

A Low Reynolds Number Discharge Coefficient Equation for Critical Flow Venturis and the Effects of Inlet Radius

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Abstract

The discharge coefficient, C_d , for a Critical Flow Venturi Nozzle (CFVN) corrects the theoretical mass flow to the actual mass flow at measured inlet conditions. The theoretical mass flow is calculated using 1-D isentropic theory and does not account for the subsonic boundary layer along the CFVN wall. For a precisely known throat area, C_d must be less than unity due to this boundary layer. Theory predicts that the geometry of the inlet to a CFVN will affect the boundary layer and therefore effect the C_d . This effect on C_d becomes more significant for smaller CFVNs which are operated at lower Reynolds Numbers. The international standard, ISO 9300 [1], for toroidal CFVNs allows inlet curvature to vary from 1.8 to 2.2 times the throat diameter, d , but limits the use of the Empirical C_d -Reynolds Number equation to Reynolds Numbers above 21000. Low Reynold's Number calibration data for hundreds of small CFVNs with $2d$ inlet curvature will be used to generate an Empirical C_d -Reynolds Number equation. This paper will also present the results of testing multiple CFVNs with varying inlet curvature at low Reynolds Numbers. These results will be used to examine the specific C_d sensitivity to this geometric component and determine if more stringent inlet curvature requirements are necessary for low Reynolds number CFVN applications. The Empirical C_d -Reynolds Number equation, along with additional inlet curvature guidelines will be presented as a method for calculating actual mass flow through a CFVN when the Reynolds Number is below the minimum value at which the ISO 9300 [1] equation can be applied.

1. Introduction

The theoretical mass flow for a CFVN is calculated using 1-D isentropic theory and does not account for the subsonic boundary or the curvature of the flow profile within the CFVN. The discharge coefficient (C_d) for a CFVN corrects the theoretical mass flow to the actual mass flow. Theory predicts that the geometry of the inlet to a CFVN will affect the boundary layer and possible other aspects of flow and therefore effect the C_d [2]. This effect on C_d becomes more significant for smaller CFVNs which are operated at lower Reynolds Numbers (Re). The international standard, ISO 9300 [1], provides an Empirical C_d - Re equation to Reynolds Numbers above 21000. Below this Re limit there is not a C_d equation available for calculating mas flow through a CFVN or for performing sizing calculation on CFVNs. There is a need for a low Re Empirical Equation for use below the lower limit of the ISO 9300 [1] equation. Additionally, the inlet curvature requirements from ISO 9300 [1] of 1.8-2.2 times the throat diameter ($1.8d$ - $2.2d$) need to be evaluated for low Re applications.

2. Low Reynolds Empirical Equation

Calibration data taken over 5 years from 184 toroidal throat critical flow venturis was gathered in order to generate a low Re empirical discharge C_d equation that could be used below the lower limit of the ISO 9300 [1] equation. The CFVNs varied in nominal throat diameter from 0.28mm to 2.36mm. They were all designed per the geometry guidelines in ISO 9300 [1] with nominal inlet curvature of 2 times their throat diameters ($2d$) and conical diffusers. All calibration data had uncertainties on C_d ($k=2$) of 0.2% or less. The 3613 total calibration points can be seen in Figure 1. The large scatter in the C_d values is due to the calibration method where a "nominal" throat diameter is used during data processing rather than a precise value. With this method the difference between the true throat diameter and the "nominal" throat diameter is then accounted for in the calibration C_d and as long as the same nominal value used during calibration is used during operation the value is correlated and no additional uncertainty needs to be included.

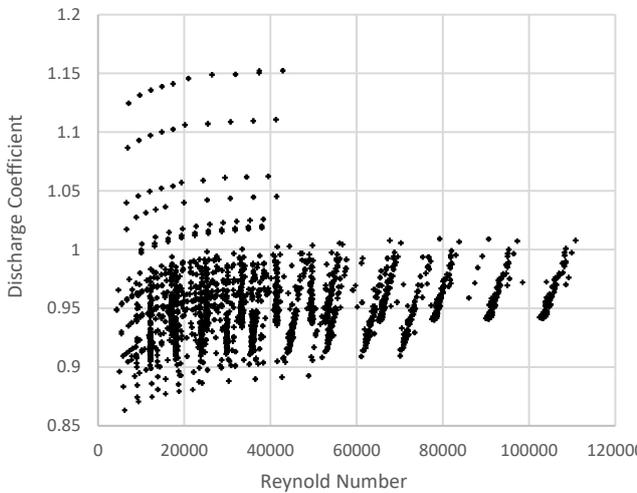


Figure 1: Calibration data from 184 CFVNs.

