Problems to note when using the nozzle to nozzle test method

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Abstract

The strange behaviors of the discharge coefficient of the critical nozzle were found when gradually changing the back pressure ratio (Pd/Pu) from the almost zero to the critical back pressure ratio in the nozzle to nozzle test method. When the test gas is Helium, the change amount of the discharge coefficients at these strange behaviors is quite large and seems significantly to influence to flow measurements. The causes of the strange behaviors are different in the ranges of Pd/Pu<0.1 and of Pd/Pu>0.1, and in either case are considered to be due to the measurement principle of the nozzle to nozzle test method that the mass flow rate through the system is constant. Under the condition that the mass flow rate is constant, the discharge coefficient of the critical nozzle on the downstream side in the nozzle to nozzle test method must definitely change in inverse proportion to its upstream pressure change. The flow models are suggested to explain the reason why the upstream pressure changes when changing the back pressure ratio. The more important point of the present results is that the equivalence of the discharge coefficient determined by the general calibration methods and by the nozzle to nozzle test method may be not always hold.

1. Introduction

A nozzle to nozzle test method is commonly used to evaluate the characteristics of critical nozzles, for example, to decide the discharge coefficient or to examine a critical back pressure ratio. In this test method, a back pressure ratio of a critical nozzle under test is constant during measurements. On the other hand, when a critical nozzle is calibrated by a PVTt method or a gravimetric method, a pressure in a constant volume tank or in a collecting tank increases continuously during calibration. Therefore, its back pressure ratio is not constant on the determination of discharge coefficient. So far, there has been no doubt about the equivalence of the discharge coefficients determined by these calibration methods at least. However, the recent experimental results have raised some doubt about their equivalence. One of them is a premature unchoking[1][2]. When a critical nozzle is calibrated by a PVTt method or a gravimetric method, whether a premature unchoking occurs cannot know in advance so that the discharge coefficient obtained is possible to include significant error. On the other hand, the nozzle to nozzle test method can decide a discharge coefficient by avoiding the range of the back pressure ratio where the premature unchoking occurs. From that respect, the nozzle to nozzle test method can be said to be a reliable and useful method. This paper describes about the strange behaviors of the discharge coefficient found in the nozzle to nozzle test method. The present results raise another and more serious problems for the equivalence of discharge coefficient determined by these calibration methods.

2. Experimental apparatus and dimensions of critical nozzles

The nozzle to nozzle test method used in the present experiments is shown in Fig.1. The standard critical nozzles are calibrated by the gravimetric system of HIRAI Co., Ltd., which is the accreditation laboratory and has the gravimetric system similar to NMIJ. The standard critical nozzle and the critical nozzle under test are connected in series and the discharge coefficient of the latter critical nozzle is calculated from Eq.(1). When the diameters of both critical nozzles and the upstream condition of the standard critical nozzle are given, the upstream pressure of the critical nozzle under test is uniquely decided from Eq.(1).

$$C_d = C_{dS} \times \left(\frac{d_S}{d}\right)^2 \times \left(\frac{P_S}{P_u}\right) \times \sqrt{\frac{T_u}{T_S}} \qquad (1)$$

Here, Cd : a discharge coefficient, d : a throat diameter, P and T : an upstream pressure and a upstream temperature of critical nozzles. Subscript "*s*" and "*u*" mean the standard critical nozzle and the critical nozzle under test, respectively. Both critical nozzles used here are the troidal throat Venturi nozzle and made of Nickel, which are manufactured by Toray Precision Co., Ltd. The throat diameters of the standard critical nozzle and the critical nozzle under test are 0.248 mm and 0.424 mm, respectively. Their inlet curvature is about 2*d*, their diffuser length is about 3 *d* and their diffuse half angle is about 3 degrees. The expanded relative uncertainty of Cds is 0.13% for Nitrogen and 0.24% for Helium.

The pressure sensors to measure Ps and Pu are YOKOGAWA 2653-S7, which are calibrated by the piston gauge of HIRAI from 50 kPa to 700 kPa and their standard uncertainties are 10 Pa. The four wire Pt resistance thermometer to measure Ts and Tu are also calibrated at HIRAI and their standard uncertainties are 0.1 deg.C. The automatic pressure controller developed by HIRAI can control the setting pressure with ± 10 Pa ~ ± 30 Pa, depending on a setting pressure and a flow rate.

