

A general approach using the thermodynamic efficiency method for absolute flow measurements

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Abstract

Manufacturers of large-scale hydraulic machines face high liquidated damages if they can not meet performance guarantees due to contractual stipulations. Accurate determination of efficiency, power and flow is therefore of major interest. For plants exceeding heads of 100 m, the thermodynamic method represents the standard method. We present a mathematical refinement that uses additional power measurements to yield an analytical solution of the flow. Under favorable conditions, the measurement uncertainty is lower compared to that of other absolute flow measuring methods. With higher heads an even lower uncertainty can be expected.

1. Introduction

The thermodynamic method represents the principal method for determining the efficiency of hydraulic machines. It makes use of the first law of thermodynamics which implies energy conservation. Any mechanical and fluid friction causes energy conversion of hydraulic energy into thermal energy. That is, hydraulic losses can be determined by means of accurate measurement of water temperatures upstream and downstream the hydraulic machine. Water will be heated up by approximately 2.4 mK per 100 m of head and per 1 % loss of hydraulic efficiency. The lower the available head the larger the geometric dimensions of the hydraulic parts and the more thermometers¹ are necessary to get reliable values of the thermal distribution. That is the reason, why the applicability of this method is limited to machines operating under net heads of more than 100 m [1].

The application of the thermodynamic efficiency method was first demonstrated by Poirson and Babbilion in 1920 [2]. It could be shown that the hydraulic efficiency η_h can be determined with low uncertainty. It took more than 30 years until essential progress in theory and experiment was made by the engineers Willm and Campmas in 1954 [3]. They set the instrumental standard of thermal insulated extraction probes

for external temperature measurement which is even nowadays in use. In the view at that time, practitioners concluded that the efficiency does not change remarkably when omitting the dynamic energy term – caused by the difference in velocity between high pressure and low pressure section – since its proportion is only a few percent of the pressure term's scale. On the other hand it would be sufficient to determine the discharge Q by means of any technique or device even with high uncertainty. In the 1950s and 1960s the determination of the discharge was therefore either done by measuring the dynamic pressure [4, 5], by estimation [6, 7] or by simply omitting the discharge value [8].

Nowadays manufacturers of hydraulic turbines and pumps are facing high liquidated damages in undergoing guaranteed efficiencies by even 0.01%. Therefore the need of high accuracy discharge evaluation is eminent. It is an exciting fact that already Willm and Campmas mentioned in their publication from 1954 the potential to calculate the discharge with thermodynamic method.² It only requires the determination of the mechanical power P_m by means of generator power measurements and the relationship $P_m = (\rho Q) \cdot E_m$. Until today, we found only one more publication, Hulaas and Rasmussen in 1990 [9], where Willm's and Campmas' mode of calculation was used. However, the majority of practitioners uses the approach to calculate the discharge by means of the turbine efficiency $\eta = P/(\rho QE)$

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¹The systematic uncertainty level of the high-precision thermometers is in an order of ± 1 mK.

²They presented the special case when applying extraction probes on both pressure sections.

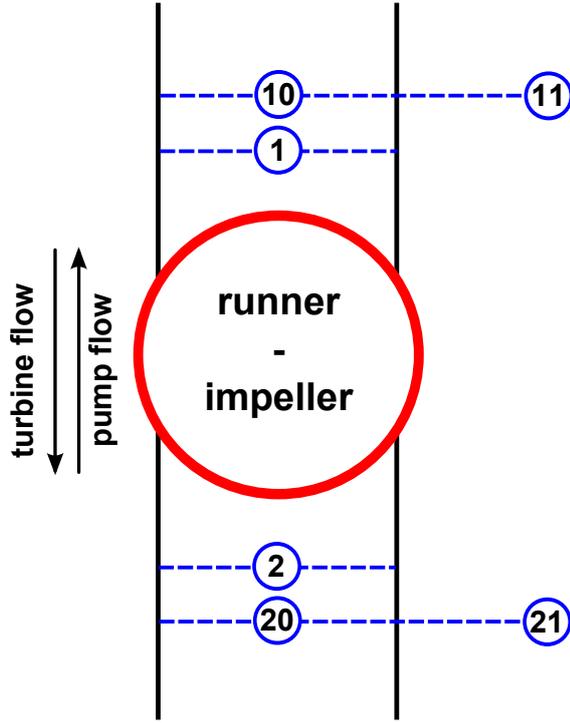


Figure 1: Measuring sections used for hydraulic machines

in iterative manner [10, 11, 12]. This proceeding provides the correct result but such an implicate formulation lacks the transparency. The widely-used test code, IEC 60041-1991, gives no exact guide lines concerning this matter and, thus, it allows some space for interpretation.

