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Theoretical analysis on the effect of divergent section with laminar boundary layer of sonic nozzles

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Abstract: ISO toroidal-throat nozzle with a divergent section differs from the nozzle used for the theoretical research of Hall & Geropp and is highly likely to affect the flow field and discharge coefficient. In this paper, the influence of ISO-type conical divergent section on discharge coefficient was analyzed basing on inviscid transonic flow model and laminar boundary layer respectively. Two coordinate systems namely cylindrical coordinates and curvilinear coordinate were applied to research the velocity distribution of core flow and calculate displacement thickness. The results indicated the influence of conical divergent section decreases with increasing of diffuser angle and Reynolds number, while increases with divergent section length increasing. It meant that the discharge coefficient is significant influence by diffuser angle and divergent section length when the Reynolds number decreases to a certain value. The sonic nozzle which is miniaturized or applied to low Reynolds number flow should be carefully designed.

Keywords: sonic nozzles; discharge coefficient; divergent section; laminar boundary layer; Reynolds number; ISO 9300

1 Introduction

A lot of studies have been made to investigate the relationship between discharge coefficient of sonic nozzle and Reynolds number by experimental measurements as well as theoretical calculations. The discharge coefficient of a circular-arc sonic nozzle is theoretically derived by Hall [1] and Geropp [2]. The nozzle studied by Hall & Geropp is called "Hall-Geropp" nozzle in this paper. The major difference between ISO toroidal-throat nozzle and Hall-Geropp nozzle is that the former has an ISO-type conical divergent section which is likely to affect the flow field and discharge coefficient.

The research of Hall-Geropp nozzle mainly focused on theoretical study. Hall [1] and Kliegel [3] obtained the approximate solutions for the inviscid non-one-dimensional flow. Geropp [2] obtained analytical solutions for the laminar boundary layer of one dimensional flow. The research results of Hall and Geropp were integrated by Ishibashi [4]. Stratford [5] divided the flow field of sonic nozzles into two regions, non-one-dimensional core flow and the boundary layer region including laminar and turbulent layer further.

On the other hand, the research of ISO toroidal-throat nozzle mainly focused on the experimental and numerical simulation [6]-[13]. Most of the research did not concern the influence of divergent section on discharge coefficient of ISO toroidal-throat sonic nozzles. Although Stratford thought that the effect of boundary layer on the position of the throat could be negligible which meant the discharge coefficient would not be affected [5]. And, Masahiro Ishibashi also put forward the difference in the conical diffuser of the ISO toroidal-throat nozzle would not affect the discharge coefficient at all [4]. However, C. H. Li [14] and J. H. Kim [15] respectively presented that influence of diffuser angle on discharge coefficient was remarkable when the sonic nozzle diameter is less than 1mm at the atmospheric condition, except that the reasons and whole factors why the divergent section can influence discharge coefficient were not found, and different inlet conditions and length of divergent section *L'* had not been considered as well.

The key of the issue is Gerrop [2] and Ishibashi [4] proposed that the boundary layer does not influence the real throat position which means core velocity of viscid flow is in accordance with non-one-dimensional inviscid flow. However because the divergent section of ISO nozzle differs from that of Hall-Gerrop nozzle, this conclusion may not be valid for ISO nozzles. In this paper, a theoretical explanation for the effect of divergent section was obtained.

2 Theoretical discharge coefficient

The research of C_d is divided into three parts [16], viscous region, namely viscous discharge coefficient C_{d1} affected by viscous effects of boundary layer. Core flow region, namely inviscid discharge coefficient C_{d2} influenced by non-one-dimensional flow (namely geometry). Virial discharge coefficient C_{d3} influenced by physical properties. The influence of C_{d3} is negligible when gas is ideal approximately [16]. Thus, discharge coefficient can be expressed by, [14]

$$C_d = C_{d1} \cdot C_{d2} = \left(1 - \frac{b_1}{\sqrt{Re}}\right) \cdot a \tag{1}$$