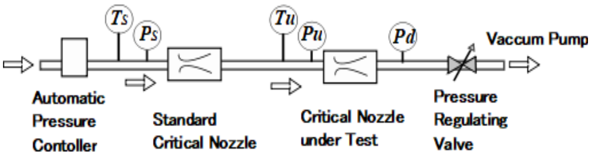


Figure 1 Experimental apparatus of the nozzle to nozzle test method

3. Experimental results

Figures 2 show the behaviors of the discharge coefficient of the critical nozzle under test when changing a back pressure ratio. Cd_0 of the symbol of the vertical axis is the discharge coefficient at Pd/Pu=0.1. Figure 2(a) is the results of Helium and Fig.2(b) is that of Nitrogen. The theoretical Reynolds numbers R_{eth} is adjusted as close as possible in comparing both experimental results, though it is difficult to adjust R_{eth} in these gases. The theoretical Reynolds number is defined as follows;

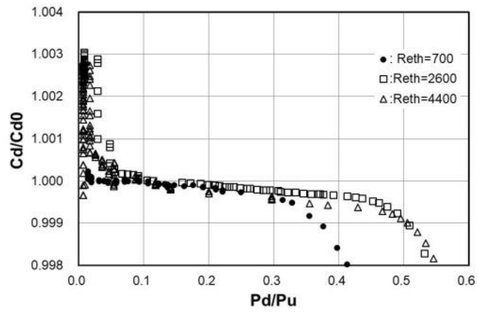
$$R_{eth} = \frac{Q_{mth} d}{A\mu}$$

Here, Q_{mth} : a theoretical mass flow rate given in Eq.(3), $A = d^2\pi/4$, μ: a viscosity of gas.

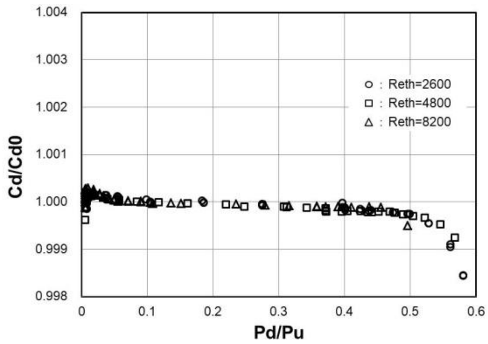
One of the strange behaviors found in these figures is the peaks of discharge coefficient in the range of Pd/Pu<0.1. The maximum peak value in Helium is quite large and reaches to about 0.3% larger than the value of Pd/Pu=0.1. On the other hand, the maximum value in Nitrogen is only 1.0004 times the value of Pd/Pu=0.1 at maximum. Another is the behaviors of the discharge coefficient in Pd/Pu>0.1 found in Helium. The discharge coefficients gradually decrease toward the critical back pressure ratio, at which the discharge coefficient decreases quickly. It should be noticed that the decrease of the discharge coefficients even in Nitrogen is slightly but definitely recognized. In this paper, the flow models are suggested to explain the reasons why these strange behaviors of the discharge coefficient occur.[3]

3.1 Behaviors of the discharge coefficient in Pd/Pu<0.1

Figures 3 show how the discharge coefficient changes with time. The numerals in Figs.3 are the values of Pd/Pu, which is kept constant in the range of the horizontal arrow, and the vertical arrows indicate the

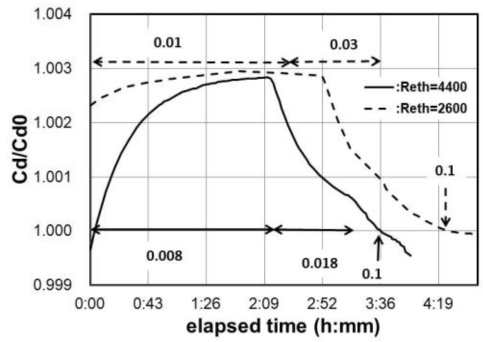


(a) Helium

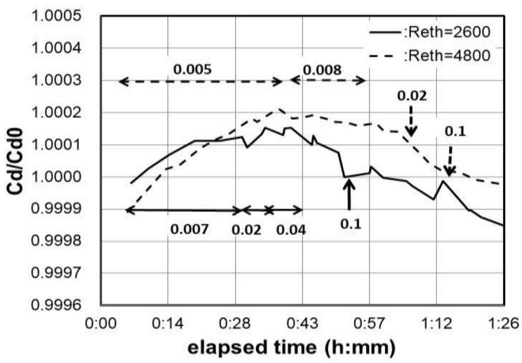


(b) Nitrogen

Figure 2 Behaviors of the discharge coefficient against the back pressure ratio



(a) Helium



(b) Nitrogen

Figure 3 Variation of the discharge coefficient with time position of the value of *Pd*/*Pu*.