A general approach for discharge determination using thermodynamic method and power measurement is made by solving the polynomial equation $Q \cdot E_m(Q) - P_m/\rho = 0$. The order of this equation depends on the metrological type of E_m -determination, and does not exceed quartic order. The main text of this publication reveals the analytically exact solutions for various types of E_m -determination. It represents therefore an absolute discharge measuring method which has not yet found acceptance by any standard test code.

2. Methods

2.1. Efficiency calculation

In the subsequent sections it will be explained how the thermodynamic efficiency method works and how it is applied in the field. Let us start with the most general

Table 1: Nomenclature

Symbol	Description	Unit
a	Isothermal factor	m^3/kg
A_i	Area of measuring section i	m^2
c_p	Specific heat capacity	$\text{J}/(\text{kg K})$
e	Specific energy of heat exchange	J/kg
E	Specific hydraulic energy	J/kg
E_i	Specific hydraulic energy at measuring section i	J/kg
E_m	Specific mechanical energy	J/kg
$E_{m,i}$	Specific mechanical energy at measuring section i	J/kg
g	Acceleration due to gravity	m/s^2
h	Specific enthalpy	J/kg
f	Relative measurement uncertainty	%
n	Rotational speed	$1/\text{s}$
p_i	Pressure at measuring section i relative to p_{amb}	Pa
P_a	Generator/motor power measured at the generator/motor terminals	W
$P_{a,L}$	Generator/motor power losses	W
P_{Lm}	Mechanical power losses	W
P_m	Mechanical power of the runner/impeller	W
q	Extracted flow to external probe referred to conditions of reference section	$1/\text{s}$
	Specific thermal exchange energy	J/kg
Q	Discharge at reference section	m^3/s
Q'	Extracted/added discharge between measuring sections	m^3/s
s	Specific entropy	$\text{J}/(\text{kg K})$
v_i	Mean flow velocity at measuring section i	m/s
T	Torque	Nm
T_i	Water temperature at measuring section i	K
z_i	Elevation of pressure transducer p_i	m
δE_m	Exchange energy between water and environment per mass of water	J/kg
ΔE_m	Change of specific energy due to extraction/addition of water	J/kg
δP_m	Exchange power between water and environment per mass of water	W
η_h	Hydraulic efficiency	1
κ	Auxiliary parameter ($= \pm 1$, $+$ for turbines, $-$ for pumps)	1
κ_i	Auxiliary parameter, $i \in \{1, 2\}$	1
ρ	Density of water at reference section	kg/m^3
ρ_i	Density of water at section i	kg/m^3

equation for the hydraulic efficiency³

$$\eta_h = \left(\frac{E_m + \Delta E_m}{E + \Delta E} \right)^\kappa \quad (1)$$

with $\kappa = \pm 1$ and the positive sign for turbines and the negative one for pumps. E_m and E denote the difference in specific mechanical energy – considering any heat exchange with the environment – and the difference in specific hydraulic energy between the high pressure section 1 and the low pressure section 2 (view figure 1). ΔE_m and ΔE take into account any extraction and/or addition of water, e.g. cooling water for generator unit, within the measuring sections⁴ [1]. In the following treatment no water will be extracted or added which makes things much easier to work through. As a consequence of this we can rewrite equation (1) by

$$\eta_h = \left(\frac{(E_{m,1} - E_{m,2}) + \delta E_{m,1-2}}{E_1 - E_2} \right)^\kappa \quad (2)$$

$E_{m,i}$ and E_i represent the measured proportions of specific energy at section i , and $\delta E_{m,1-2}$ considers any heat exchange between water and the environment [13] within section 1 and 2. The latter is obtained by summation of contributing exchange power values $\delta E_{m,1-2} = \delta P_{m,1-2}/(\rho Q)$. In practice, the sections for the measurement of specific mechanical energy are denoted by 10 and 20 and they do not need to coincide with those ones used for the measurement of the specific hydraulic energy. Because the law of energy conservation implies $E_{m,10} = E_{m,1} + \delta E_{m,10-1}$ and, respectively, $E_{m,20} = E_{m,2} + \delta E_{m,20-2}$. The latter terms in each case may then be combined to express the corrected exchange energy term $\delta E_{m,10-20} = \delta E_{m,1-2} + (\delta E_{m,10-1} - \delta E_{m,20-2})$ and to reshape equation (2) by

$$\eta_h = \left(\frac{(E_{m,10} - E_{m,20}) + \delta E_{m,10-20}}{E_1 - E_2} \right)^\kappa \quad (3)$$

The parameters in (3) are calculated by

$$E_{m,i} = ap_i + gz_i + \frac{v_i^2}{2} + c_p T_i \quad (4)$$

$$E_i = \frac{p_i}{\rho} + gz_i + \frac{v_i^2}{2} \quad (5)$$

³A detailed derivation of the efficiency of hydraulic machines in terms of thermodynamics is given in Appendix A.

