

STUDY ON GEOMETRIC PARAMETER MEASUREMENT AND WEIGHT COEFFICIENT CORRECTION OF ULTRASONIC FLOWMETER

He-ming Hu, Tao Meng, Feng Gao, Liang Zhang, Chi Wang
Flow Lab, National Institute of Metrology, P. R. China
E-mail: huhm@nim.ac.cn

Abstract

Geometric parameter measurement system for multi-path ultrasonic flowmeters based on Leica total station has been established, which can directly measure the 3D coordinates of the conduit points and sensor points and automatically calculate geometric parameters of the flowmeter. Compared with the traditional method using the theodolite and steel tape, the first advantage of this system is the high efficiency and good accuracy, and the second is the path heights can be re-measured after the sensor mounted. Cylinder fitting is its key problem. Lukács algorithm is introduced for the cylinder fitting, and Monte-Carlo simulation method is introduced for its uncertainty estimation. To illustrate the rationality and validity of this measurement system, the calibration data and its uncertainty of 3 flowmeters in three gorges power station have been comparative analysed. Besides, the influences of the water temperature and pressure on the diameter of three gorges steel penstock and then on the other parameters have been estimated. At last, the effect on the flow rate of the path heights which are different from the design values has been analysed. When this effect is not ignorable, the analytical formulas has been deduced for the weight coefficient correction, which can get satisfactory weight coefficient.

Key words: ultrasonic flowmeter, geometric parameter, total station, Faro arm, cylinder fitting, uncertainty, weight coefficient

Introduction

With advantages of no pressure loss, high accuracy and the possibility to be installed on existing conduits, multi-path ultrasonic flowmeter has become one of the best choices for large diameter water flow measurement.

Geometric parameters are very important for the ultrasonic flowmeter which calculate the flow velocity and the flow rate based on time measurement and length measurement. According to the basic principle of multi-path ultrasonic flowmeter, the flow rate is directly related to the geometric parameters of flowmeter as follows,

$$q_v = R^2 \sum_{i=1}^N W_i (h_i) \frac{L_i}{\cos \phi_i} f(t_{di}, t_{mi}) \quad (1)$$

where R is the radius of the conduit; h_i is the relative height of acoustic path; W_i is the weight coefficient related to h_i ; L_i is acoustic path length; ϕ_i is acoustic path angle; $f(t_{di}, t_{mi})$ is a function related to the ultrasonic propagation times. The Subscripts i denotes the different acoustic paths. There is a strict installation requirement of the multi-path

ultrasonic flowmeter. The transducers must be installed carefully on the design positions of the conduit to guarantee the accuracy of flow rate measurement, and the geometric parameters should be measured accurately and then be input into the flowmeter console. For the large diameter flowmeter, theodolite is usually used to locate the transducers, then steel tape is used to measure the geometric parameters. The acoustic path lengths and conduit diameter are measured by steel tape, and the acoustic path angles are measured by theodolite. The inaccurate measurement of geometric parameters will affect the flowmeter accuracy. Currently, some manufacturers declare that the accuracy of their large flowmeters can reach up to 0.5%, and this will only be possible if the accurate measurement of geometric parameters is done with scientific method and sophisticated instruments.

Measurement System and Method

Main instrument and method

The typical layout of acoustic paths of the flowmeter is shown as Fig.1. It is a flowmeter with a configuration of two planes with nine paths each. The geometric parameters to be measured include radius of conduit section R , length of acoustic path L_i which is the distance between a pair of transducers, angle of acoustic path ϕ_i which is the acute angle between acoustic path and flow direction, and height of acoustic path h_i which is the spatial distance between acoustic path and conduit axis. At present, the first three parameters of R , L_i and ϕ_i are generally measured, and the path height h_i is usually regards as its predetermined value after the completion of installation because of the measurement difficulties.