Where, C_{d2} equals to parameter a which is influenced by geometry. b_1 is undoubtedly influenced by Re and geometry factors. According to previous studies, the discharge coefficient can be commonly described using two constants as

$$C_d = a - \frac{b}{\sqrt{Re}} \tag{2}$$

Where, $b = a \cdot b_1$. According to Hall's equation, [1]

$$C_{d2} = a = 1 - \frac{\kappa+1}{\left(2R/d\right)^2}\left(\frac{1}{96} - \frac{8\kappa+21}{4608\left(2R/d\right)} + \frac{754\kappa^2+1971\kappa+2007}{552960\left(2R/d\right)^2}\right) \tag{3}$$

In this paper, the radius of curvature R is always $2d$. For air, κ=1.4, C_{d2}=0.99859. According to theory of Geropp, [2] the discharge coefficient C_{d1} can be written as

$$C_{d1} = \left(1-\frac{2\delta_1}{L}\right)^2 \approx 1-\frac{4\delta_1}{L} = 1-\frac{b_1}{\sqrt{Re}} = 1-\frac{4}{\sqrt{Re\cdot m}}\left(\frac{\kappa+1}{2}\right)^{\frac{1}{2(\kappa-1)}}\left(3\sqrt{2}-2\sqrt{3}+\frac{\kappa-1}{\sqrt{3}}\right) \tag{4}$$

Where, parameter m defined by Geropp is determined by the nozzle geometry [2]. Masahiro Ishibashi [4] put forward that, due to the non-one-dimensional flow, Hall's equation should be applied as the core velocity field to calculate m.

In order to analyze the effect of divergent section on discharge coefficient for any gas composition and at different operating conditions, the important dimensionless parameters are determined by normalizing the governing gas dynamic equations [17]-[19]. The function of discharge coefficient is given by the following dimensionless parameters [16],

$$C_d = f_1\left(Re_i, \kappa, \mu^0, \Lambda^0, Pr, Z, c_p^0, \beta T, T_{wall}/T^*, geometry\right) \tag{5}$$

Where, Λ is the heat conductivity coefficient. βT is coefficient of volumetric expansion. In this paper, the effect of roughness is not considered. The gas is considered to be a perfect ideal air and the nozzle wall is adiabatic. Thus, the function can be simplified to

$$C_d = f_1\left(Re, geometry\right) \tag{6}$$

C. H. Li [14] and J. H. Kim [15] found that the influence of diffuser angle on the flow of sonic nozzle with small diameter cannot be neglected, namely there exist differences in velocity distribution of ISO toroidal-throat nozzle and Hall's equation. Thus, parameter m and discharge coefficient change correspondingly. Nevertheless, no further research for the influence of divergent section on velocity field and discharge coefficient is conducted yet.

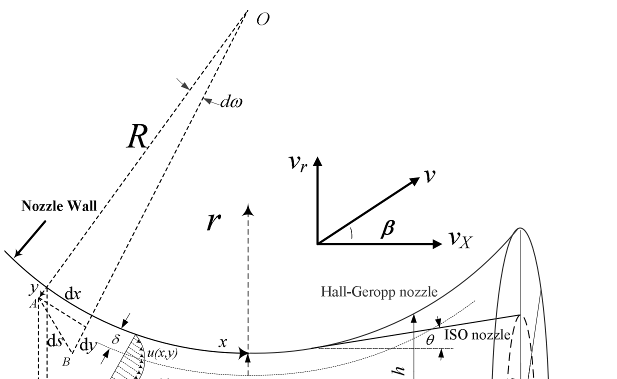


Fig. 1 Sonic nozzles configuration and two types of coordinates

To research the influence of divergent section, in this paper, a laminar boundary layer model combined with the non-one-dimensional core flow for the axially-symmetric nozzle was presented. As is shown in

Fig. 1, two coordinates namely cylindrical coordinates (r, φ, X) and curvilinear coordinate (x, y, φ) were applied to analyze the velocity distribution of core flow and calculate displacement thickness of ISO toroidal-throat nozzle, where X and r are axial and radial of nozzle respectively. φ is rotation angle. x and y are wall surfaces and surface normal, respectively. In addition, the back pressure ratio must be low enough to achieve the critical flow at throat section and avoid shock at divergent section of nozzle [20]-[22].