⁴The arrangement of pressure relief pipelines within high and low pressure section is often used in single-regulated reaction turbines, e.g. Francis turbines, pump-turbines and pumps. The same amount of water is extracted there in a region of high pressure and returned into lower pressure. The occurring friction losses belong to the machine's hydraulic losses. Therefore, $\Delta E_m = 0$ and only any additional heat exchange with the environment along the relief pipelines needs to be considered.

If we could measure the three parameters (p_i , v_i and T_i) of each section we would be able to determine the efficiency. It is desirable to avoid any direct measurement of the overall flow velocity $v_i = Q/A_i \cdot \rho/\rho_i$ due to tremendous efforts and low accuracy. Another challenge is to use precision thermometers in presence of flow velocities exceeding 1 m/s. Firstly, the high-sensitive measuring zone of such devices is very fragile. Secondly, a higher flow velocity causes more friction and, therefore, heating up of the thermometer which results in overestimating the actual temperature [14, 15].

2.2. Temperature measurement

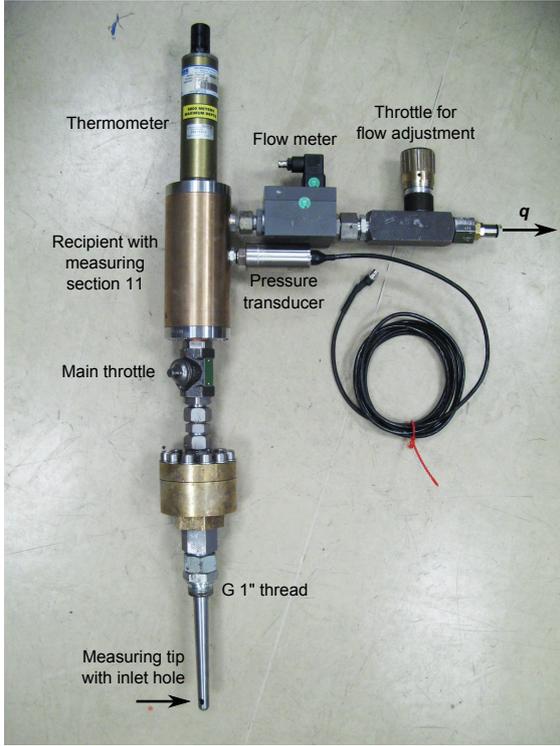
2.2.1. Exterior temperature measurement

One way out of the dilemma described in the antecedent paragraph is given by using extraction probes as depicted in figure 2(a). Here, a hollow rod of metal with a tap hole on the tip is inserted into the water-bearing system. This hole is positioned in such a manner that it is facing the main flow and that any impact of the turbulent boundary layer is avoided. Penetration depths are usually between 10 and 50 cm. Then water with $q = 0.1 \dots 0.5$ l/s is continuously extracted through that hole and led to a measuring section outside the system (section 11 for high pressure section, section 21 for low pressure section) where pressure, velocity and temperature are measured. The application of commercially available flowmeters is sufficient to determine such little flow rates. The velocity can be kept small to avoid excessive friction of the thermometer. In many cases heat exchange between extraction water and environment within sections 10 and 11 – as well as within sections 20 and 21 – can not be reduced to zero even with proper thermal insulation. Therefore, the impact has to be investigated and – in case of need – a correction has to be applied on the results. One may assume that at a certain operating point, the power of heat exchange within section 10 and 11 remains constant⁵ and is not affected by variation of extraction flow

$$P_{m,10-11} = \text{constant} = (\rho \cdot q) \cdot \delta E_{m,10-11}(q) \quad (6)$$

We see that the specific energy of heat exchange $\delta E_{m,10-11}(q)$ depends on the discharge of extraction. The sum of measured specific mechanical energy $E_{m,11}(q)$ and specific energy of heat exchange equals the correct value of the specific mechanical energy of measuring

⁵With good thermal insulation the exchange power rises up to a few watts.