Next, let's look at the results of Helium at *Reth*=4400 in Fig.3(a) in details. When *Pd*/*Pu*=0.008, the discharge coefficient increase gradually to the maximum value over about two hours. When the back pressure ratio changes from 0.008 to 0.018, the discharge coefficient begins to decrease toward *Pd*/*Pu* = 0.1. The situation is similar in the case of *Reth* = 2600. In Nitrogen of Fig.3(b), the changes of the discharge coefficient are much gentle compared with that in Helium. In both R_{eth}=2600 and R_{eth}=4800, the discharge coefficients increase gradually and reach to their maximum value after about 40 minutes, and then the discharge coefficients begin to decrease gradually.

Figures 4 (a) and (b) show how the pressure changes with time in Helium at *Reth*=4400 and in Nitrogen at *Reth*=4800, respectively. *Ps* is the upstream pressure of the standard critical nozzle and *Pu* is the upstream pressure of the critical nozzle under test. The change of *Ps* is about 0.05% in Helium and 0.02% in Nitrogen so that *Ps* in both gases is considered to be constant during measurements. *Pu* in Helium changes largely and its change amount reaches to 0.3 %. On the other hand, the change of *Pu* in Nitrogen is only 0.04% and is about the same as that of *Ps*, but the aspect of the change of *Pu* is similar to that of Helium, that is, increases after decreasing. It is understood from Fig.3 and Fig.4 that the change of the discharge coefficient of the critical nozzle under test and that of its upstream pressure are linked each other.

The behaviors of the discharge coefficient found in Figs.3 might be considered to be able to explain as the thermal non-equilibrium of a nozzle body and a flow field. However, the discharge coefficients of Figs.3 increase and then decrease taking long time as the back pressure ratio changes and does not seem to approach to a certain steady state in the range of *Pd*/*Pu*<0.1. Furthermore, the pressure changes shown in Figs.4 would not be explained by the thermal non-equilibrium.

Therefore, another idea is proposed to explain the reason that the discharge coefficient changes.

Equation (2) is the definition of *Cd*.

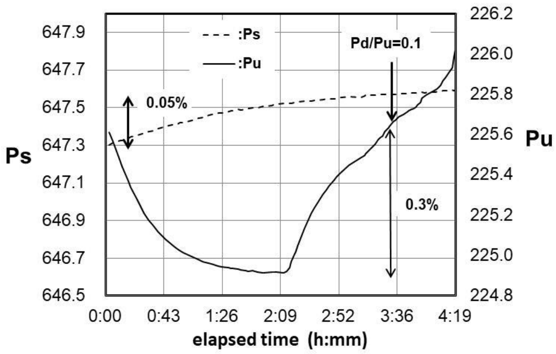
$$C_d = \frac{Q_m}{Q_{mth}} \quad (2)$$

Qmth in Eq.(2) is a theoretical mass flow rate and given as follows;

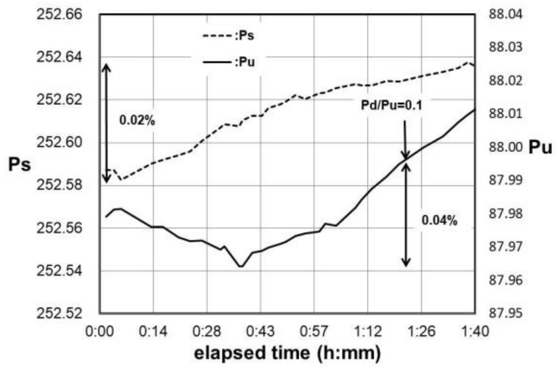
$$Q_{mth} = P_u A^* C^* \sqrt{\frac{N_g}{\mathcal{R} T_u Z_u}} \quad (3)$$

Here, *Qm* : a mass flow rate, *A**: a cross sectional area of throat, *C**: a critical flow function, *Ng*: a molecular weight of gas, $\mathcal{R}$: an universal gas constant, *Pu*, *Tu*: a pressure and a temperature upstream of critical nozzle and *Zu*: a compressible factor at upstream condition.

