{\mathrm{D}}$ is local drag coefficient of a blade, $s$ is rotor blade spacing, and $C_{\mathrm{Di}}$ is drag coefficient due to finite wing spread, secondary fluid loss and blade tip clearance drag.

Based on airfoil theory and its application in turbomachinery [23], $C_{\mathrm{Di}}$ is:

$$C_{\mathrm{Di}} = \frac{C_{\mathrm{L}}^2}{\pi(AR)} + 0.04 C_{\mathrm{L}}^2 \sigma \frac{s}{R_{\mathrm{t}} - R_{\mathrm{h}}} + \frac{1}{4} C_{\mathrm{L}}^2 \sigma \frac{R_{\mathrm{o}} - R_{\mathrm{t}}}{R_{\mathrm{t}} - R_{\mathrm{h}}} \frac{1}{\cos\beta_2}$$

Where first item is the effect of finite wing spread, second item is the effect of secondary fluid loss, the last item is the effect of blade tip clearance drag.

Where

$$\xi_V = \frac{2\theta}{\cos^2\beta_{2\text{corr}}}, \quad \delta_u = (1+\Delta^* - \theta)tg\beta_{2\text{corr}} - tg\beta_1$$

$$\Delta^* = \frac{\delta^*_{\text{TS}} + \delta^*_{\text{TP}}}{s\cos\beta_{2\text{corr}}} \quad \theta = \frac{\theta_{\text{TS}} + \theta_{\text{TP}}}{s\cos\beta_{2\text{corr}}}$$

Where $\beta_{2\text{corr}}$ is angle made by the exit velocity with the meter axis calculated by airfoil theory ignoring the effect of the boundary layer. $\theta_{\text{TS}}$ is momentum thickness of boundary layer at suction side of blade, $\theta_{\text{TP}}$ is momentum thickness of boundary layer at pressure side of blade, $\delta^*_{\text{TS}}$ is the displacement thickness of boundary layer at suction side of blade, and $\delta^*_{\text{TP}}$ is the displacement thickness of boundary layer at pressure side of blade.

Where

$$\begin{cases} \theta_{\text{TS}}/c = \theta_{\text{TP}}/c = 0.664R_c^{-\frac{1}{2}} \\ \delta^*_{\text{TS}}/c = \delta^*_{\text{TP}}/c = 1.721R_c^{-\frac{1}{2}} \end{cases} \left(R_c < 2.5\times10^5\right) \quad \begin{cases} \theta_{\text{TS}}/c = \theta_{\text{TP}}/c = 0.0463R_c^{-0.2} \\ \delta^*_{\text{TS}}/c = \delta^*_{\text{TP}}/c = 0.036R_c^{-0.2} \end{cases} \left(R_c \geq 2.5\times10^5\right)$$

$$R_c = \frac{U_{\infty c}c}{\nu}, \quad U_{\infty c} = \frac{V_z}{\cos\beta_{\infty c}\left(1 - t_{\text{b}}/\left(s\cos\beta_{\infty c}\right)\right)}, \quad tg\beta_{\infty c} = \frac{tg\beta_1 + tg\beta_{2\text{corr}}}{2}$$

Where $U_{\infty\text{C}}$ is the mean flow velocity relative to blade calculated by airfoil theory ignoring the effect of the boundary layer, $\beta_{\infty\text{C}}$ is the angle between the mean flow velocity direction and the meter axis ignoring the effect of the boundary layer, $\nu$ is fluid kinematic viscosity, and $\beta_1$ is angle made by inlet velocity with the meter axis.