(a) Exterior temperature measurement: A typical probe using continuous water extraction q is shown without thermal insulation. Temperature, pressure and velocity will be determined outside the water-bearing system.



(b) Interior temperature measurement: Here, a measuring frame is lowered down into the tailrace (measuring section 20) of a Pelton-unit. Several thermometers and pressure transducers are inserted into the vertical perforated pipes.

Figure 2: Temperature measurement in practice

section 10

$$\begin{aligned} E_{m,10} &= E_{m,11}(q) + \delta E_{m,10-11}(q) \\ &= E_{m,11}(q) + \frac{P_{em,10-11}}{\rho} \cdot \frac{1}{q} \end{aligned} \quad (7)$$

Since the slope $P_{em,10-11}/\rho$ is unknown, measurements with at least three different extraction flow q need to be done (view figure 3). A linear regression analysis will finally provide slope and offset for the following model function

$$E_{m,11}(q) = a_0 - a_1 \cdot \frac{1}{q} \quad (8)$$

where $E_{m,10} = a_0$.

The difference in specific mechanical energy can thus be calculated by (3) in using extraction probes on the high pressure and on the low pressure side and in considering (8). It is reasonable that more than one probe per section is used for bigger pipes. The downstream side requires a higher number of probes since the energy distribution there is usually more disturbed than that one

at the upstream side. The above-written procedure is known as the direct operating procedure for extraction probes and represents the principal method nowadays. The partial expansion procedure is also still in use but it has become less important due to technical improvements of absolute temperature measuring systems. The interested reader should be forwarded to Ref.[1] for a more detailed description of this procedure.

2.2.2. Interior temperature measurement

Good experience has been made by immersion of thermometers into perforated, smooth pipes on a frame which is temporarily mounted into the low pressure side (figure 2(b)). This arrangement represents the preferred alternative procedure whenever the application of extraction probes is not appropriate (large dimensions, open channels, draft tube is embedded in concrete). Such a temperature measuring section is only seldom used on the high pressure side since the higher flow velocity there can provoke overestimation of water temperatures. The static pressure in this section p_i is simply mea-

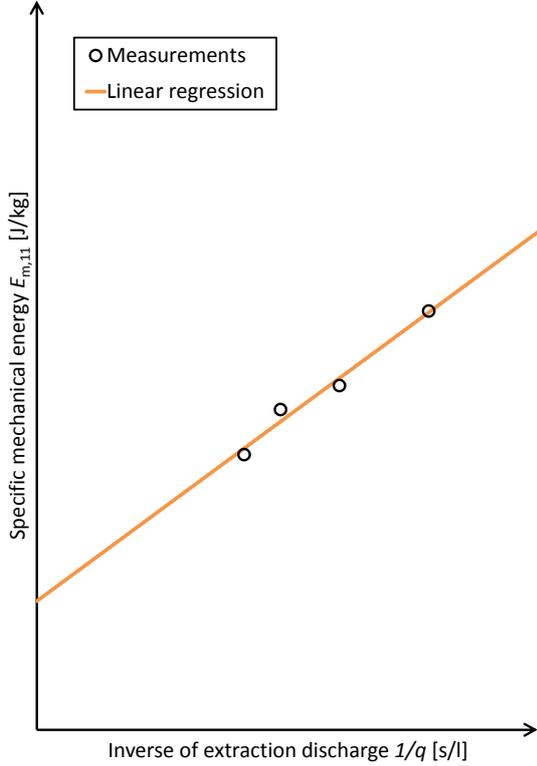


Figure 3: Determination of the specific mechanical energy in measuring section 10 when using an extraction probe: The measured energies $E_{m,11}$ in dependence of the inverse extraction flow yields with the aid of a linear regression $E_{m,10} = E_{m,11}(0)$

sured by several transducers additionally inserted into the thermometer-equipped pipes. The mean velocity v_i – which is proportional to the overall discharge Q – can not be measured in such an arrangement. So the discharge needs to be calculated first with (10), then v_i can be determined.