3 Inviscid transonic flow in ISO toroidal-throat nozzle

Firstly, for the inviscid critical flow, the effect of divergent section on flow field, especial on flow-rate of ISO toroidal-throat nozzle was studied. The steady irrotational compressible axisymmetric Euler equations described by Eq. (7) are established in (r, φ, X) coordinate system. The velocity in direction φ is 0.

$$\begin{cases} \left(c^2-v_X^2\right)\dfrac{\partial v_X}{\partial X} - 2v_X v_r\dfrac{\partial v_X}{\partial r} + \left(c^2-v_r^2\right)\dfrac{\partial v_r}{\partial r} + \dfrac{c^2 v_r}{r} = 0 \\ \dfrac{\partial v_r}{\partial X} = \dfrac{\partial v_X}{\partial r} \\ c^2 = c_0^2 - \dfrac{(\kappa-1)}{2}\left(v_X^2+v_r^2\right) \end{cases} \tag{7}$$

Where, speed of sound $c_0 = \sqrt{(\kappa+1)/2}\cdot c^*$, and c^* is the critical speed of sound.

For ISO toroidal-throat nozzle, the toroidal surface before divergent section is the same with Hall-Geropp nozzle while the divergent section is conical which is different from Hall-Gerrop nozzle, half-height h can be gained by the equation below,

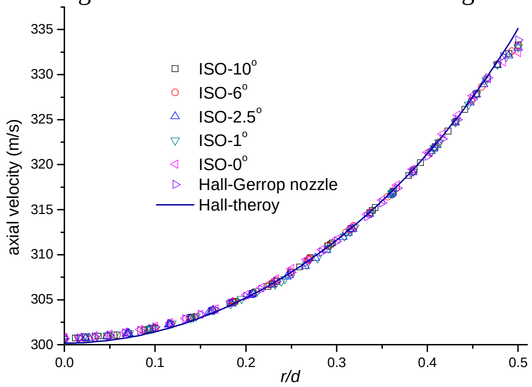
$$h = \begin{cases} \dfrac{d}{2} + \dfrac{X^2}{2R} + \dfrac{X^4}{8R^3} + O\left(X^4\right), X \le R\cdot\sin\theta \\ \dfrac{d}{2} + R\left(1-\cos\theta\right) + \left(X - R\sin\theta\right)\tan\theta, X > R\cdot\sin\theta \end{cases} \tag{8}$$

Therefore, the wall boundary conditions of velocity can be described by

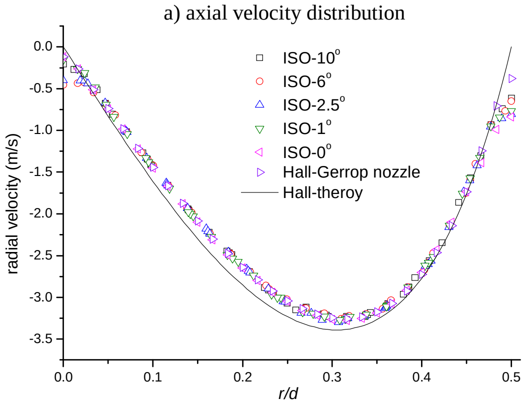
$$\frac{dh}{dX} = \frac{v_r}{1+v_X} = \begin{cases} \dfrac{X}{R} + \dfrac{X^2}{2R^3} + O\left(X^3\right), X \le R\cdot\sin\theta \\ \tan\theta, \quad X > R\cdot\sin\theta \end{cases} \tag{9}$$

Since the first and second derivative of v_r at ISO-type conical divergent section is 0, velocity solutions similar with Hall which converts the velocity into the inverse powers of R cannot be obtained. Thus, analytic solution of axial and radial velocity is unavailable. In this paper, the velocity distribution can be gained by using discrete numerical method.

Fig. 2 shows the velocity distribution at throat with 0°, 1°, 2.5°, 6° and 10° diffuser angles. The velocities at throat are unchanged with the variation of diffuser angles.