In order to generate an empirical equation based on this data the results needed be adjusted to correct for the difference between the nominal and actual throat diameters. In order to do this the calibration Cd-Re values must be compared against known Cd-Re values and the variation used to adjust the nominal throat diameter to a true throat diameter. Only calibrations that included Cd data within and below the ISO 9300 [1] equation Re limits was used. This allowed the points within the limits to be used to correct the throat diameter. The corrected throat diameter was then used to reprocess the calibration points below the ISO 9300 [1] limit while maintaining the curvature of the calibration fit. With the throat diameters corrected the calibration data collapses as seen in Figure 2.

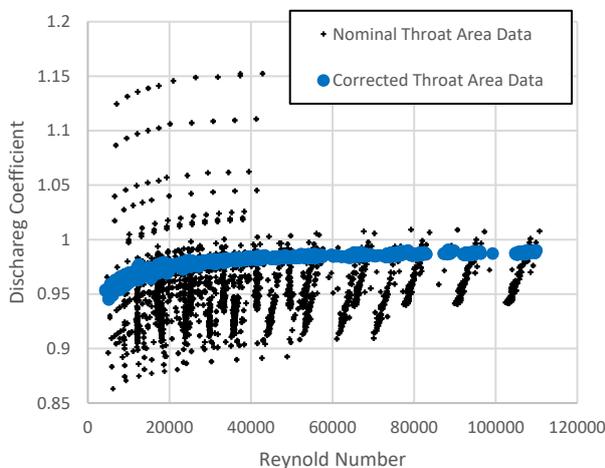


Figure 2: Calibration data from 184 CFVNs with the throat area corrected

The corrected calibration data was then compared to the extrapolated ISO 9300 [1] below the minimum Re of 21000 to see how well it represents the corrected data. Figure 3 displays the results and shows that when extrapolated, the ISO 9300 [1] equation over predicts Cd by 0.5-1.0% and a new equation should be generated.

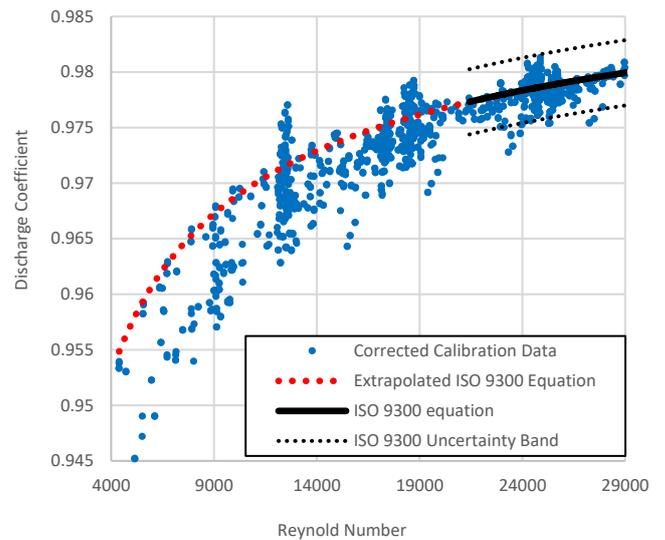


Figure 3: Extrapolated ISO 9300 Equation

To generate a best fit equation for the corrected calibration data the Cd values were plotted against $Re^{-0.5}$ which partially linearize the results as shown in Figure 4.

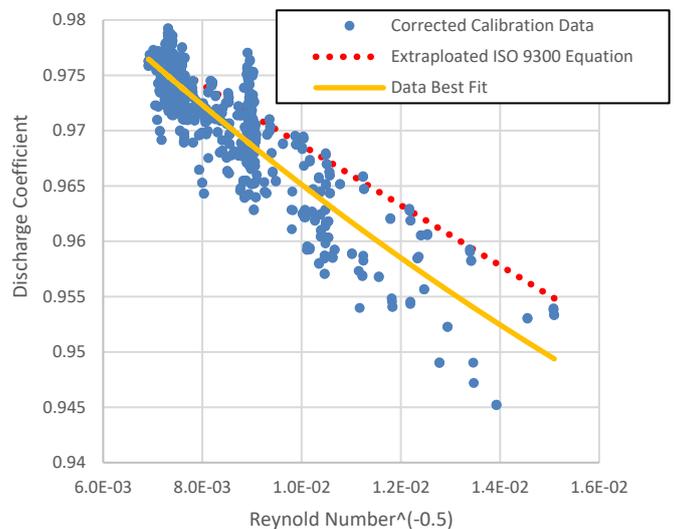


Figure 4: Best fit of corrected calibration data

The plot provides the best fit Equation 1 for the corrected calibration data.

