CHOKING PRESSURE RATIO GUIDELINES FOR SMALL CRITICAL FLOW VENTURIS AND THE EFFECTS OF DIFFUSER GEOMETRY

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Abstract

The ratio of maximum exit static pressure to inlet stagnation pressure that maintains sonic velocity at the throat of a Critical Flow Venturi (CFV) is referred to as the Maximum Back Pressure Ratio (MBPR). Current standards only provide MBPR equations for CFVs operated at throat Reynolds Numbers (Re_{nt}) above 200,000. This paper will provide MBPR guidelines for operating CFVs below a Re_{nt} of 200,000. Additionally this paper will examine the causes of "premature unchoking" and how diffuser geometry contributes to this effect.

Introduction

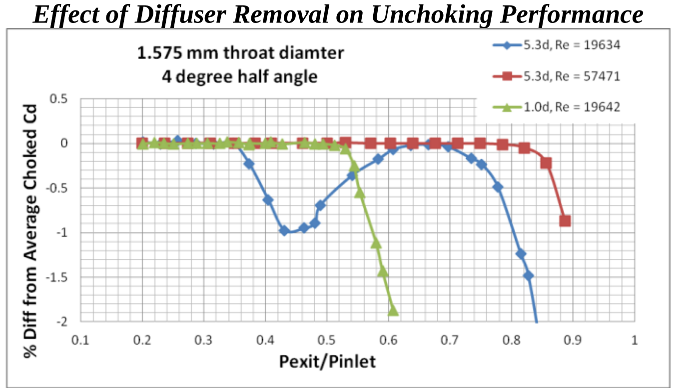
Flow Systems (FSI) primarily provides air flow test and calibration systems which utilize CFVs as the working standards. Some of the industries that employ these systems include aircraft engine, industrial gas turbine and automotive. The first two industries primarily use dry, filtered, and compressed air as the test fluid while the latter chiefly uses ambient air. CFV throat diameters may be as small as 0.4 mm and minimum inlet stagnation pressures range from 17 to 150 kPa. These parameters can result in Re_{nt}'s almost two orders of magnitude lower than the current MBPR equation limit in ISO 9300 [1].

Previous work by Britton, et al. [2], suggests MBPR values for similar CFVs and operating conditions. This work shows the increase of MBPR with increasing Re_{nt} and the lesser effect of decreasing MBPR with increasing isentropic exponent. FSI has continued to optimize diffuser geometry (half angle and length) for use in their air flow systems and the latest results are presented herein.

Most interesting to the authors here are the mechanisms behind what has previously been called "premature unchoking." This phenomenon is analogous to a reduction in CFV discharge coefficient at MBPR values below the Critical Pressure Ratio (r_*) as predicted by isentropic theory (0.528 for dry air). Britton, et al. [2], shows these trends with Re_{nt} and diffuser area ratio and geometry. FSI desired to expand upon this work with particular emphasis on diffuser efficiency for application on CFVs in air flow test systems.

To embark on our study an experiment was conducted on a CFV with a throat diameter of 1.575 mm and a diffuser with a 4 degree half and 5.3 throat diameter length (diffuser area ratio of 3.0). First an unchoking test was performed at a Re_{nt} of 57,471, and as expected the MBPR was greater than r_*. However, when the Re_{nt} is reduced to 19,634, the CFV exhibits "premature unchoking." Next the diffuser of the CFV was machined to a length of one throat diameter, the minimum value required by ISO 9300 for a toroidal throat device. The result of the re-test was, as expected, in that the MBPR found was very close to the r_* and no "premature unchoking" was observed. These first test results are shown in Figure 1. The obvious conclusion is that while a certain geometry diffuser may normally be beneficial (MBPR>r_*), at some Re_{nt}, the same diffuser may be detrimental (MBPR<r_*).