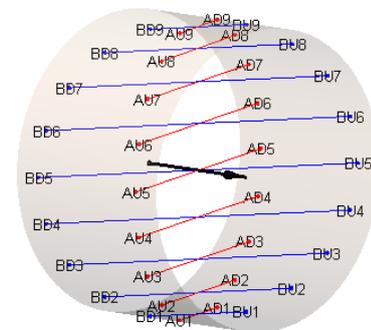


Fig.1 Schematic Drawing of Acoustic Path Layout

Total station is a new generation instrument which could directly obtain the 3-D coordinates of the target points. Leica TCRM1201+R400 total station has been adopted by our measuring system. It can operate in ranging mode

without prism with accuracy of 2mm+2ppm, and its angle measurement accuracy is 1". First, the total station is used to collect the coordinates of wall points of flowmeter conduit, and cylinder fitting is conducted to obtain the conduit radius and conduit axis. Next, central-point coordinates of the transducers' end faces are measured one by one. The lengths of acoustic paths are calculated directly and the angles and heights of acoustic paths are calculated based on the conduit axis and transducer points.

Traditional theodolite method needs a reference axis determined by two target gauge points. The process of arranging this axis is quite complicated and the accuracy is difficult to control. However, the total station method could take advantage of enough conduit wall points to fit the cylindrical surface. It promotes the measurement accuracy of radius of conduit section and angles of acoustic paths.

The software set up in the total station itself is very powerful, but its most functions would not be used in our application. The measurement may be low-efficient and the erroneous measurement data could not be identified timely. Aiming at the application in geometric parameter measurement of ultrasonic flowmeter, we have developed a dedicated software for the total station.

Software and procedure

Dedicated software has been developed for automatic scanning of the conduit points with total station and automatic calculation of all the geometric parameters. The software runs on portable notebook, which can control the total station to scan the conduit wall points and receive data from it. It is easy to perform the cylinder fitting and geometric parameters calculation accurately, and it is not only compatible with the total station, but also possible to

import the data from other 3-D coordinate measurement instruments for only parameter calculation. The main interface is shown in Fig.2 as follows.

Cylinder fitting is performed based on these conduit wall points, as shown in Fig.3(a). The fitting algorithm is used for the calculation based on the 3 points on the cross section. The fitting result is obtained directly. The distance between the measured wall points and the fitted wall could be marked in different colors, and the outliers by n times of standard deviation can be removed automatically.

There is a special design for the measurement of the central point of the transducer, as shown in Fig.3(b). Those points could not be scanned automatically, and could only be measured by aiming the telescope at the center of the transducer. According to the layout information of acoustic paths of the flowmeter, the planar positions and serial numbers of the transducer points could be generated automatically. The measurement starts after aiming at the corresponding position and clicking the serial number. In this way, the confusion caused by manual numbering can be avoided, and the efficiency of the measurement is improved.

Algorithm of cylinder fitting

Cylinder fitting is the core algorithm of this measurement system. It is the problem of how to achieve the minimum of the square sum of distances between all sampling points and the cylindrical surface. Since the cylindrical surface equation is a nonlinear quadratic equation, solving the least square through quadratic equation may not result in convergence correctly. So it is crucial to parameterize the cylindrical surface reasonably.