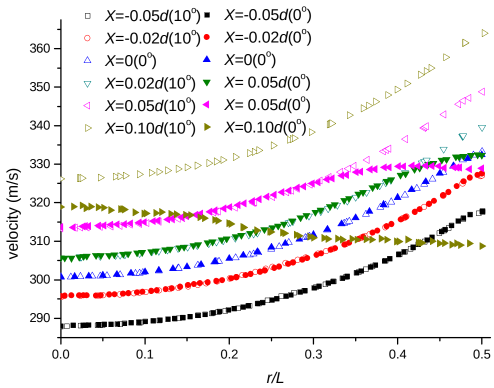


a) axial velocity distribution

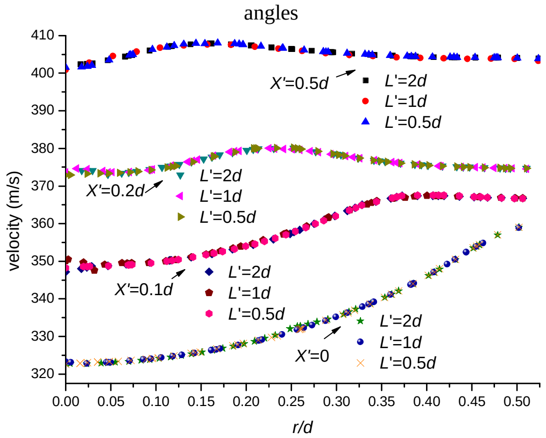


b) radial velocity distribution

Fig. 2 Velocity at throat with different diffuser angles



a) velocity distribution at different positions with 0° and 10° diffuser angles



b) Velocity distributions at different positions with 0.5d, 1d and 2d divergent section length, 2.5° diffuser angles

Fig. 3 Velocity distributions with different diffuser angles and divergent section length

Fig. 3 a) shows the velocity distribution of different positions X with 0° and 10° diffuser angles. Fig. 3 b) shows the velocity distributions at different positions ($X'=X-R\sin\theta$) with the same diffuser angle 2.5° but different divergent section lengths L'. As are shown in the figures, the velocities at throat and convergent section stay unchanged with variations of divergent section (including diffuser angle θ and divergent section length L'), while the velocity at divergent section depends on diffuser angle.

When the flow is inviscid, the divergent section of ISO toroidal-throat nozzle does not influence the flow ahead of the throat, which means the flow-rate is not affected. However, due to the viscousity of the real gas, the velocity near wall of non-one-dimensional flow is influenced by the divergent section which undoubtedly will affect the boundary layer thickness [2]. The conclusion 'the real throat position are not influenced by the boundary layer and the core velocity distribution has no difference with non-one-dimensional inviscid flow' presented by Gerrop and Ishibashi probably are not tenable.

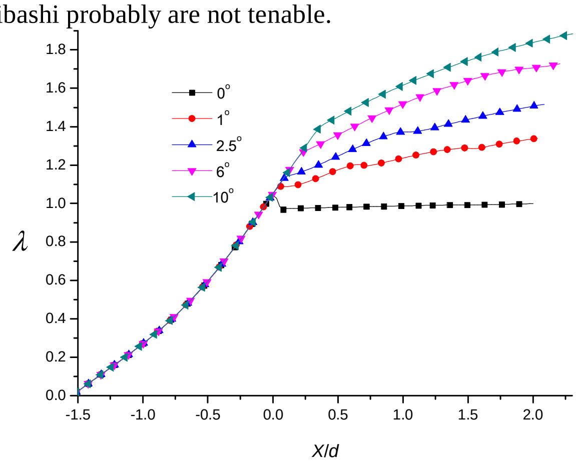


Fig. 4 Velocities at the nozzle wall with different diffuser angles under critical condition

When the boundary layer thickness continues to increase behind the throat which leads to the equivalent flow area behind the throat is less than the equivalent flow area of the throat, the real equipotential surface of velocity is bound to move towards the exit of nozzle to ensure it reaches sonic speed at the minimum cross-section. Theoretical analysis will be carried out in the following chapter to verify the correctness of this assumption. As is shown in Fig. 4, the non-one-dimensional inviscid dimensionless velocity distribution λ at the nozzle wall normalized by c^* with different diffuser angles is given in order to analyze the influence of divergent section on real gas later.