2.3. Power measurement

The direct measurement of mechanical power P_m , i.e. individual measurement of rotational speed n and torque T (do not mistake with temperature), does not provide enough accuracy on large machines. It is rather used on small units [1]. On large machines generator/motor efficiency tests are usually conducted which provide low uncertainty. That is the reason why it is more common to measure the electrical power P_a and use the generator/motor test results to obtain the mechanical power in an indirect way. The mechanical power provided to a pump or from a turbine thus yields by summation of relevant power losses and the electrical power

$$P_m = P_a + \kappa \cdot (P_{a,L} + P_{Lm}) \quad (9)$$

The maximum tolerance for the generator/motor loss determination yields 10% of the total losses [16]. That is, the relative uncertainty of the generator/motor losses needs to be $|f(P_{a,L})| \leq 10\%$. However, typical values are $f(P_{a,L}) = \pm 2 \dots 3\%$ [1]. The bearing losses P_{Lm} , which typically do not exceed 1% of the rated mechanical power, are measured with an uncertainty of $f(P_{Lm}) \pm 10\%$ or better. In cases where measurements can not be done, empirical formulae are used with increased uncertainty, e.g. $f(P_{Lm}) \cong \pm 30\%$. Finally, one may assume $f(P_m) \cong f(P_a)$ without loss of significance for large-scale machines.

3. Absolute flow calculation

The representative discharge Q running through a hydraulic machine is obtained by solving the polynomial equation

$$\mathcal{P}(Q) = 0 \quad (10)$$

with Q as the independent variable and

$$\mathcal{P}(Q) = Q \cdot [E_m(Q) + \Delta E_m(Q)] - P_m/\rho \quad (11)$$

3.1. Schematic approach

The dependent parameters $E_m(Q)$ and $\Delta E_m(Q)$ can be represented schematically by

$$E_m(Q) = a_0 + a_2 \cdot Q^2 + \frac{a_{-1}}{Q} \quad (12)$$

$$\Delta E_m(Q) = \frac{Q'}{Q} \cdot \left(b_0 + b_2 \cdot Q^2 + \frac{b_{-1}}{Q} \right) \quad (13)$$

with flow independent parameters a_i and b_i . The extracted/added discharge Q' between high and low pressure measuring section is typically less than 1% of the rated flow. The last term in each of the previous two formulae considers any heat exchange with the environment. As a consequence, equation (10) yields with (12) and (13) in its most general representation a polynomial equation of quartic order which can be solved analytically. However, it is very tedious doing it that way. Thus, reducing the order of equation (10) to cubic order by negligence of the $b_{-1}Q'/Q^2$ -term simplifies slightly this work. Nevertheless, it is more reasonable to use a numerical iteration algorithm than to derive an exact analytical solution – especially due to the numerous cases, in practice, how to arrange $\Delta E_m(Q)$.

Table 2: Analytical solution for equation (10) in dependence of κ_1 and κ_2 (definition in equation 21) and $\Delta E_m = 0$

		κ_2	
		0	1
κ_1	0	<p>$A_{10} = A_{20}$: polynomial of 1st order</p> $Q = \frac{(P_m - \delta P_{m,10-20})/\rho}{[ap + gz + c_p T]_{11}^{21}} \quad (14)$ <p>$A_{10} < A_{20}$: 3rd order, 1 real solution</p> $Q = \sqrt[3]{-\frac{\alpha_0}{2} + \sqrt{\left(\frac{\alpha_1}{3}\right)^3 + \left(\frac{\alpha_0}{2}\right)^2}} + \sqrt[3]{-\frac{\alpha_0}{2} - \sqrt{\left(\frac{\alpha_1}{3}\right)^3 + \left(\frac{\alpha_0}{2}\right)^2}} \quad (15)$ <p>$A_{10} > A_{20}$: 3rd order, 3 real solutions; physically correct solution yields</p> $Q = \sqrt{-\frac{4\alpha_1}{3}} \cos \left[\frac{1}{3} \arccos \sqrt{-\frac{27\alpha_0^2}{4\alpha_1^3} - \frac{2\pi}{3}} \right] \quad (16)$	<p>3rd order, 1 real solution</p> $Q = \sqrt[3]{-\frac{\alpha_0}{2} + \sqrt{\left(\frac{\alpha_1}{3}\right)^3 + \left(\frac{\alpha_0}{2}\right)^2}} + \sqrt[3]{-\frac{\alpha_0}{2} - \sqrt{\left(\frac{\alpha_1}{3}\right)^3 + \left(\frac{\alpha_0}{2}\right)^2}} \quad (17)$
	1	<p>3rd order, 3 real solutions; physically correct solution yields</p> $Q = \sqrt{-\frac{4\alpha_1}{3}} \cos \left[\frac{1}{3} \arccos \sqrt{-\frac{27\alpha_0^2}{4\alpha_1^3} - \frac{2\pi}{3}} \right] \quad (18)$	<p>1st order</p> $Q = \frac{(P_m - \delta P_{m,11-21})/\rho}{[ap + gz + c_p T + v^2/2]_{11}^{21}} \quad (19)$