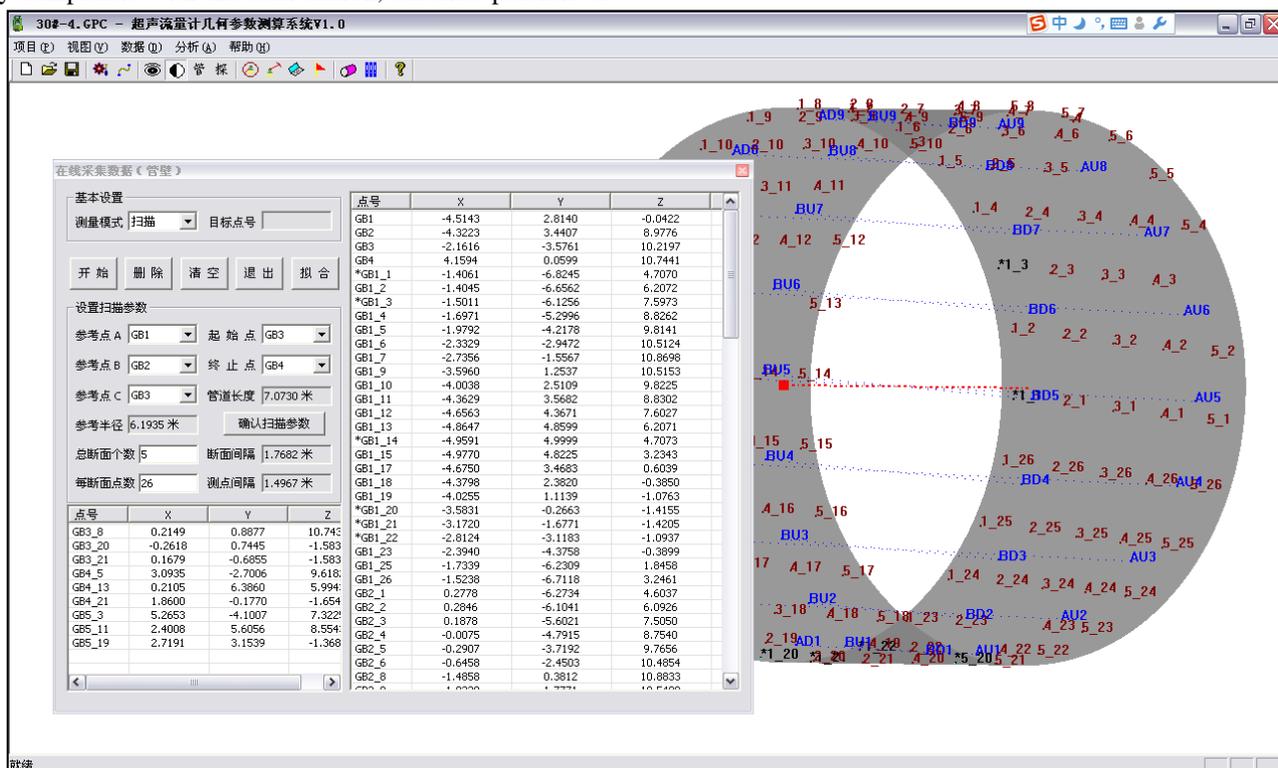
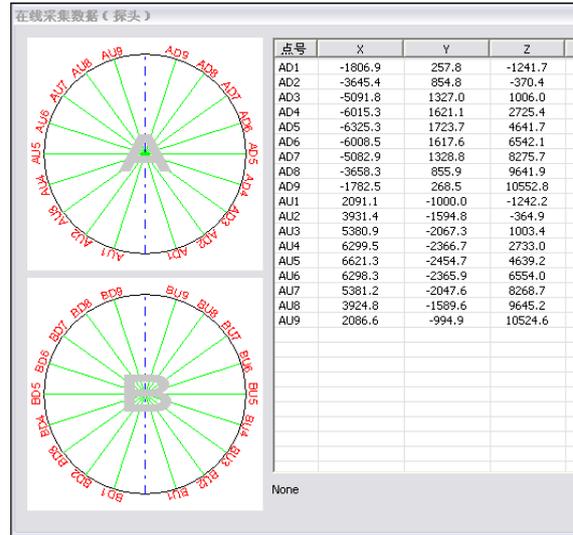