$$Cd = 1.0068 - 4.8720 \times Re^{-0.5} + 70.895 \times Re^{-1}$$

with a range of $7000 \leq Re \leq 21000$

Equation 1: Low Re Empirical Cd Equation with Re limits

Due to the sparse data at very low Reynold Numbers the equation bounds were limited. It was decided to bound the equation by excluding the lowest 4% of the data and setting a minimum Re range of 7000. With the high concentration of data at the higher Reynold Number range no reduction in the upper bound was necessary and the equation can be used up to the ISO 9300 [1] Re limit of 21000.

In order for the new equation to be properly applied, an associated uncertainty needs to be provided. Three primary components were used to conservatively calculate the uncertainty in the empirical Cd equation. The first uncertainty component U_c , is due to the uncertainty Cd from the original 184 CFVN calibrations. All the calibration Cd values had established uncertainties less than 0.2% so that value was conservatively used for all data points. The next uncertainty component U_d , comes from the throat diameter correction. This correction was based on the ISO 9300 Cd equations which has a published uncertainty of 0.3%. The final uncertainty component U_f , is established from the residuals from the curve fit and the corrected data. In order to determine this, a +/- band on the curve fit line was establish to bound 95% calibration data. The three-primary uncertainty components were then combined using the Root-Sum-Squared method show by Equation 2.

$$U = [U_c^2 + U_d^2 + U_f^2]^{-0.5}$$

Equation 2: Uncertainty of Empirical Cd Equation

The resulting combined uncertainty of the equation was 0.65% ($k=2$). The corrected data, curve fit, and uncertainty band can be seen in Figure 5.

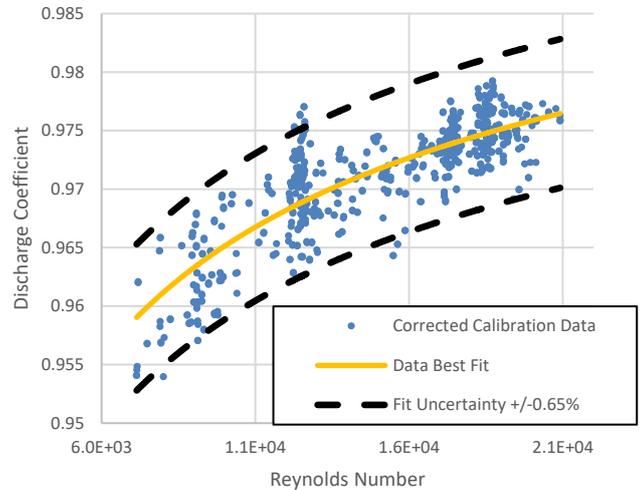


Figure 5: Empirical Cd Curve Fit with uncertainty bands

3. Inlet Curvature Effects on Cd

The corrected calibration data collapsed and provided a useful low Re empirical equation but there was still significant scatter in the data. It was hypothesized that while the inlet radius of the all the CFVNs was nominally 2 times the throat diameter, due to the difficulty of machining CFVNs with throats as small as 0.28mm, the true inlet radius may deviate significantly from nominal. In order to determine if variations in actual inlet curvature would invalidate the equation or were the primary cause of the scatter, 8 new 1.47mm throat CFVNs were manufactured and calibrated. Two each with 1.5d, 1.8d, 2.2d, and 2.5d inlet radius CFVNs shown in Picture 1.



Picture 1: CFVNs with varying inlet curvature.

The calibration results for these CFVNs are shown in Figure 6 and 7 and compared to the empirical Cd equation.