Figure 1
Effect of Diffuser Removal on Unchoking Performance



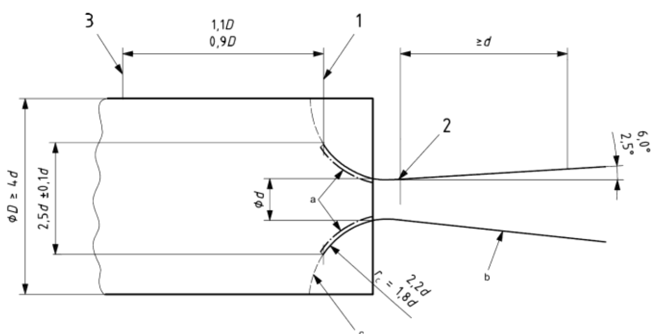
At this juncture the authors propose to introduce the term Diffuser Performance Inversion (DPI) in lieu of "premature unchoking." This term is proposed as a more accurate way to describe the above phenomenon since the sonic surface near the throat of the CFV is maintained and the effect is a result of the diffuser behavior.

As a result of the above observations, the current study focuses on diffuser performance and the effects on CFV behavior. While multiple mechanisms are involved in diffuser performance, one of the dominant parameters, which is already familiar to CFV users, is the throat Reynolds number, Re_{nt}.

Test Program

For this study the unchoking characteristics of 55 CFVs were tested. The tested CFVs were fabricated with a toroidal throat design in accordance to the design specification found in Figure 2 and in Reference [1]. Thirty three of these CFVs were designed with a variety of diffuser lengths and diffuser half angles so the unchoking effects of multiple geometries could be examined. These CFVs were comprised of 18 CFVs used in a previous study (Reference [2]) and 15 new CFVs. These CFVs include three throat diameters; 0.787 mm, 1.575 mm, and 3.175 mm. The diffuser half angles ranges from 2.5 to 6 degrees and diffuser lengths from 4.8 to 20 throat diameters long. These CFVs were tested at inlet pressures of 100 kPa, 200 kPa, and 300 kPa . Additionally 22 CFVs with an "optimized geometry" that had a shallow half angle and long diffuser were tested in order to establish recommendations for CFVs operating below a Re_{nt} of 200,000 . These CFVs had throat diameters ranging from 0.406 mm to 12.7 mm and were tested at four inlet pressures: 100 kPa, 150 kPa, 200 kPa, and 300 kPa. Three identical CFVs were fabricated in each size to check for performance differences that result from manufacturing tolerances.

Figure 2
ISO 9300, Toroidal Throat CFV Design



Unchoking tests were conducted using an automated system utilizing ambient temperature, dry, compressed air as the test fluid. The Gas Flow Meter Calibration System, described in Reference [3], was used to control inlet stagnation pressure and record all required measurements. This system was not submerged as shown in Reference [3] but long test run times were used to allow temperature stabilization and low uncertainty measurements. Back pressure to the Test CFV was controlled by a series of manifolded control valves ranging in CV from 1.2 to 420. Inlet pressure was held constant to the Standard CFV and the manifolded control valves were adjusted to reach a desired pressure ratio across the Test CFV. Once stable flow conditions were attained a data point was collected and the test moved to the next pressure ratio. All tests were initiated with the back pressure control valves open. Previous studies have shown that hysteresis effects are negligible so all tests were performed with increasing pressure ratios only.

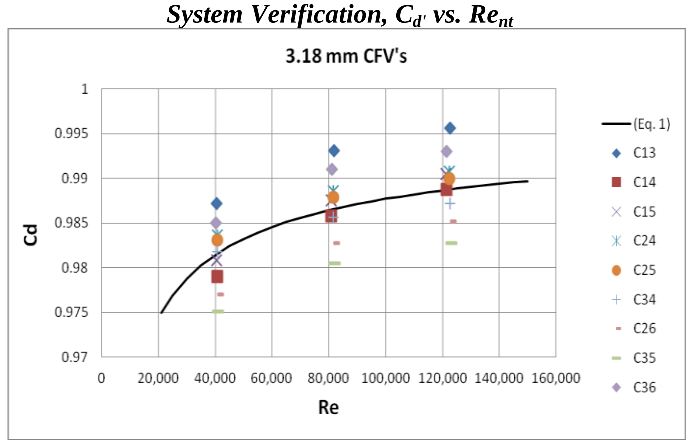
Test Results

Before and during testing, multiple protocols were used to insure low uncertainty and reproducible results. The non-submerged Gas Flow Meter Calibration System was tested over 8 months and demonstrated a k = 2 reproducibility of +/- 0.07% for flows as low as 0.08 grams/second. The System also demonstrated a short term repeatability of +/- 0.04% at low flows. Periodically during testing, specific unchoking tests would be performed twice to check repeatability. Additionally, the performance of all CFVs of the same throat diameter was compared. The comparison can be found in Figure 3 and is the Discharge Coefficient ($C_{d'}$) at a back pressure ratio (BPR) of 0.2 versus Re_{nt}. Equation 1, found in Reference [1], gives the excepted $C_{d'}$ values for the given Re_{nt}. The scatter in the $C_{d'}$ values for the CFVs is a result of differences in the actual throat diameter caused by manufacturing tolerances. The maximum spread in $C_{d'}$ value equates to a 0.01 mm deviation from the nominal throat diameter.