Fig.2 Main Interface of the Measurement Software



(a) Fitting Process of Cylindrical Surface

(b) Collection of Transducer Points

Fig.3 Main Functional Interface of the Measurement Software

As shown in Fig.4, the coordinate origin is point O_0 and the direction vector of cylindrical surface axis C_1C_2 is \mathbf{a} . The foot of perpendicular of origin O_0 on axis C_1C_2 is O' , and direction vector of O_0O' is \mathbf{n} . Obviously, \mathbf{n} is the normal vector of cylindrical surface, $\mathbf{a} \cdot \mathbf{n} = 0$. $\rho\mathbf{n}$ is the nearest point on the cylindrical surface from origin O_0 . κ is the largest curvature value at point $\rho\mathbf{n}$, and $1/\kappa$ denotes the radius of cylindrical surface. \mathbf{a} and \mathbf{n} are two mutually perpendicular unit vectors, which possess 3 degrees of freedom, and could be represented by three angles as follows,

$$\mathbf{n} = (\cos\varphi \sin\vartheta, \sin\varphi \sin\vartheta, \cos\vartheta) \quad (2)$$

$$\mathbf{a} = (\cos\varphi \cos\vartheta, \sin\varphi \cos\vartheta, -\sin\vartheta) \cdot \cos\alpha + (-\sin\varphi, \cos\varphi, 0) \cdot \sin\alpha \quad (3)$$

where ϑ is the angle between \mathbf{n} and z-axis; φ is the angle between the projection of \mathbf{n} on surface $z=0$ and x-axis; α is the angle between \mathbf{a} and vector $(\cos\varphi \cos\vartheta, \sin\varphi \cos\vartheta, -\sin\vartheta)$. In this way, cylindrical surface is parameterized into $S = (\varphi, \vartheta, \alpha, \rho, \kappa)$ with 5 degrees of freedom.

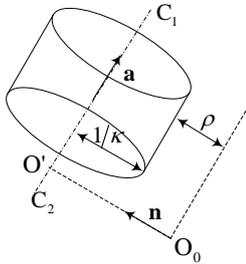


Fig.4 Parameterization of Cylindrical Surface

Levenberg-Marquardt algorithm is adopted in the minimizing process. The algorithm converges rapidly, but it is very sensitive to the initial values of the parameter especially the three angles related to axis. If the initial values are chosen inappropriately, it is difficult to converge, and even go to the incorrect local minimum solution. For

the geometric parameter measurement of flowmeter, it is more efficient to calculate the initial values by using a group of measurement points. This group of points should be located on a certain vertical cross-section of cylindrical surface. There are three points at least, and the normal vector of the plane where the points are located is the axis of conduit given by linear least square method, and the initial values of other parameters could be obtained further.

Determination of the wall point number

Obviously, the more the measured points on the wall, the better the fitting would be. A appropriate point number should be decided to ensure the fitting accuracy. Monte-Carlo simulation is employed to analyze the relation between the number of measured points and the fitting effect.

For any given parameter of the cylindrical surface S , the axis length is set as L_a . A group of n precise wall points (x_i, y_i, z_i) would be generated by rotating the matrix about an axis. The accuracy of coordinate measuring instrument could be converted into the standard uncertainty of the coordinates, which is $\delta(\Delta)$. The measurement error is assumed to obey normal distribution. A group of n random numbers $\Delta x_i, \Delta y_i$ and Δz_i that obey the normal distribution and have the standard deviation $\delta(\Delta)$ could be generated. Thus, coordinate $(x_i + \Delta x_i, y_i + \Delta y_i, z_i + \Delta z_i)$ is a group of possible measured data. Cylindrical surface simulation could be conducted to obtain radius R and axis $\mathbf{a} = (x_a, y_a, z_a)$ by using this group of data. In order to make the axis result more intuitive, $\delta(\mathbf{a})$ has been converted to a angle value $\delta(\theta_a)$.

Simulation data prove that the values of $\delta(R)$ and $\delta(\theta_a)$ are related with $\delta(\Delta)$ and n . The values of n are shown in Table 1. It could be derived that the uncertainty of radius

$\delta(R)$ caused by coordinate measuring is smaller than the uncertainty $\delta(\Delta)$ of the coordinate itself. Both $\delta(R)$ and $\delta(\theta_a)$ increase with $\delta(\Delta)$, and decrease with the increasing of n . There is an approximate linear relation between $\delta(R)$, $\delta(\theta_a)$ and $\delta(\Delta)$. It should be noted that $\delta(\theta_a)$ is related to the length of the measured points on wall surface projected in the axial direction, which is L_a , showing an approximately inverse proportion. However $\delta(R)$ and L_a have no association.