The change of *Cd* can simply be explained as the change of the effective sonic plain area, which is caused by the change of the boundary layer thickness at the throat. That is, when the boundary layer becomes



(a) Helium : R_{eth} = 4400



(b) Nitrogen : R_{eth} = 4800

Figure 4 Variation of the upstream pressure in *Pd*/*Pu* < 0.1

thinner, the effective sonic plain area increases. At this time, if the upstream condition is maintained, more mass flow flows and *Cd* increases. If the boundary layer becomes thicker, the opposite occurs.

However, in the nozzle to nozzle test method, the mass flow rate through the system is equal to that obtained from the standard critical nozzle and is constant as long as the upstream condition of the standard critical nozzle does not change. If the mass flow rate through the critical nozzle is constant, when the boundary layer becomes thinner and the effective sonic plain area increases, *Pu* must decrease to keep a mass flow rate constant and resultantly *Cd* must increases. The problem is why the boundary layer becomes thinner and why *Pu* decreases taking a long time.

The flow field at the nozzle exit is simply considered to consist of a core flow having almost constant Mach number and a boundary layer having subsonic velocity. When a back pressure ratio is less than *Pd*/*Pu*=0.1, a flow field outside a nozzle exit is in state of an under expanded jet and a strong expansion wave generates from the edge of nozzle exit. At this time, a flow along a boundary layer, which develops over the diffuser wall including the throat area, is strongly accelerated so that the boundary layer on the nozzle wall becomes thinner and resultantly the effective sonic plain area becomes larger. Therefore, as mentioned above, when the effective sonic plain becomes larger, the upstream pressure decreases and resultantly the discharge coefficient increases under the condition that the mass flow rate is constant like in a nozzle to nozzle test method. It might take a long time to reach the stable upstream pressure condition because the flow filed changes slowly by the pressure wave traveling from the downstream through the thin boundary layer.

When a flow field at a critical nozzle exit is an under expanded jet, a core flow in a diffuser is an isentropic flow from a throat to a nozzle exit so that the Mach number, the pressure and the cross sectional area of the diffuser have a following relations;

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/(\gamma-1)} \quad (4)$$

$$\frac{P_0}{P_e} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \quad (5)$$

Here, A and M : a cross sectional area of diffuser section at any place and the Mach number at that place, respectively, Ae : a cross sectional area of a nozzle exit, $P0$: a stagnation pressure, Pe : a nozzle exit pressure, γ : a specific heat ratio.

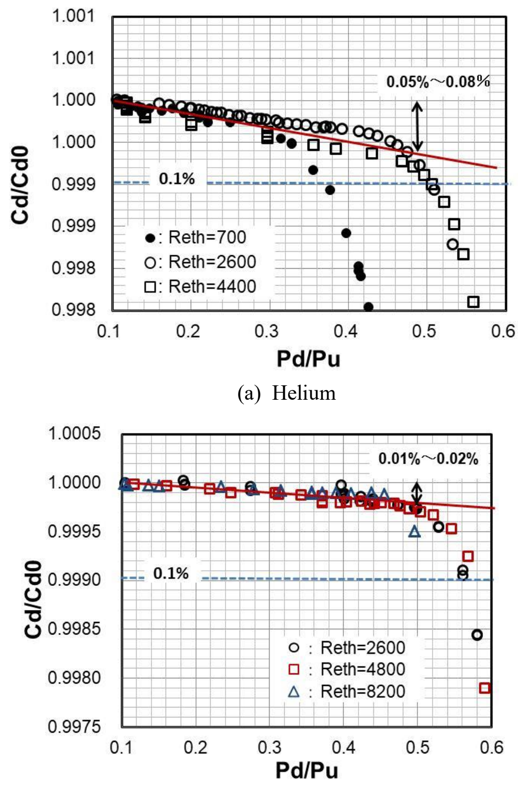
As (Ae/A^*) of the critical nozzle under test used here is 1.72, Me and $Pe/P0$ are 2 and 0.12 in Nitrogen, and 2.2 and 0.09 in Helium from Eq.(4) and Eq.(5). The back pressure ratio at the nozzle exit, $Pe/P0$, is almost coincident to the back pressure ratio that the strange behaviors of the discharge coefficient found in Figs.3 disappear. This simple calculation results might support qualitatively the explanation that the behaviors of the discharge coefficient in the region of Pd/Pu<0.1 is caused by the strong expansion fun generated at the nozzle exit. The differences of the behaviors of the discharge coefficient between Nitrogen and Helium would come from the difference of the characteristics of the boundary layer due to thermal properties of these gases. However, it is difficult quantitatively to confirm this explanation by experiments.