Equation 1

$$C_{d'} = 0.9985 - 2.720 / \sqrt{(Re_{nt})}$$

Figure 3
System Verification, $C_{d'}$ vs. Re_{nt}



The unchoking results are presented as the percent difference from the average choked $C_{d'}$ value versus BPR. A 0.05% change in $C_{d'}$ was used as the criteria for unchoking. Figure 4 shows the performance of a 0.787 mm CFV with a 4 degree half angle and 7.2 diamter length diffuser. This CFV clearly demostrates that decreased inlet stagnation pressure, and therefore decreased Re_{nt}, resuls in a lower MBPR and increased DPI severity. This behavior is expected and has been demostrated in previous studies.

Figure 4
Re_{nt} Effect on Unchoking Performance

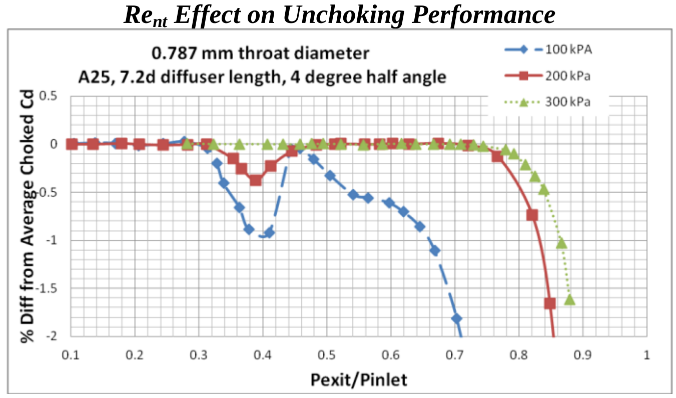


Figure 5 shows three 1.575 mm CFVs all with diffuser that are 10 throat diameters long but have 3, 4, and 6 degree half angles. This plot demostrates the increase in MBPR for lower diffuser half angles.

Figure 5
Diffuser Half Angle Effect on Unchoking Performance

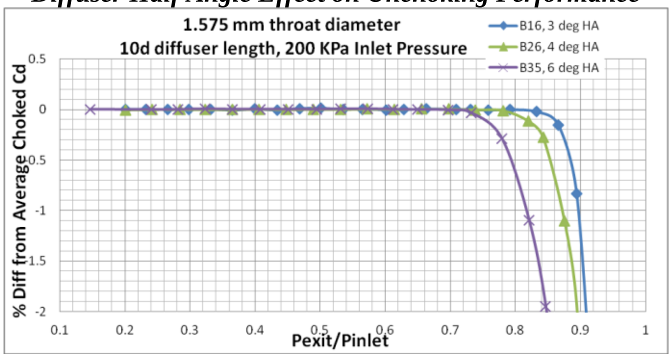
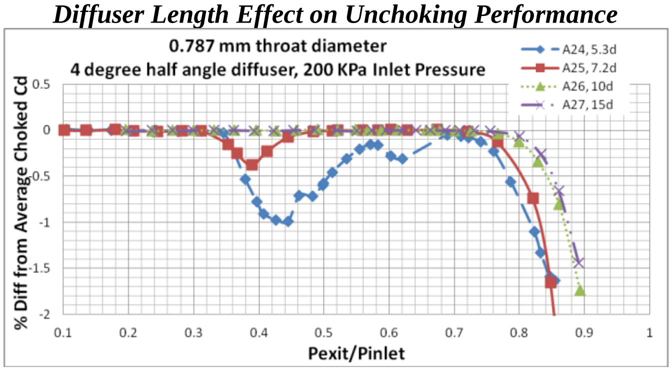


Figure 6 shows a significant trend in diffuser design. This plot shows the unchoking performance of four 0.787 mm CFVs all with 4 degree half angles but with varying length diffusers. A small increase in MBPR is observed with the increased length but more importantly the DPI effect is reduced as the diffuser length is increased. This behavior was observed for all CFV throat sizes that displayed DPI and suggests increasing the diffuser length to over 10d. Results for the "optimized geometry" CFVs will be presented in the MBPR Guidelines section of this paper.