Table 1 Uncertainty of Radius and Axis Direction ($L_a = 2m$)

Point number n	$\delta(\Delta)=1mm$		$\delta(\Delta)=2mm$		$\delta(\Delta)=5mm$	
	$\delta(R)$ /mm	$\delta(\theta_a)$ /°	$\delta(R)$ /mm	$\delta(\theta_a)$ /°	$\delta(R)$ /mm	$\delta(\theta_a)$ /°
10	0.47	0.10	0.92	0.21	2.31	0.52
20	0.26	0.05	0.51	0.11	1.27	0.27
30	0.20	0.04	0.39	0.08	0.98	0.20
50	0.15	0.03	0.30	0.06	0.74	0.15
100	0.10	0.02	0.20	0.04	0.51	0.10

The fitting is performed based on the simulation data,

$$\delta(R) = \delta(\Delta) \times (4.15/n + 0.06) \quad (4)$$

$$\delta(\theta_a) = \delta(\Delta) / L_a \times (4.15/n + 0.06) \times 420^\circ \quad (5)$$

The equation could be utilized for guiding the selection of the number of measured points on the wall. Generally, about 50 measured points are enough.

Analysis of Three Gorges Flowmeter

Measurement detail

The measurement system is exploited to calibrate the three flowmeters of the Three Gorges Hydropower Plant. All of them are with a diameter of about 12.4m. Fig.5(a) is a picture of transducers installation with scaffolding using theodolite. Fig.5(b) is a picture of conducting the calibration of flowmeter geometric parameters with total station. When measuring the transducer point, a bright flashlight is used to light up the transducer area. An accurate alignment of the center position of the transducers is performed with the theodolite telescope. The telescope is not available for alignment when measuring the area near the zenith. Therefore, a diagonal eyepiece is used to solve the aiming problem of the blind area.

There are welding seams on the conduit of the Three Gorges flowmeter, which are taken as the reference objects. The precision of the cylinder fitting would be improved by scanning the points on each ring. In the scanning process of conduit wall points and the measurement of transducer points, a fixed datum point has been kept watch on repeatedly for several times. The variation of the coordinates of this point should not exceed 2mm, or it would mean that the total station might have been moved or some other problems have occurred.



(a) Transducer Installation with Theodolite



(b) Geometric Measurement with Total Station

Fig. 5 Multi-path Flowmeter of Three Gorges Power Station

Analysis of result and its uncertainty

As the geometric parameters of flowmeters used for the Three Gorges are the data measured by the manufacturer, the flow rate measurement errors brought by the difference between manufacturer data and calibration data are analyzed. The flow error due to radius measurement is two times of that of radius error, and the errors of path length and path angle are related to the flow rate distribution and weight coefficients corresponding to the acoustic paths. According to $u_m = (1-r)^{1/9}$, the formula of flow rate distribution of fully-developed turbulent flow in the circular pipe, together with the weight coefficients of different paths of the corresponding flowmeters, the flow measurement errors caused by factory defaults of geometric parameters of the three flowmeters are respectively calculated, as shown in Table 3. From the data we can see radius error is most possibly the main error of the geometric parameters. The path angle errors are not so large because of compensation effect of the two-plane path configuration.

Table 2 Flow Errors due to Geometric Parameter Measurements

Flowmeter	Radius	Path length	Path angle	Total error
A	0.12%	0.03%	0.01%	0.16%
B	0.13%	0.01%	0.00%	0.14%
C	0.03%	0.01%	0.03%	0.07%

As all of the geometric parameters are measured by the same instrument, the path lengths are strongly correlated and the path angles are also strongly correlated. Moreover, the time-related terms can be expressed as the relation between path velocity u_{mi} and length of acoustic paths,

$$f(t_{di}, t_{ui}) = \frac{1}{t_{di}} - \frac{1}{t_{ui}} = \frac{c + u_{mi} \cos \phi_i}{L_i} - \frac{c - u_{mi} \cos \phi_i}{L_i} = \frac{2u_{mi} \cos \phi_i}{L_i} \quad (6)$$