$$tg\beta_\infty = \frac{tg\beta_1 + tg\beta_2}{2}$$

$$tg\beta_2 = (1+\Delta^* - \theta)tg\beta_{2\text{corr}}, \quad tg\beta_1 = \frac{r\omega}{V_z}, \quad tg\beta_{2\text{corr}} - tg\beta_1 = \frac{2q}{1+q}\left(\frac{2\pi r}{L} - \frac{r\omega}{V_z}\right), \quad q = \frac{2R}{R^2+1}\cos\alpha$$

$$\begin{cases} tg\alpha = (tg\beta)\dfrac{R^2-1}{R^2+1} \\ \dfrac{s}{c} = \dfrac{1}{\pi}\left\{\cos\beta\ln\left(\dfrac{R^2+2R\cos\alpha+1}{R^2-2R\cos\alpha+1}\right) + 2\sin\beta\left(tg^{-1}\dfrac{2R\sin\alpha}{R^2-1}\right)\right\} \end{cases}, \quad tg\beta = \frac{2\pi r}{L}, \quad c = \frac{L_{\text{h}}}{\cos\beta} - \left(r - R_{\text{h}}\right)\cdot tg\left(\frac{\pi}{2} - \gamma\right)$$

$$AR = \frac{R_{\text{t}} - R_{\text{h}}}{c_{\text{a}}}, \quad c_{\text{a}} = \frac{c_{\text{t}} + c_{\text{h}}}{2}, \quad \sigma = \frac{c}{s}, \quad s = \frac{2\pi r - Nt_{\text{b}}}{N}$$

Where $\beta_2$ is angle made by the exit velocity with the meter axis considering the effect of the boundary layer, $\omega$ is rotor speed of a real meter, $R$ is position of the sources and sinks in conformal mapping, $\alpha$ is angle in potential flow solution, $AR$ is blade aspect ratio, $L_{\text{h}}$ is length of rotor hub, $c_{\text{a}}$ is average chord of blade, $c_{\text{h}}$ is chord at the root of blade, $c_{\text{t}}$ is chord at the tip of blade, and $\sigma$ is solidity ratio.

According to the accurate solution of Navier-Stockes equation to the problem of steady flow in two turning coaxial cylinders [24], $T_{\text{b}}$ can be presented as:

$$T_{\text{b}} = \frac{4\pi R_1^2 R_2^2}{R_2^2 - R_1^2} L_{\text{b}}\rho\nu\omega \tag{7}$$

Where $R_1$ is the rotor axis radius, $R_2$ is the journal bearing inner radius, and $L_{\text{b}}$ is the length of friction part between the rotor axis and the journal bearing.

Based on the research of fluid drag due to skin friction on a flat plate in the same direction as a flow, $T_{\text{h}}$ is expressed as:

$$T_{\text{h}} = \frac{1}{2}\rho V_{z\text{h}}^2 A_{\text{h}} R_{\text{h}} C_{\text{h}} \frac{tg\beta_{\infty\text{h}}}{\cos\beta_{\infty\text{h}}} \tag{8}$$

$$C_{\mathrm{h}} = 1.328 R_{e\mathrm{h}}^{-\frac{1}{2}} \quad (R_{e\mathrm{h}} < 2.5\times 10^5), \quad A_{\mathrm{h}} = 2\pi R_{\mathrm{h}} L_{\mathrm{h}} - N t_{\mathrm{bh}} c_{\mathrm{h}}, \quad R_{e\mathrm{h}} = \frac{U_{\infty\mathrm{h}} c_{\mathrm{h}}}{\nu}$$
$$C_{\mathrm{h}} = 0.074 R_{e\mathrm{h}}^{-0.2} \quad (R_{e\mathrm{h}} > 2.5\times 10^5)$$

Where $V_{\mathrm{zh}}$ is axial component of absolute flow velocity at hub, it can be regarded as $V_{\mathrm{z}}$, $U_{\infty\mathrm{h}}$ is average flow velocity relative blade at hub, $\beta_{\infty\mathrm{h}}$ is angle between average flow velocity direction and meter axis at hub, $t_{\mathrm{bh}}$ is thickness of blade at hub.