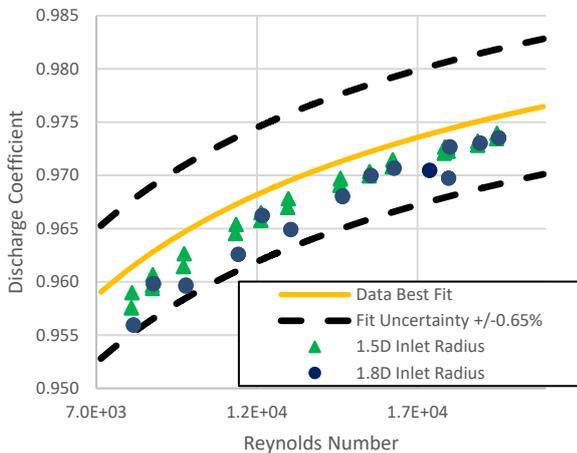


Figure 6: 1.5D and 1.8D Inlet Curvature

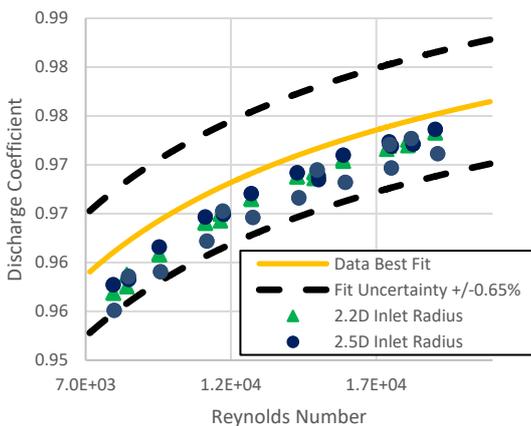


Figure 7: 2.2D and 2.5D Inlet Curvature

The results from calibrating the 1.5d, 1.8d, 2.2d and 2.5d inlet curvature CFVNs show Cd values consistently low but within the uncertainty expectations of the empirical equation. This is an unexpected result as theory predicts that the 1.5d and 1.8d inlet curvature should result in higher Cd values than 2d CFVNs as shown by the theoretical CFV flow models developed by Johnson and Wright [2]. The Johnson and Wright [2] model predicts that below a Re of 150,000 a decreased inlet curvature results in larger Cd values. These results suggest that the Low Re number scatter is due to a factor other than inlet curvature. Manufacturing variations in surface finish or the transition from the inlet curvature to the diffuser cone are other possible causes of this scatter. The results do however show that data taken from with inlet curvature from 1.5D

to 2.5D falls within the uncertainty expectations of the empirical Cd equation allowing reduced curvature restriction than the ISO 9300 [1] equation.

4. Conclusion

Calibration data was collected from 184 CFVNs in order to determine an Empirical Cd-Re curve fit that could be used below the ISO 9300 [1] equation minimum Reynolds Number of 21000. All the calibrations included data within the ISO 9300 [1] range which was used for throat diameter corrections from nominal to actual.

The following equation was established:

$$Cd = 1.0068 - 4.8720 \times Re^{-0.5} + 70.895 \times Re^{-1}$$

with a range of $7000 \leq Re \leq 21000$ and an uncertainty of 0.65% ($k = 2$)

Equation 3: Low Re Empirical Cd Equation with Re limits and Uncertainty

The equation is valid for toroidal throat CFVNs built per the ISO 9300 requirements and seems to allow for an inlet curvature range as wide as 1.5d to 2.5d.

This equation can be used as a more accurate tool for sizing low Reynold number CFVNs than extrapolating the ISO 9300 equation. It can also be combined with a standard throat measuring dimensional technique such as pin gauges to provide a Cd*A value for higher uncertainty flow rate calculations. As an example; for a 5mm throat CFVN using a +0.00/-0.02 mm gauge pin set and the developed empirical equation an uncertainty in Cd*A of approximately 1.2% ($k=2$) is achievable. Further research into the effects of manufacturing techniques on low Re CFVN Cds could be used to further reduce the scatter and uncertainty in the Empirical Cd equation.

References

- [1] ISO, Measurement of Gas Flow by Means of Critical Flow Venturi Nozzles, ISO 9300, 1st edition, 1990, 2nd edition, 2005.
- [2] Johnson, A., and Wright, J., "Comparison Between Theoretical CFV Flow Models and NIST's Primary Flow Data in the Laminar, Turbulent, and Transition Flow Regimes", Journal of Fluids Engineering, Vol 130, 071202-1 through -11, 2008.