Therefore, the compositing formula of the uncertainty related to the geometric quantity is simplified into

$$\left(\frac{u_c(q_v)}{q_v}\right)^2 = \left(2\frac{u(R)}{R}\right)^2 + \left(\sum_{i=1}^N W_i u_{mi} \frac{u(L_i)}{L_i}\right)^2 + \left(\sum_{i=1}^N W_i u_{mi} \tan \phi_i u(\phi_i)\right)^2 \bigg/ \left(\sum_{i=1}^N W_i u_{mi}\right)^2 \quad (7)$$

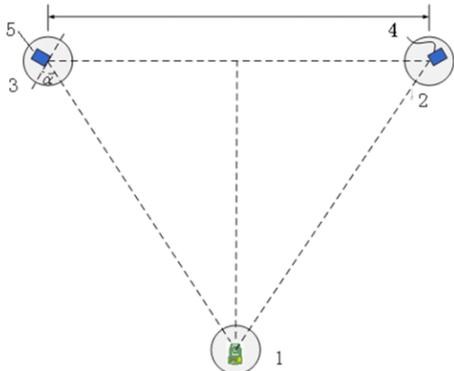
Based on formula (7), uncertainty contribution of geometric parameter to flow rate measurement is calculated. The standard uncertainty is about 0.10%, and the contribution magnitudes of each source are in table 3 as below. In the short-distance application of total station, angle measurement error has a small contribution to measurement uncertainty, and there is only the distance measurement error with 2mm+2ppm. When the transducer central points are measured, telescope is required to aim at the measured point, and an additional aiming error will be included.

Table 3 Flow Uncertainty Introduced by Geometric Parameters

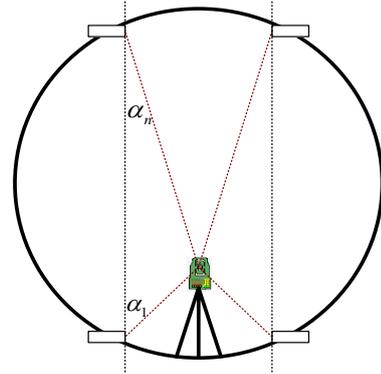
Uncertainty components		Flowmeter		
		A	B	C
Conduit radius	Repeatability	0.01%	0.01%	0.02%
	Total station	0.07%	0.07%	0.07%
	Cylinder fitting	0.01%	0.01%	0.03%
Path length	Repeatability	0.01%	0.00%	0.01%
	Total station	0.03%	0.03%	0.03%
Path angle	Repeatability	0.01%	0.00%	0.04%
	Total station	0.03%	0.03%	0.03%
	Telescope aiming	0.04%	0.04%	0.04%
Combined uncertainty		0.09%	0.09%	0.10%

Confirmation of uncertainty of total station

Our length measurement department only provides certification of angle measurement and prism ranging for total stations, which cannot reflect the accuracy of ranging without prism. Therefore, a set of evaluation system for comparison is designed. The centers of end faces of two small cylinders are adopted to determine a standard distance. FaroArm platinum is used to measure the standard distance, with accuracy of 0.035mm. The standard distance is compared to the measured value by total station in non-prism mode to analyze the ranging error as in fig.6(a).



(a) Test for Non-prism Mode



(b) Location of Total Station in Conduit
Fig.6 Schematic Diagram of Ranging in Non-prism Mode

Fig.7 shows the ranging error of the total station under different incident distances and angles, including the measured data over a long period. It can be found that the incident angle hardly affects the ranging error when the incident distance is larger. But when the incident distance is smaller, the ranging accuracy tends to be worse with small incident angle. This is lucky that only the large incident distances and small incident angles can meet together as shown in fig.6(b). On the whole, the reasonable use of the total station should ensure 2mm ranging accuracy in non-prism mode.