Figure 6
Diffuser Length Effect on Unchoking Performance



<u>Background and Explanation</u>

One characteristic of CFVs which facilitates their practical use for flow measurement is the high MBPR that can be achieved without effecting $C_{d'}$. It is the proper performance of a diffuser which allows the MBPR to be higher than the r_* that exists at the throat of a CFV. The MBPR for a converging nozzle without a diffuser is equal to the critical pressure ratio of the gas and is determine solely by the isentropic exponent. This ratio is always below 0.59 but higher MBPR values, from above 0.6 to as high as 0.95, occur in many CFV applications as the result of pressure recovery in the diffuser. Toroidal throat CFVs designed and used in accordance with ISO 9300 [1], have reliably high MBPR values when the Re_{nt} is greater than 200,000. In this context, MBPR is the highest value of the BPR at which flow in a CFV is choked. Data in this study show that for optimized geometry CFVs with moderate half angles and sufficient length, reasonably high MBPRs can be maintained down to Re_{nt}'s as low as 12,000.

A number of previous publications (References [2], [4], [5], [6], [7], and [8]) have addressed the values of MBPR for small size and low Re_{nt} CFVs. One significant finding, identified by Caron and Britton (References [2] and [4]), is that some small CFVs operated at low pressures experience DPI. The unexpected drop in discharge coefficient that results from DPI is observed at pressure ratios from approximately 0.3 to 0.6. Data shown in Figures 1, 4 and 6 of this study demonstrate situations where DPI occurs. In Figure 4 data for one CFV at different inlet pressures and hence different Re_{nt}, show that the drop in discharge coefficient at low pressure ratios is in part a function of Re_{nt}. The role of Re_{nt} on the MBPR for a CFV and the occurrence of DPI will be discussed later in this paper. Data shown in Figure 6 represents four CFVs with the same diameter, half angle, and inlet pressure but with different diffuser lengths and hence different area ratios. Figure 6 clearly shows that with all other conditions being equal, DPI is dependent on diffuser length or area ratio.

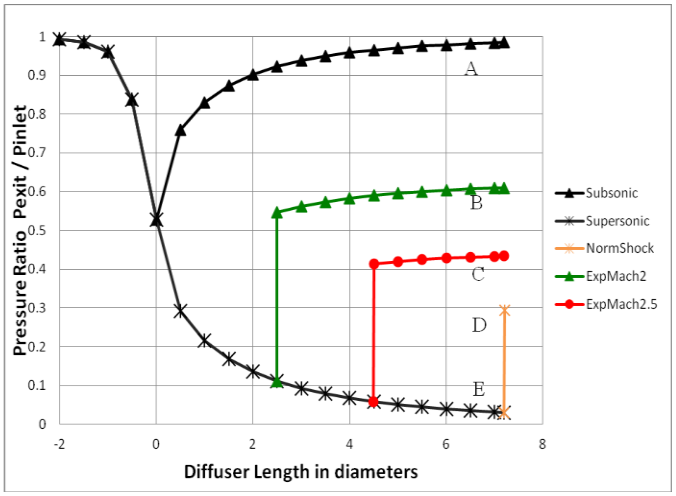
Related conclusions have been reached in a number of previous publications. In Reference [4] Caron and Britton investigated CFVs of one size with different half angles and similar diffuser lengths and identified what they called “premature unchoking”, now believed to be DPI, was less severe at higher inlet pressures (higher Re_{nt}) and with smaller half angles. The work by Nakao and Takamoto (References [5] and [6]) used CFVs with mostly constant half angels, relatively short diffuser lengths, and some very low Re_{nt}. As a result of their work, Nakao and Takamoto, showed that the MBPR for CFVs, either without or with the DPI behavior, is a function of Re_{nt}. Although Nakao and Takamoto commented on the effects of diffuser geometry they only tested relatively short diffusers and where not able to reach strong conclusions about the effects of diffuser length or area ratio. In Reference [7], researchers from the National Institute of Metrology in China performed careful tests and found a clear correlation between the MBPR, without DPI, and Re_{nt}, however they tested different sizes of a single CFV geometry, including one, half angle, length, and area ratio.