3.2. Analytical solution without usage of extracted or added water

In many thermodynamic efficiency measurements balancing of power is not present or need not be considered, e.g. only negligible leakage through shaft sealings. That is,

$$\Delta E_m(Q) = 0 \quad (20)$$

and an analytical solution for (10) is easier to derive. We introduce another parameter, which will help in the subsequent derivation.

$$\kappa_i = \begin{cases} 0 & \dots \text{interior measuring section} \\ 1 & \dots \text{exterior measuring section} \end{cases} \quad (21)$$

$i \in \{1, 2\}$ where index 1 denotes the high pressure side, index 2 the low pressure side. Let us rewrite equa-

tion (12) with more details using (4) and (21) yielding

$$E_m(Q; \kappa_1, \kappa_2) = [ap + gz + c_p T]_{10+\kappa_1}^{20+\kappa_2} + \frac{1}{2} (\kappa_1 v_{11}^2 - \kappa_2 v_{21}^2) + \frac{Q^2}{2} \left(\frac{1 - \kappa_1}{A_{10}^2} - \frac{1 - \kappa_2}{A_{20}^2} \right) + \frac{1}{Q} \delta P_{m,(10+\kappa_1)-(20+\kappa_2)}/\rho \quad (22)$$

The first term in squared brackets on the right hand side of the previous equation represents the Q -independent specific energies between measuring section $(10 + \kappa_1)$ and $(20 + \kappa_2)$. Consequently, one obtains with (11) the subsequent depressed cubic function

$$\mathcal{P}(Q; \kappa_1, \kappa_2) = Q^3 + \alpha_1 Q + \alpha_0 \quad (23)$$

with

$$\alpha_0 = -\frac{2(P_m - \delta P_{m,(10+\kappa_1)-(20+\kappa_2)})/\rho}{\frac{1-\kappa_1}{A_{10}^2} - \frac{1-\kappa_2}{A_{20}^2}} \quad (24)$$

$$\alpha_1 = \frac{2\left[ap + gz + c_p T\right]_{10+\kappa_1}^{20+\kappa_2} + (\kappa_1 v_{11}^2 - \kappa_2 v_{21}^2)}{\frac{1-\kappa_1}{A_{10}^2} - \frac{1-\kappa_2}{A_{20}^2}} \quad (25)$$

which possesses one real and two complex conjugate roots if $(\alpha_1/3)^3 + (\alpha_0/2)^2 > 0$ and three real roots otherwise [17]. Depending on the values of κ_1 and κ_2 the solution for (10) can be derived exactly. Table 2 reflects the individual solutions based on typical numerical values for the involved physical parameters.

3.3. Uncertainty estimation

Deriving an analytically exact expression for the relative discharge uncertainty $f(Q)$ for formulae given in the previous subsection can be really cumbersome and the efforts therefore do not stand for any improvement in knowledge. Since its value represents a guess, it seems to be sufficient to calculate the uncertainty from the subsequent implicit formula by means of an iterative algorithm

$$f(Q) = \pm \left[\frac{E_m^2 \cdot f^2(E_m) + \Delta E_m^2 \cdot f^2(\Delta E_m)}{(E_m + \Delta E_m)^2} + f^2(P_m) + f^2(\rho) \right]^{\frac{1}{2}} \quad (26)$$

Example: A turbine with 100 m of head and a three-phase synchronous generator is given where no water is neither extracted nor added between high and low pressure measuring section ($\Delta E_m = 0$). Under favorable conditions, i.e. excellent thermalization and steady pressure conditions, the specific mechanical energy can be determined within $f(E_m) = \pm 0.90\%$ using a statistical confidence level of 95% [1]. The generator power is measured by means of a power analyzer ($f_P = \pm 0.10\%$) connected to the relevant current transformers ($f_I = \pm 0.20\%$ for accuracy class 0.2) and voltage transformers ($f_V = \pm 0.20\%$, a.c. 0.2) of the generation unit in three-wattmeter circuit [1]. Neglecting the uncertainty of the mechanical and electrical power losses yields for the mechanical power

$$f(P_m) \cong f(P_a) = \pm \sqrt{f_P^2 + f_I^2/3 + f_V^2/3} = \pm 0.19\%$$

A typical value for the uncertainty of the water density is $f(\rho) = \pm 0.10\%$. Finally, the uncertainty of the discharge can be estimated with (26) by

$$f(Q) = \pm \sqrt{f^2(E_m) + f^2(P_m) + f^2(\rho)} = \pm 0.93\%$$

The uncertainty is lower for higher heads since the specific mechanical energy can be measured with more accuracy.⁶

4. Conclusions

The relationship between power and specific energy provides the underlying polynomial equation to be solved. The order of this polynomial depends on how the specific mechanical energy E_m will be determined in a metrological point of view. It can be shown that this polynomial does never exceed quartic order whereupon functions of first and third order are present in the majority of cases.