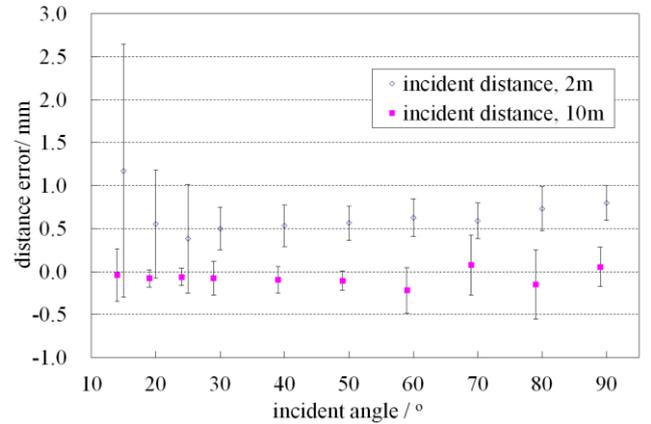


Fig.7 Distance Measurement Error of Total Station

Errors caused by vacant conduit measurement

It is worth noting that conduits of the flowmeters are vacant when measuring geometric parameters, but are full with pressure in the operating condition. The changes of upstream water head will lead to moderate deformation of the conduits. The parts outside steel penstocks where the flowmeters are located are concrete and there are cushions made of other materials in local area. The complex structure will lead to irregular deformation. Here a preliminary estimation of the influence of water pressure on the radius change is given as follows,

$$\Delta R = \frac{P \cdot R^2}{E_s t} = \frac{(0.8 \sim 1.18) \text{MPa} \times (6.2 \text{m})^2}{210 \text{GPa} \times 60 \text{mm}} = (3 \pm 0.6) \text{mm} \quad (8)$$

where P is the internal pressure of the conduit; E_s is the elastic modulus of the material; t is the conduit thickness. Due to large difference in internal pressure, the expansion of the radius is about 3mm, with relative value of 0.048%,

and the radius value needs to be corrected based on the actually measured value.

The expansion of conduits also impacts on the lengths and angles of acoustic paths. Under hydraulic expansion, only radial expansion is considered but not axial expansion, and the relative increment of length of acoustic path is 0.039%. The angle of acoustic path will also increase by 0.011°, and the corresponding flow rate increment is $\tan \phi \cdot \Delta \phi = 0.039\%$. Therefore, the radius is increased by 3mm due to internal pressure, which further increases the actual flow rate by 0.18% as shown in table 4. Due to the possible change of the internal pressure and temperature, the uncertainty of the flow rate measurement should add a term of 0.04%.

Table 4 Influence of the Change of Pressure and Temperature

Factor of influence	radius	Path length	Path angle	Total	
Flow rate error	-0.097%	-0.039%	-0.039%	-0.176%	
Relative standard uncertainty	Pressure	0.023%	0.009%	0.009%	0.026%
	Temperature	0.026%	0.013%	/	0.029%
	Total	0.034%	0.016%	0.009%	0.039%

Analysis of Path Height error and Weight Coefficient Correction

Assessment of installation error of path height

The measured acoustic path heights can be used to assess the flow rate measurement error caused by the difference between the measured acoustic path height and the design value based on the possible velocity distribution in the conduit of flowmeter. The influence of height deviation is not only relevant to weight coefficients, but also to the gradient of velocity distribution curve of acoustic paths at the heights. Although the weight coefficient of acoustic path near the edge is small, its velocity gradient is usually larger, and the influence of installation errors of acoustic path height at this point is also greater.

Based on the above formula for fully-developed turbulent flow, the influence caused by the height deviation is preliminarily assessed as shown in table 5. It is found that the absolute values of acoustic path height of flowmeter A and B are larger or smaller than the design value, and the resulting flow rate measurement errors are small. However, all the path height of flowmeter C are smaller than the designed values, which leads to ineligible positive error of flow rate measurement.