In an evaluation of extensive data from several sources, Mickan, Kramer, and Li in Reference [8], developed a correlation of MPBR to not only Re_{nt} but to their form of a Hagen number and an area ratio dependent pressure term. Although the derivation of the Hagen number in Reference [8] is unclear, this parameter seems to reduce the spread of the plotted data. The Mickan, Kramer, and Li treatment does not, however, appear to explain why some CFVs experience DPI and others do not. The work in Reference [8] addresses issues not observed in our testing and does not explain how geometry of a diffuser affects the occurrence of DPI or the MBPR. The previously published studies do contribute to knowledge of what performance can be expected and what factors play a role but they leave much to be discovered before improved guidelines can be developed and applied.

Descriptions and some equations covering the isentropic performance of converging – diverging nozzles can be found in a number of textbooks on compressible flows or gas dynamics. From these analyses it is known that the ideal pressure recovery in a diffuser can take place along a subsonic path that follows the choked condition at the throat or along a supersonic path on which flow is accelerated to high Mach numbers and lower pressures before a normal shock occurs to return the flow to a subsonic and higher pressure conditions. Several of these possible paths for flow velocity and pressure within a diffuser are shown in Figure 7.

Figure 7

CFV Diffuser Pressure Paths



It is important to realize that the pressure recovery in a diffuser will never be as high as indicated in Figure 7 due to the boundary layers and viscous losses in real flows that are not accounted for in ideal calculations. For example, in small CFVs, particularly at low Re_{nt}, the boundary layers in the throat and along the walls of the diffuser are more significant than for larger size CFVs, at higher Re_{nt} such that pressure recovery along the subsonic path towards Point A will be noticeable lower for small CFVs than shown in Figure 7. Most of the MBPRs measured in the current testing fall well below Point A but above Point B and are in the range of 0.7 to 0.9. A likely situation for most of the well behaved diffusers in this study is that a normal shock occurs shortly after the throat and then a further pressure recovery, at a lower than ideal rate, takes place in the remainder of the diffuser. A key question regarding the drop in $C_{d'}$ at low pressure ratios in the range of 0.3 to 0.6 is what occurs in the diffuser.

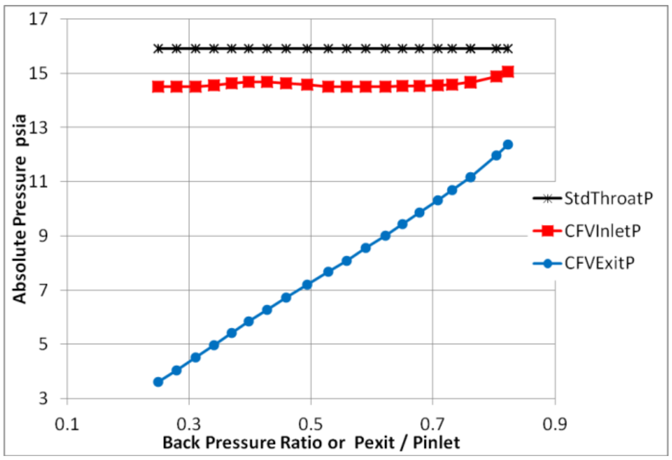
Figure 7 represents the ideal behavior of a CFV with the same geometry, half angle, length, and area ratio of 4.0278, as the CFV test results shown in Figure 4. The lowest pressure ratio that can ideally be achieved with a subsonic exit velocity is 0.3 with a normal shock at the exit of the diffuser as shown at Point D in Figure 7. To reach an exit pressure ratio of 0.3 at Point D the diffuser becomes fully expanded at Point E, where the Mach number is over 2.9. The normal shock at the exit of the diffuser would result in an exit Mach number of 0.4784 and an exit pressure ratio of nearly 0.3 as show at Point D, which appears, in Figure 4, as the lowest BPR at which the DPI starts. To reach a BPR of approximately 0.45 (corresponding to an upper DPI point in Figure 4) at Point C in Figure 7, the flow would expand to a Mach number of approximately 2.5, experience a normal shock to a Mach number of 0.5124 and pressure ratio of approximately 0.414 before ending at Point C. For this particular CFV the region where the pressure ratio is between 0.3 and 0.45 is the area where the drop in $C_{d'}$ due to DPI occurs. It is clear from the data in Plot 2 that this small size, 4 degree half angle, diffuser can sustain this expansion of gas to high Mach numbers with an inlet pressure of 300 kPa. However, as the inlet pressure is reduced it appears that the diffuser no longer behaves in the expected manner and that its performance in this high Mach number region breaks down. With the low inlet pressures and highly expanded flow just prior to where a normal shock should occur, the absolute pressure is very low such that there is little momentum or energy in the flow stream. Perhaps a separation, an oblique shock, or some other loss mechanisms takes place and the diffuser is no longer able to recovery pressure. In any case it can be expected that when the diffuser performance breaks down or inverts, less pressure recovery occurs and the $C_{d'}$ of the CFV is reduced.