The value of $T_{\mathrm{m}}$ is dependent on the parameters of the magnetoelectricity detector. For a specified design of the turbine flowmeter, $T_{\mathrm{m}}$ can be regarded as a constant:

$$T_{\mathrm{m}} = \mathrm{Const} \tag{9}$$

According to the equation of blade tip retarding torque by Tuskamoto [10], $T_{\mathrm{t}}$ can be presented as:

$$T_{\mathrm{t}} = \frac{1}{2}\rho(\omega R_{\mathrm{t}})^2 c_{\mathrm{t}} R_{\mathrm{t}} t_{\mathrm{bt}} N C_{\mathrm{Dt}} \tag{10}$$

$$C_{\mathrm{Dt}} = 2/\mathrm{Re}_{\mathrm{t}} \quad (\mathrm{Re}_{\mathrm{t}} < 1000), \quad \mathrm{Re}_{\mathrm{t}} = \omega R_{\mathrm{t}}(R_{\mathrm{o}} - R_{\mathrm{t}})/\nu$$
$$C_{\mathrm{Dt}} = 0.016 \big/ \mathrm{Re}_{\mathrm{t}}^{0.25} \quad (\mathrm{Re}_{\mathrm{t}} > 1000)$$

Where $t_{\mathrm{bt}}$ is thickness of blade tip

According to the accurate solution of Native-Stokes equation to the problem of retarding torque to surface friction of turning round disks in a flow [24], $T_{\mathrm{w}}$ is:

$$T_w = \tfrac{1}{2}\rho\omega^2 R_h^5 C_M \tag{11}$$

$$C_M = 3.87 R_w^{-\frac{1}{2}} \quad (R_w < 3\times 10^5), \quad R_w = \frac{R_h^2 \omega}{\nu}$$
$$C_M = 0.146 R_w^{-0.2} \quad (R_w > 3\times 10^5)$$

Equations (5)-(11) form the mathematical model of the turbine flowmeter. In the model, there is not any parameters need to be regulated by person or depend on the experimental data of the meter. The variable $\omega$ in equation (1) was solved with the binary search method [25]. The definite integral in equation (2) was solved by Legendre-Gaussian quadrature method [25].

According to the definition of meter factor $K$ [1], The value of meter factor $K$ is dependent on the rotor rational speed $\omega$ and volume flow rate $Q$. $K$ can be expressed as:

$$K = \frac{\omega * N}{Q} * 3.6 \tag{12}$$

## 5. Objective function and optimization algorithm

### 5.1. Objective function

In this paper, the aim of optimizing the geometric parameters of the rotor is to reduce the meter's linearity and then improve the meter's accuracy. The linearity is expressed using the as following equations [1, 26]:

$$J = \mathrm{Min}\{\delta_1\} \tag{13}$$

$$\delta_1 = \frac{K_{\mathrm{Max}} - K}{K}\times 100\%, \; K = \frac{K_{\mathrm{Max}} + K_{\mathrm{Min}}}{2}, \; K_{\mathrm{Max}} = \mathrm{Max}\{K_i\}, K_{\mathrm{Min}} = \mathrm{Min}\{K_i\}, \; K_i = \sum K_i'/n$$

Where the meter factor $K_{\mathrm{i}}$ is obtained according to equation (12)

From equation (13), better linearity can be obtained when objective function $J$ is minimized. Then, the optimum values of the characteristic parameters of the rotor can be derived.

### 5.2. Optimization algorithm

From the characteristic parameters and their constraint conditions and objective function, it can be seen that the problem of optimizing rotor's geometric parameters is to obtain the four-dimensional minimum solution under the variable constraint conditions. So the complex configuration optimization algorithm [25] under *n* dimensions constraint conditions is selected as the optimization algorithm. The flowchart of calculation is shown in Figure 2.