Analytical expressions for discharge evaluation with the thermodynamic method and power measurement are derived in this paper. Due to the diversity of partial discharge extraction/addition between high and low pressure measuring section we recommend to use an iterative algorithm in such cases to find the solution. The measurement uncertainty is expected to fall below $\pm 1.0\%$ under favorable flow conditions and – for this method – typical heads higher than 100 m.

Appendix A. Derivation of the hydraulic efficiency by means of thermodynamics

The subsequent derivation makes use of specific energy terms having a unit [J/kg]. The specific hydraulic energy E is measured between measuring section 1 and 2, the specific mechanical energy E_m between 10 and 20. The latter sections are chosen that way to be close to sections 1 and 2, respectively. Any significant extraction or injection of water between the measuring sections 10 and 20 needs to be considered. However, it is avoided in this section to keep the derivation simple. Let us assume an infinitesimal distance between the high pressure section 1 and the low pressure section 2 as well as between section 10 and 20 (view figure A.4). The hydraulic efficiency of a turbine or a pump between that small states yields

$$\eta_h = \left(\frac{dE_m}{dE} \right)^k = \left(\frac{dh_{\text{poly}} + v \cdot dv + g \cdot dz + \delta e}{dh_{\text{ad}} + v \cdot dv + g \cdot dz} \right)^k \quad (\text{A.1})$$

⁶The higher the available head the higher the ratio $a(p_1 - p_2)/E_m$. Since pressure can be measured with very low uncertainty the uncertainty of E_m decreases too.

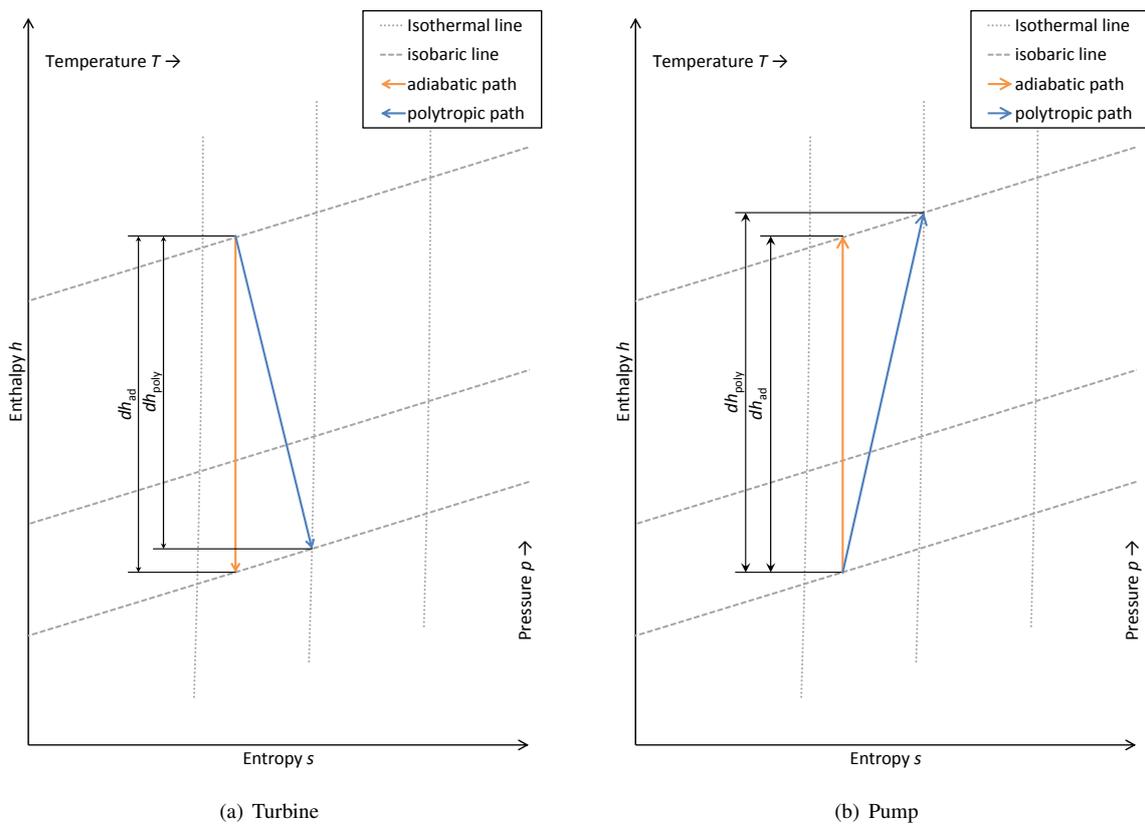


Figure A.4: Thermodynamic processes of hydraulic machines in the enthalpy-entropy diagram