The influence of this undesirable behavior of the discharge coefficient found in Pd/Pu<0.1 would be able to avoid by using a critical nozzle under the condition of Pd/Pu>0.1. However, this problem is not so simple. When a critical nozzle is calibrated by a PVTt method or a gravimetric method, a test gas flows into a constant volume tank or a measuring tank from an almost zero back pressure ratio to a near critical back pressure ratio so that the discharge coefficient of the critical nozzle obtained should be noticed to be the value at an average value of back pressure ratios. The change of discharge coefficient found in Pd/Pu<0.1 in Nitrogen is at most 0.03% so that its change may be negligible within measurement uncertainty. However, in the cases that a test gas is Helium or the high precision measurement is required in Nitrogen, this change of the discharge coefficient in Pd/Pu<0.1 would be unable to neglect and the discharge coefficient obtained might be suspicious.

3.2 Behaviors of discharge coefficient in Pd/Pu>0.1

Figures 5 are the enlarged view of the part of Pd/Pu>0.1 in Figs.2. The discharge coefficients in both gases decrease gradually toward the critical back pressure ratio that the discharge coefficient decreases quickly, its value depending on the theoretical Reynolds number. In the measurement in the range of Pd/Pu>0.1, the back pressure ratio is kept for about 10 minutes, during which the discharge coefficient does not change at all. The change amount of the discharge coefficient between Pd/Pu = 0.1 and Pd/Pu = 0.5 is only 0.01% ~ 0.02% in Nitrogen, but is 0.05% ~ 0.08% in Helium. This change amount in Helium would be unable to be overlooked for flow measurement.

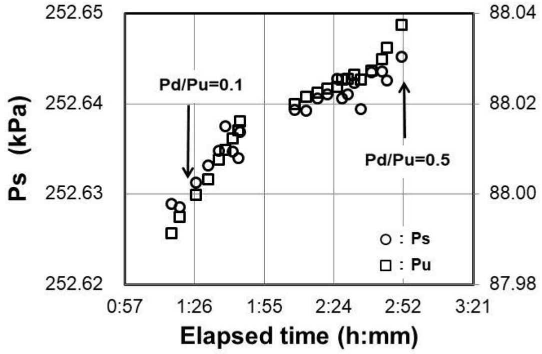
Figures 6 show the variations with time of the upstream pressures and the upstream temperatures of the standard critical nozzle and the critical nozzle under test in Nitrogen at R_{eth}=4800. The change amounts of Ps and Pu are about 15 Pa. Tu and Ts are stable within 0.1 deg.C. These are the conditions of pressure and temperature normally expected in the nozzle to nozzle test method. Figures 7 show the variations with time of Ps, Pu, Ts and Tu in Helium at R_{eth}=4400. Ps is quite stable and its change amount is only about 5 Pa between Pd/Pu=0.1 and Pd/Pu=0.5, but Pu increases about 5 kPa in the same range. Ts is stable within 0.03 deg.C, and the change of Tu is about 0.1 deg.C.