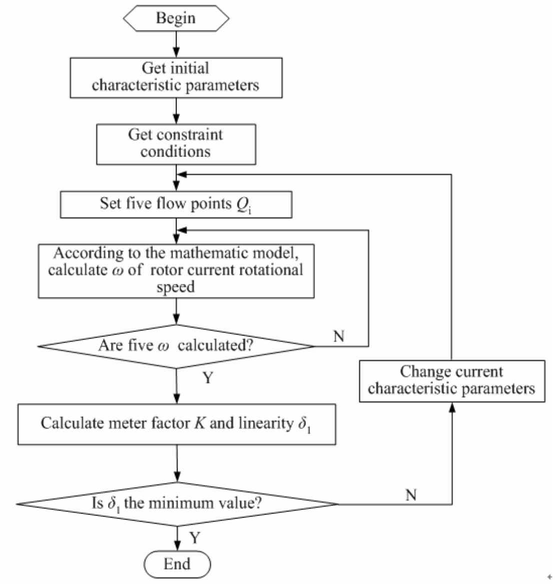


*Figure 2. Flowchart of calculation*

## 6. Results and discussion

### 6.1. Optimization results

The measurement ranges of existed 50mm and 25mm caliber turbine flowmeter is from 4 to 40 $m^3/h$ and 1 to 10 $m^3/h$. The main geometric parameters of the two existing rotors are listed in Table 2. The characteristic parameters of the two existing rotor are listed in Table 3.
According to the optimization method mentioned above, the characteristic parameters of existed rotor were optimized, and the optimal characteristic parameters are listed in Table 3. The calculated linearity of existed rotor and optimal rotor are also listed in Table 3.
According to the characteristic parameters of optimal rotor listed in Table 3, the parameters of optimal rotors were calculated inversely. The main parameters of optimal rotor are listed in Table 2 (The blank location in Table 2 mean that the value is same to the existing rotor).

*Table 2. Main geometric parameters of the two existing rotors*

| Geometric parameters | DN50 | | DN25 | |
|---|---|---|---|---|
| | Existing rotor | Optimized rotor | Existing rotor | Optimized rotor |
| $N$ | 6 | | 6 | |
| $L$ /m | 0.157 | 0.212975 | 0.05 | 0.06294 |
| $R_1$ /m | 0.0015 | | 0.00095 | |
| $R_2$ /m | 0.00151 | | 0.00105 | |
| $L_b$ /m | 0.0045 | | 0.004 | |
| $L_h$ /m | 0.008 | 0.0062 | 0.0075 | 0.00679 |
| $t_b$ /m | 0.0008 | | 0.0004 | |
| $t_{bh}$ /m | 0.0008 | | 0.0011 | |
| $c_h$ /m | 0.009 | | 0.0075 | |
| $R_t$ /m | 0.024 | 0.02425 | 0.01195 | 0.01203 |
| $R_h$ /m | 0.01275 | 0.013338 | 0.00525 | 0.005471 |
| $b_h$ /m | 0.0045 | | 0.004 | |

*Table 3. Characteristic parameters and calculated linearity of the existing rotor and optimized rotor*

| Characteristic parameters and $\delta$ | DN50 | | DN25 | |
|---|---|---|---|---|
| | Existing rotor | Optimized rotor | Existing rotor | Optimized rotor |
| $\tau$ /% | 4.72 | 3.01 | 4.4 | 3.8 |
| $\theta$ | 0.5247 | 0.55 | 0.4393 | 0.455 |
| $\overline{\beta}$ /° | 37.28 | 30.1 | 47.216 | 43 |
| $\sigma$ | 0.3995 | 0.3 | 0.5543 | 0.66 |
| $\delta_1$ /% | 0.6151 | 0.353 | 0.5481 | 0.2466 |

### 6.2. Experimental evaluations

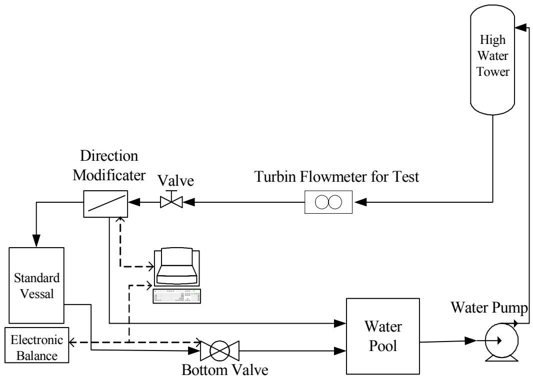


*Figure 2. The schematic of the water flow calibration facility*

Figure 3 shows the schematic of the water flow calibration facility. The pressure in the pipe was steady in nature as it was originated from a high water tower (height of the tower is 32 m). The accuracy of the electronic balance is ±0.0167%. The accuracy of the overall calibration facility is found to be ±0.0692%. Weighing method is adopted at the water flow calibration facility [1]. According to the national standard [26], the meter factor $K$, the linearity $\delta_1$, and the repeatability $\delta_2$ of the turbine flowmeter are calculated and listed in Table 4. The $K$-$Q$ performance curves of different rotors are plotted in Figure 4 and Figure 5. Repeatability $\delta_2$ is calculated as follows:

$$\delta_2 = MAX\left[\frac{1}{K_i}\left[\frac{1}{n-1}\sum_{j=1}^{n}(K_{ij}-K_i)^2\right]^{1/2}\times 100\%\right] \tag{14}$$