Another CFV tested in this study has nearly the same area ratio, 4.121, as the CFV in Figure 4 and 7, and a very similar drop in $C_{d'}$ due to DPI. This CVF has a throat diameter of 1.575 mm, a 6 degree half angle, and a 4.9 throat diameter length diffuser. Pressure data from a low inlet pressure test of this CFV is shown in Figure 8 where the middle pressure of 3 is the CFV Inlet pressure. During this testing the pressures and mass flow rate in the upstream standard CFV were held constant and the exit pressure from the CFV under test was varied, as shown in Figure 8, to create the change in BPR. In Figure 8 the highest pressure shown is the upstream CFV throat pressure, calculated from the standard CFV inlet pressure, and shows that the pressure and flow conditions at the standard CFV are steady and constant throughout this test. The middle pressure in Figure 8 is the measured inlet pressure at the CFV being tested and on examination shows an increase at precisely the pressure ratios at which the drop in $C_{d'}$ due to DPI occurs. It is important to note that the mass flow in this test is constant and hence the same mass flow is passing through the tested CFV when the BPR is 0.7 or 0.4. Thus the tested CFV is not "unchoked" at the 0.4 BPR condition but rather because its diffuser is not able to sustain an expansion to high Mach numbers with an efficient pressure recovery, both inlet and throat pressures increase such that gas density increases and mass flow is maintained but with a lower $C_{d'}$.

A diffuser performance plot, similar to Figure 7 but for a typical optimized geometry CFV would show that although the final area ratio and hence pressure expansion are nearly the same, the rate at which Mach number and pressure changes is lower over the longer length of the optimized CFVs. Test results indicate that because the optimized geometry CFVs are longer than the CFV represented in Figures 1, 4, and 6, there are fewer conditions under which DPI occurs. In fact as shown in later figures, for the optimized CFV the drop in $C_{d'}$ due to DPI only happen at the lowest Re_{nt}, specifically below 12,000. Data from the current testing show that longer diffusers, with similar area ratios, appear to provide more opportunity for a normal shock followed by a pressure recovery than shorter diffusers and hence experience fewer cases of DPI.

Figure 8

Pressure Data from Unchoking Test

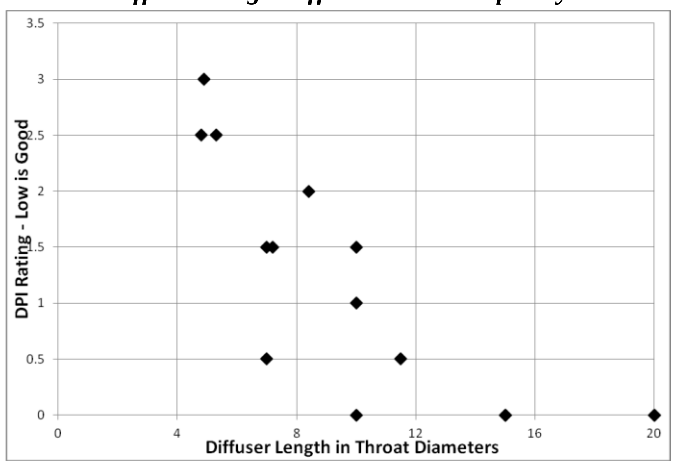


If all of the test results in this study in which no change in the $C_{d'}$ is seen at low pressure ratios are plotted, it is clear that there is a relationship between the MBPR and Re_{nt} with only minor influences from the diffuser length and area ratio. This trend is particularly true for the optimized geometry CFVs. On the other hand, if test results with drops in $C_{d'}$ due to DPI are evaluated there appears to be little correlation with Re_{nt} other than the lower the Re_{nt} the more DPI events occur and that for any single CFV the magnitude of the drop in $C_{d'}$ is greater at lower Re_{nt}. In an analysis of the 26 DPI events found during the current testing, there appears to be a weak affect of area ratio on where between 0.3 and 0.5 the drop in $C_{d'}$ occurs. This would seem reasonable given the explanation shown in Figure 7.