4 Viscid transonic flow with laminar boundary layer

To analyze the boundary layer distribution of the surface and keep the same form with Prandtl boundary equations, curvilinear coordinate system (x, y, φ) in Fig. 1 is applied. It is assumed that velocity in rotation angle φ is 0, and u and v are velocities in x and y direction. Thus, the axisymmetric laminar boundary layer equations can be simplified as

$$\begin{cases} \rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \\ \frac{\partial}{\partial x}(r\rho u)+\frac{\partial}{\partial y}(r\rho v)=0 \\ \rho\left(u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}\right)=\mu\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\frac{\Lambda}{c_p}\frac{\partial h}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \\ p=R^*\rho T \end{cases} \tag{10}$$

With Howarth-type coordinate transformation [2], the velocity of boundary layer after coordinate transformation can be written as

$$\int_{\lambda_S}^{\lambda}\frac{d\lambda}{\lambda^2\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{2\kappa-1}{\kappa-1}}\left(\frac{r}{r_{nt}}\right)^2}=\frac{x}{L}m \tag{11}$$

Where, λ_S is dimensionless velocity at initial position (x=0). $L=N_1^2T_0^2/(v_0a_0)\sqrt{(\kappa+1)/2}$ is the feature size of nozzle. Since N_1 is arbitray, L can be the nozzle throat namely L=d.

4.1 One-dimensional core flow

According to the equations of one-dimensional flow, under critical conditions, the following equation for cross profile of nozzle can be obtained.

$$\frac{A(x)}{A^*}=\left(\frac{r}{r_{nt}}\right)^2=1\Big/\lambda\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{1}{\kappa-1}} \tag{12}$$

Substituting (12) into (11), the velocity distribution of axisymmetric swirl nozzle is as follows

$$\left(\frac{\kappa+1}{2}\right)^{\frac{1}{\kappa-1}}\int_{\lambda_S}^{\lambda}\frac{d\lambda}{\lambda\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^2}=\frac{x}{L}m \tag{13}$$

According to Geropp study, the displacement thickness of boundary layer for one-dimensional flow is

$$\frac{\delta_1}{L}=\frac{3\sqrt{2}-2\sqrt{3}+\frac{2}{\sqrt{3}}\frac{\frac{\kappa-1}{\kappa+1}\lambda^2}{1-\frac{\kappa-1}{\kappa+1}\lambda^2}}{\sqrt{Re\cdot m}\cdot\lambda^{\frac{1}{2}}\left[1-\frac{\kappa-1}{\kappa+1}\lambda^2\right]^{\frac{1}{2(\kappa-1)}}} \tag{14}$$

Thus, when the flow at throat is critical, the displacement thickness of nozzle is as follow,

$$\frac{\delta_1}{L}=\frac{1}{\sqrt{Re\cdot m}}\left(\frac{\kappa+1}{2}\right)^{\frac{1}{2(\kappa-1)}}\left(3\sqrt{2}-2\sqrt{3}+\frac{\kappa-1}{\sqrt{3}}\right) \tag{15}$$

So the discharge coefficient C_{d1} is as shown in Eq.(4). According the analysis in this paper as well as Masahiro Ishibashi [4], parameter m changes with different nozzle structures. Parameter m of ISO toroidal-throat nozzle before the divergent section is the same as that of Hall-Geropp nozzle. While for ISO-type divergent section, the derivative of r with respect to the wall curvilinear coordinate x can be written as below

$$\frac{d(r/r_{nt})}{d(x/L)}=\frac{d(r/r_{nt})}{d\lambda}\frac{d\lambda}{d(x/L)}=2\sin(\theta) \tag{16}$$

With the relation of the swirl radius r and dimensionless velocity λ in Eq. (12),

$$\frac{d(r/r_{nt})}{d\lambda}=-\frac{1}{2}\left[\lambda\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{1}{\kappa-1}}\right]^{-\frac{3}{2}}\cdot \left\{\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{1}{\kappa-1}}-\lambda^2\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{2-k}{\kappa-1}}\right\} \tag{17}$$

Using Eq. (13), gives

$$\frac{d\lambda}{d(x/L)}=m\left(\frac{\kappa+1}{2}\right)^{-\frac{1}{\kappa-1}}\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^2\lambda \tag{18}$$