δe considers any heat exchange between the water and the environment within within infinitesimal states. Equation (A.1) reveals that the change in specific enthalpy h is of most significance. h is a state function and can therefore be represented as a total differential of two intensive parameters. It is common to use the intensive parameters T and p [18]:

$$dh = \left(\frac{\partial h}{\partial T}\right)_p \cdot dT + \left(\frac{\partial h}{\partial p}\right)_T \cdot dp \quad (\text{A.2})$$

On the other hand, the change of enthalpy can be represented by

$$\begin{aligned} dh &= d\left(u + \frac{p}{\rho}\right) = du - \frac{p}{\rho^2} \cdot d\rho + \frac{dp}{\rho} \\ &= du - \delta w + \frac{dp}{\rho} \end{aligned} \quad (\text{A.3})$$

using the specific internal energy u and the specific volumetric energy w . The δ -sign gives notice that w is not a state function and can not be represented as a total differential. Finally, equation (A.3) can be reshaped yielding

$$dh = \delta q + \frac{dp}{\rho} = T \cdot ds + \frac{dp}{\rho} \quad (\text{A.4})$$

Here, the specific thermal exchange energy q and the entropy s are employed. q is no state function.

The adiabatic change of enthalpy dh_{ad} may be calculated easiest with (A.4) exploiting that $\delta q = T ds = 0$ and it yields

$$dh_{\text{ad}} = \frac{dp}{\rho} \quad (\text{A.5})$$

The derivation of the polytropic change of enthalpy dh_{poly} requires more efforts. The first partial derivative on the right-hand side of equation (A.2) is known as the isobaric thermal capacity c_p , the second one is the isothermic factor a leading to

$$dh_{\text{poly}} = c_p \cdot dT + a \cdot dp \quad (\text{A.6})$$

The isothermic factor a is almost equal to the reciprocal value of the density ρ . Doing it in an exact manner

gives with (A.3)

$$\begin{aligned} a &= \left(\frac{\partial h}{\partial p}\right)_T \\ &= \left(\frac{\partial u}{\partial p}\right)_T + \frac{1}{\rho} \left(\frac{\partial p}{\partial p}\right)_T - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial p}\right)_T \\ &= \frac{1}{\rho} \left[1 - \frac{p}{\rho} \left(\frac{\partial \rho}{\partial p}\right)_T\right] \end{aligned} \quad (\text{A.7})$$

The internal energy does not depend on the pressure and, thus, in the second line of (A.7) the first term on the right nullifies. The hydraulic efficiency between infinitesimal small measuring sections yields with (A.5) and (A.6)

$$\eta_h = \left(\frac{adp + vdv + gdz + c_p dT + \delta e}{\frac{dp}{\rho} + vdv + gdz}\right)^k \quad (\text{A.8})$$

Integration of the numerator between states present at measuring sections 10 and 20 and integration of the denominator between states present at measuring sections 1 and 2 yields the exact result

$$\eta_h = \left(\frac{\int adp + \int vdv + \int gdz + \int c_p dT + \int \delta e}{\int \frac{dp}{\rho} + \int vdv + \int gdz}\right)^k \quad (\text{A.9})$$

In practice, one uses constant average values for a , c_p , g and ρ . That is,

$$\begin{aligned} \eta_h &= \left(\frac{\bar{a}(p_{10} - p_{20}) + \frac{v_{10}^2 - v_{20}^2}{2} + \bar{g}(z_{10} - z_{20})}{\frac{p_1 - p_2}{\bar{\rho}} + \frac{v_1^2 - v_2^2}{2} + \bar{g}(z_1 - z_2)}\right)^k \\ &\quad + \frac{\bar{c}_p(T_{10} - T_{20}) + e}{\frac{p_1 - p_2}{\bar{\rho}} + \frac{v_1^2 - v_2^2}{2} + \bar{g}(z_1 - z_2)} \end{aligned} \quad (\text{A.10})$$

Vitae

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