*Table 4. Experimental results of the existing rotor and optimized rotor*

| Experimental results | DN50 | | DN25 | |
|---|---|---|---|---|
| | Existing rotor | Optimized rotor | Existing rotor | Optimized rotor |
| Linearity $\delta_1$ /% | 0.5944 | 0.3657 | 0.5017 | 0.2830 |
| Repeatability $\delta_2$ /% | 0.1032 | 0.1041 | 0.0767 | 0.0631 |

A comparison between the experimental linearity (Table 4) and the calculated linearity (Table 3) for both the existing rotors and the optimized rotors indicates that the experimental and modeling results have the same trend and agree well in general. According to Table 4, the linearity of the two optimized rotors has been found to be 0.3657% and 0.283%. Compared with the two existing rotors, the linearity of the two optimized rotors was reduced about 38.48% and 43.59% respectively, and the repeatability of the optimized rotors was not changed much. The two optimized rotors are clearly better linearity than the existing rotors.

It is evident that the method for optimizing rotor's geometric parameters has improved the linearity of the two turbine flowmeter.

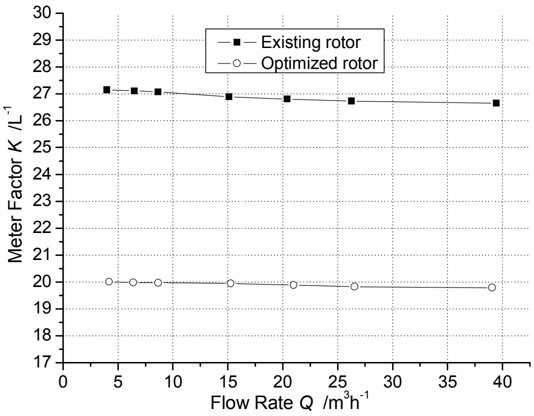


*Figure 4. K-Q performance curve of optimized rotor and existing rotor (DN50)*

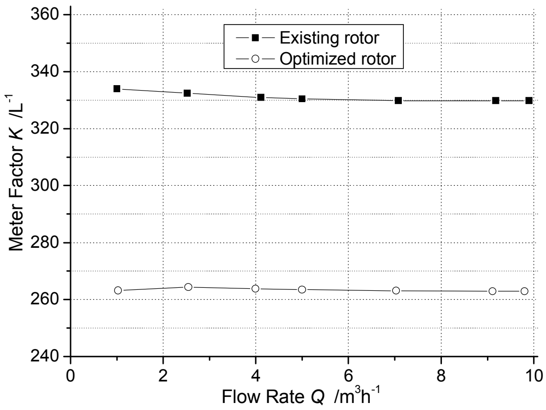


*Figure 5. K-Q performance curve of optimized rotor and existing rotor (DN25)*

## 7. Conclusions

This paper has described a method for optimizing the geometric parameters of a turbine flowmeter's rotor for better linearity. The linearity of the turbine flowmeter was selected as an objective function in the optimization process. Based on the mathematic model of the turbine flowmeter and *n* dimension extremum complex configuration optimization algorithm, new characteristic parameters of optimal rotors are achieved, which are $\tau$ =3.01, $\theta$ =0.55, $\overline{\beta}$ =30.1°, $\sigma$ =0.3(DN 50) and $\tau$=3.8, $\theta$ =0.455, $\overline{\beta}$ =43°, $\sigma$ =0.66(DN 25). The direct comparison between the experimental results of the optimized rotors and the existing rotors has verified the validity of the method. The linearity of the two optimized rotors has been found to be 0.3657% and 0.283%. Compared with the existing rotors, the repeatability of the optimized rotors was not changed much.

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