Substitute Eq. (17) and Eq. (18) into Eq. (16), the parameter m of ISO toroidal-throat nozzle at divergent section can be described by

$$m=\frac{-4\sin(\theta)\left(\frac{\kappa+1}{2}\right)^{\frac{1}{\kappa-1}}\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^{-2}}{\lambda\left[\lambda\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{1}{\kappa-1}}\right]^{-\frac{3}{2}}\left\{\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{1}{\kappa-1}}-\lambda^2\left[1-\frac{\kappa-1}{2}(\lambda^2-1)\right]^{\frac{2-k}{\kappa-1}}\right\}} \tag{19}$$

4.2 Non-one-dimensional core flow

Eq. (19) is obtained under one-dimensional flow while the real flow is non-one-dimensional. Thus, velocity distribution for non-one-dimensional inviscid flow should be applied for calculating m. Eq. (12) about the relation of r/r_{nt} and λ at the nozzle wall is for one-dimensional flow which cannot be used further. So, ・this relation for Hall-Geropp nozzle can be obtained directly by Hall's equation. While for ISO toroidal-throat nozzle, this relation for divergent section must be obtained through numerical simulation as is shown in Fig. 4. Meantime, the first derivative of $\lambda(x/L)$ are calculated by the original Eq. (11) instead of Eq. (13). Therefore, Eq. (18) become

$$\frac{d\lambda}{d(x/L)}=m\lambda^2\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{2k-1}{k-1}}\left(\frac{r}{r_{nt}}\right)^2 \tag{20}$$

Substituting in Eq. (16) using Eq. (20), parameter m for ISO-type divergent section can be written

$$m=\frac{2\sin(\theta)}{\lambda^2\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{2k-1}{k-1}}\left(\frac{r}{r_{nt}}\right)^2\frac{d(r/r_{nt})}{d\lambda}} \tag{21}$$

Fig. 5 shows the value of m with velocity filed of inviscid flow for IS0 toroidal-throat nozzle. The diffuser angles are including 1°, 2.5°, 6° and 10°. According to Fig. 4, the velocity gradient at the nozzle wall for ISO-type divergent section significantly less than that before divergent section. Therefore, the value of m for divergent section drops significantly, which means that the boundary layer might become sufficiently thicker that the effective flow area of divergent section is smaller than that of throat.

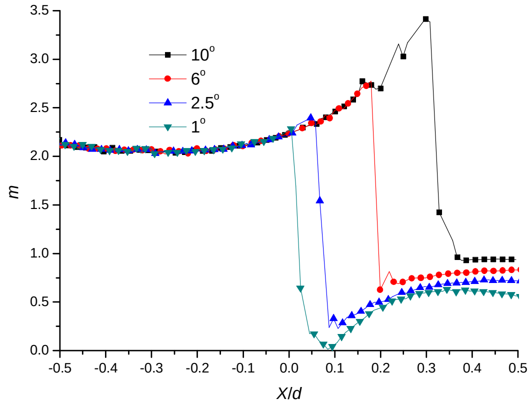


Fig. 5 *m* with velocity filed of inviscid flow for IS0 toroidal-throat nozzle

5 The effect of divergent section on flow flied and discharge coefficient

Proceeding as in section 4.1, thickness of boundary layer can be obtained by the equation below

$$\frac{\delta_1}{L}=\frac{\left(\frac{\kappa+1}{2}\right)^{-\frac{1}{2(k-1)}}\left(3\sqrt{2}-2\sqrt{3}+\frac{2}{\sqrt{3}}\frac{\frac{\kappa-1}{\kappa+1}\lambda^2}{1-\frac{\kappa-1}{\kappa+1}\lambda^2}\right)}{\sqrt{Re\cdot m}\cdot\lambda\cdot\left(\frac{r}{r_{nt}}\right)\cdot\left(1-\frac{\kappa-1}{\kappa+1}\lambda^2\right)^{\frac{1}{(\kappa-1)}}} \tag{22}$$

Where, the value of m can be calculated by Eq. (21). When the flow at throat is critical, the form of discharge coefficient C_{d1} is in accordance with Eq. (4), except the value of m is different. If using Hall's equation, at throat, m=2.2272. According to Eq. (4), b_1=3.3982, b=3.3934, which is basically in accordance with the discharge coefficient equation for ISO accurate machined toroidal-throat (b=3.412).