Table 5 Assessment of Installation Errors of Acoustic Path Height

Flowmeter	Installation error	Standard deviation
A	-0.01%	0.03%
B	-0.01%	0.01%
C	0.05%	0.01%

Correction of weight coefficients

In addition to the overall correction of the measured flow rate, the installation errors of acoustic path height can be reduced by using the new weight coefficients of acoustic

paths. In the Gauss integral, the weight coefficient W_i can be calculated through the following integral expression:

$$W_i = \int_{-1}^1 \rho(h) \cdot \prod_{k=0, k \neq i}^N \frac{h-h_k}{h_i-h_k} \cdot dh \quad (9)$$

where $\rho(h)$ is the weight function. Since the integral expression of the formula is complex to use, the explicit expression of the weight coefficients is deduced as below:

$$W_i = \frac{\sum_{j=1}^{(N+1)/2} g_j(\kappa) \cdot f_j(\{h_{k=1, \dots, N \text{ and } k \neq i}\})}{(1-h_i^2)^{\kappa-0.5} \prod_{j=1, j \neq i}^N (h_i-h_j)} \quad (10)$$

where for Gauss-Jacobi method, $\kappa=0.5$, but for OWICS method, $\kappa=0.6$. $f_j(\{h_k\})$ is the term with the $N+1-2j$ th power after the expansion of the polynomial $\prod_{m=1}^{N-1} (h-h_m)$, and can be expressed as

$$f_j(\{h_k\}) = \text{sum}(\text{prod}(\text{nchoosek}(\{h_k\}, j), 2)) \quad (11)$$

in matlab. In addition, $g_j(\kappa) = \int_{-1}^1 (1-x^2)^\kappa x^{2(j-1)} dx$, and the data in table 6 can be used directly.

Table 6 The Coefficient $g_j(\kappa)$ in Formula (10)

	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$\kappa=0.5$	1.5707963	0.3926991	0.1963495	0.1227185	0.0859029
$\kappa=0.6$	1.5133647	0.3603249	0.1743508	0.1063115	0.0729588

Although formula (10) is still complex in form, it could be read by the software program. For the flowmeters with 3 acoustic paths, the formula of weight coefficient can be expanded into

$$\omega_i = \frac{g_2(\kappa) + g_1(\kappa) \prod_{k=1, k \neq i}^3 h_k}{(1-h_i^2)^\kappa \prod_{k=1, k \neq i}^3 (h_i-h_k)} \quad i=1,2,3 \quad (12)$$

For the flowmeters with 4 acoustic paths, the formula of weight coefficient can be expanded into

$$\omega_i = \frac{-g_2(\kappa) \sum_{k=1, k \neq i}^4 h_k - g_1(\kappa) \prod_{k=1, k \neq i}^4 h_k}{(1-h_i^2)^\kappa \prod_{k=1, k \neq i}^4 (h_i-h_k)} \quad i=1, \dots, 4 \quad (13)$$

and so on. For the flowmeter C, the weight coefficients can be corrected by the above algorithm, and the flow rate measurement error after correction can reduce from 0.05% to below 0.01%.

Conclusion

The installation and geometric parameter measurement of transducers of ultrasonic flowmeters are very important. The installation personnel should take a serious attitude and strictly follow the procedures, otherwise inaccurate measurement of geometric parameters will affect the accuracy of flowmeters.

A set of automated measurement system of geometric parameters for large ultrasonic flowmeters is built based on Leica total station. It can scan conduit wall surface and transducer location, yielding an integrated description of geometric morphology of flowmeters. The conduit radius, as well as angles and lengths of acoustic paths are calculated, and the accuracy and efficiency of geometric parameter measurement are improved. Especially, the weight coefficients could be corrected in line with the actually measured height of acoustic path.

At present, this system can only be applied to geometric parameter measurement of flowmeters in circular conduit. It will be applied to the location and installation of transducers after slight modification, and the algorithm will be extended to flowmeters in square conduit and open channel.

Acknowledgements

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