In order to provide an overall evaluation of DPI, the 55 CFVs that were tested were categorized in 15 geometrically similar groups, that is the CFVs with the same half angle and length, without regard to the size were considered together. A qualitative rating based on the number of DPI events in each group was assigned with 0 if no DPI events occurred and 3 if every CFV in the group experienced DPI events. If an average or approximately half of the CFVs in the geometrically similar group experience DPI events during testing a rating of 1.5 was assigned to the group. The result of this global evaluation is that there is almost no correlation between half angle and the DPI rating of the geometric groups other than the one highest rated group has the largest half angle tested, 6 degrees. As to area ratio the general trend is that the highest DPI event rating occurred for area ratios between 2 and just over 4, moderate ratings occurred for area ratio above 4 to over 6, and CFVs with area ratios near 10 or higher experienced no DPI events. A correlation does appear between the subjective DPI Rating and CFV diffuser length as shown in Figure 9 where the highest ratings clearly occur for the shortest CFVs and no DPI events (a 0 rating) occur for CFV diffusers over 12 diameters in length. It should be remembered that the results in Figure 8 include a full range of Re_{nt} and other variables. Despite this, per the current test results, if CFVs were constructed with very long diffusers of more than 14 diameters then DPI events in the range of sizes and Re_{nt} testing could be avoided.