The thickness of boundary layer with velocity filed of inviscid flow for IS0 toroidal-throat nozzle could be calculated by using Fig. 5 and Eq. (22). Additionally, since the thickness of boundary layer is along y direction of wall in curvilinear coordinate system, so the angular relation should be taken into consideration when the real thickness is calculated. Namely,

$$\frac{\delta_1'}{\delta_1}=\begin{cases}\sqrt{1-(X/R)^2}, & X\le R\cdot\sin\theta\\ \cos(\theta), & X>R\cdot\sin\theta\end{cases} \tag{23}$$

Where, δ_1' is the equivalent boundary layer thickness in r direction.

Fig. 6 shows the displacement thickness of the boundary layer with velocity filed of inviscid flow for IS0 toroidal-throat nozzle with different diffuser angles. Due to the boundary layer effect, the real minimum flow area is not at the throat. The minimum area will decrease with the drop of diffuser angle and move towards the exit of nozzle. Based on the gas dynamics, critical condition will appear in the minimum area of nozzle, while in Fig. 6 the minimum area is not at nozzle throat, which is contradictory with the assumption above. Thus, compared with velocity filed of non-one- dimensional inviscid flow, the real velocity field is bound to move backwards and the final velocity field coincides with the minimum cross profile, which will lead to the decrease of discharge coefficient definitely. Further, the smaller the diffuser angle is, the bigger the effect mentioned above is and the smaller the discharge coefficient is. The shorter the divergent section length is, the smaller its effect is and the bigger the discharge coefficient is. In addition, according to Eq. (22), the result that the larger Reynolds number is, the smaller its effect is can be obtained.

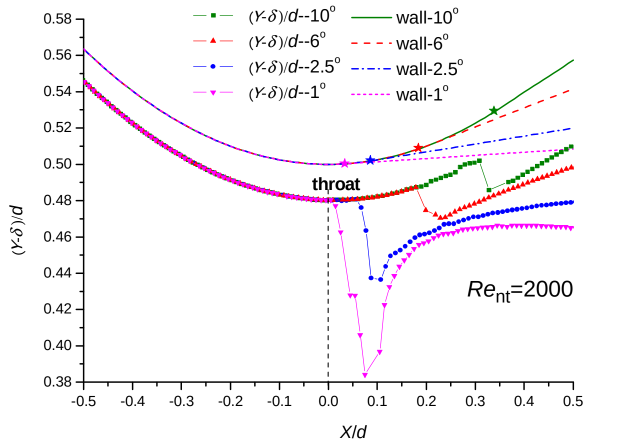


Fig. 6 The displacement thickness of the boundary layer with velocity filed of inviscid flow for IS0 nozzle with different diffuser angles

6 Conclusion

The influence of divergent section on discharge coefficient of ISO toroidal-throat nozzle was analyzed with inviscid transonic flow model and laminar boundary layer respectively. According to theoretical analysis, the real velocity field is bound to move backwards which will lead to the decrease of discharge coefficient definitely and the minimum effective area of the flow will be behind the throat due to the effect of boundary layer thickness. The influence of divergent section on flow field and discharge coefficient of ISO toroidal-throat nozzle can be concluded as below.

Flow field:

1. With the decrease of diffuser angle θ or the increase of divergent section length L', the velocity field and the equivalent throat position moves towards the exit.
2. The thickness of boundary layer is in inverse relation with square root of pressure or Reynolds number Re_{nt}.

Discharge coefficient:

1. Discharge coefficient decreases with the drop of diffuser angle θ.
2. Discharge coefficient decreases with the rise of divergent section length L'.
3. Discharge coefficient increases with the rise of Reynolds number Re_{nt}.

It means that the sonic nozzle which is miniaturized or working at low Reynolds number flow should be carefully designed with respect to the diffuser angle θ and divergent section length L'.

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