Figure 9

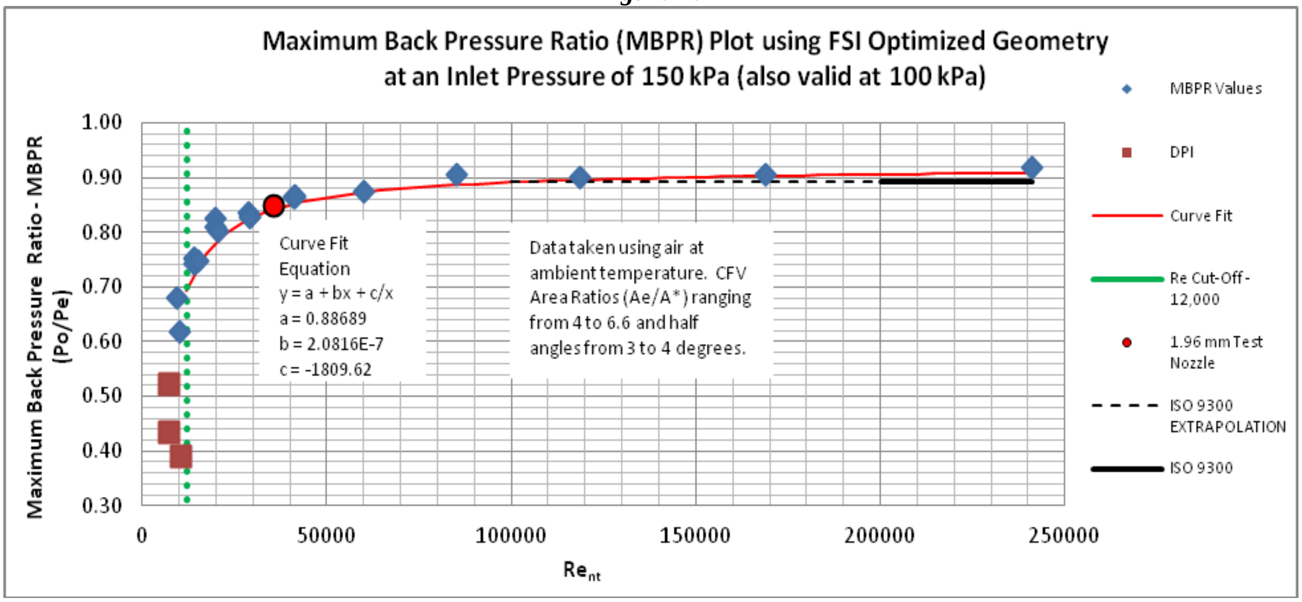
Diffuser Length Effect on DPI Frequency



MBPR Guidelines For Practical Application

As previously discussed, FSI builds air flow test systems using CFVs where minimum inlet stagnation pressures vary from 17 to 150 kPa. Most common is the range of 100 to 150 kPa. It was desired to establish a predictive equation that could be used in both system design and application for MBPR of our CFVs when servicing ambient temperature air and for throat diameters ranging from 0.4 to 13 mm. Additionally, we wanted to see how low in Re_{nt} it may be possible to extrapolate the MBPR equation from ISO 9300 (Reference [1]) with our CFVs. The following figure reflects our results.

Figure 10



A Re_{nt} cutoff of 12,000 was applied to Figure 10 where the slope of the curve fit equation was felt to be too large or where DPI occurred. An additional CFV was constructed and tested for MBPR. The results suggest the equation presented will work for intermediate throat diameters, above the Re_{nt} cutoff, providing optimized diffuser geometries. For the CFVs tested the ISO 9300 (Reference [1]) equations from section 8.5 may be extrapolated down to a Re_{nt} of 100,000 with good results. Both equations agree well in the overlapping Re_{nt} region and testing supports use of this predictive equation for "optimized" CFVs down to inlet stagnation pressures of 100 kPa for ambient temperature air.

Given the strong dependence on Re_{nt}, it is thought that the MBPR equation suggested may be applied successfully to other common industrial gasses with the exception of helium and hydrogen. Both these gases have low molecular mass, and therefore low Re_{nt}, as compared to air for the same CFV at the same inlet stagnation temperature and pressure.

Since throat diameter determination for small CFVs can be a major source of uncertainty, we recommend flow calibration. For all numerical calculations the user should use the same throat area used in calibration with the calibrated $C_{d'}$. This offers users of CFVs a perfect opportunity to determine the MBPR and minimum Re_{nt} where DPI is either not present or significant given the application. With care and skill, the diffuser of a CFV exhibiting DPI can be reduced to the minimum ISO 9300 requirements without effecting the calibration.

Conclusions

The following conclusions result from data taken during the current extensive test program, an analysis of that data with respect to MBPR and diffuser performance, and an effort on the part of the authors to provide guideline for the effective use of optimized geometry CFVs.

Testing demonstrated that the drop in $C_{d'}$ that occurs for some CFVs between BPR of 0.3 and 0.6, previously called “premature unchoking” is a result of a loss of pressure recovery in the diffuser, that is, a Diffuser Performance Inversion (DPI).

A total of 55 toroidal throat CFVs designed per ISO 9300, with different half angles and different diffuser lengths including 22 of an optimized geometry were tested over a Re_{nt} range of approximately 4,660 to 241,240 in order to demonstrate the MPBR behavior and the conditions at which DPI events occurred. The occurrence of DPI events in CFVs, is affected by Re_{nt}, by half angle, and from analysis of the results, is strongly influenced by diffuser length.

For the optimized geometry CFVs, testing shows that MPBR is predictable as a function of Re_{nt} to values of 12,000 and that no DPI breakdowns occur above this Re_{nt}. An equation that conforms to an extrapolation of the ISO 9300 MPBR equation down to Re_{nt} of 100,000 and predicts the MBPR that should be used for the optimized geometry CFVs down to Re_{nt} of 12,000 is presented and fits the measured data closely.

DPI is more likely to occur in a CFV when relatively low BPR is required, expansion of the internal diffuser flow to low pressures and high Mach Number is necessary along the diffuser path, and when the diffuser half angle is relatively large and / or the diffuser length is too short.

It is clear that future investigations directed towards understanding what takes place in a diffuser when DPI occurs, quantifying the controlling parameters for DPI, and studying the operation of CFVs on even lower pressure ambient temperature air down to inlet stagnation pressures as low as 17 kPa would be beneficial. It would also be of value for MPBR predictions and DPI avoidance to investigate a range of other gases with different isentropic exponents, and the effects other similar geometry CFVs, including longer diffusers and diffusers with backward facing steps.

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