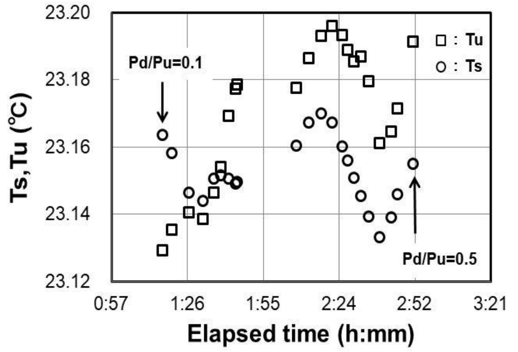
(a) Helium

(b) Nitrogen

Figure 5 Enlarged view of the discharge coefficient in Pd/Pu>0.1

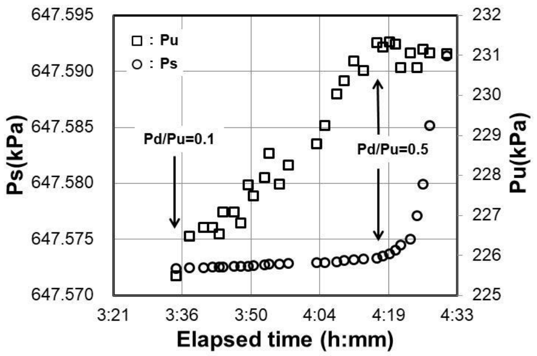


(a) Pressure

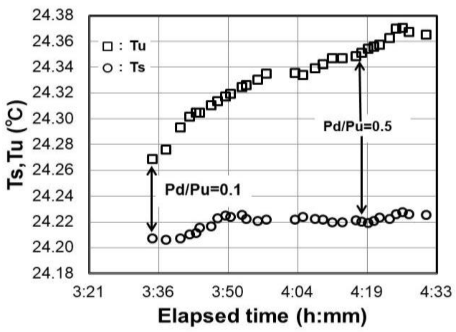


(b) Temperature

Figure 6 Variations of the pressure and the temperature with time
Nitrogen: $Pd/Pu>0.1$, $R_{eth}=4800$



(a) Pressure



(b) Temperature

Figure 7 Variations of the pressure and the temperature with time
Helium : $Pd/Pu>0.1$ $R_{eth}=4400$

As the temperature change is similar to that in Nitrogen, the large change of the discharge coefficient in Helium is considered to be caused by the upstream pressure change. Therefore, it is necessary to explain the reason why *Pu* increases largely as sown in Fig.7 (a).

The choked flow is defined as an upstream side and a downstream side of a critical nozzle are completely separated by a sonic plain and a flow field of an upstream side is not affected by that of a downstream side. However, actually, the pressure wave travels through the boundary layer from the downstream side of a critical nozzle to its upstream side or vice versa. Therefore, the upstream pressure changes definitely when a flow field changes by a change of a back pressure ratio, even if there is a difference in degree, depending on a kind of gases or on the theoretical Reynolds number. Normally, an upstream pressure change of a critical nozzle appears as a change of a mass flow rate. However, as discussed in 3.1, in the nozzle to nozzle test method that the mass flow rate is constant, the upstream pressure change results in the change of the discharge coefficient. The behaviors of the discharge coefficient shown in Figs.5 would be explained as follows; when the back pressure ratio changes, since the thickness of the boundary layer at the throat changes, the upstream pressure changes to the value of the new state of the flow filed. And the discharge coefficient changes according to the pressure change. This change of the discharge coefficient caused by the pressure change is usually small and can be negligible like the case of Nitrogen, but the change of the upstream pressure in Helium of Fig.5 (a) is larger than expected and resultantly the discharge coefficient will show a significant change.

There is not any definite evidence quantitatively to explain the differences of the change amount of the discharge coefficient found in Helium and in Nitrogen, besides the differences of the characteristics of the boundary layer due to thermal properties of gases. Unfortunately, there have been few researches investigating the characteristics of the boundary layer for gases other than Air.

4. Conclusion

The strange behaviors of the discharge coefficient reported here are considered to be due to the measurement principle of the nozzle to nozzle test method, in which the critical nozzle under test is evaluated under the condition that the mass flow rate is constant. Therefore, even the slight change of the upstream pressure must come out as the change of the discharge coefficient. On the other hand, in the PVTt method or in the gravimetric method, when the upstream pressure changes, since the mass flow rate changes simply in proportion to the upstream pressure, the discharge coefficient does not necessarily need to change. This is an important and essential difference between the general calibration methods and the nozzle to nozzle test method.

The strange behaviors of the discharge coefficient might appear only under the limited measurement conditions, for example, a gas such as Helium or Hydrogen, and at low or moderate theoretical Reynolds number, etc. Also, the flow models suggested to explain these behaviors might be not correct because there are not any clear experimental evidences to support the flow models. Although, the important thing of the present results is to raise doubts about the equivalence of the discharge coefficient determined by the general calibration method like a PVTt method and that determined by the nozzle to nozzle test method. The nozzle to nozzle test method is not only easy to use, but also is definitely a reliable test method that can investigate the characteristics of critical nozzle. On the other hand, the results obtained by this test method must be examined carefully because the discharge coefficient obtained might be not coincident to that determined by other calibration method depending on calibration conditions.

Afterword

In this paper, the interesting experimental results about the discharge coefficient of the critical nozzle were shown and also the flow models to explain these phenomena were suggested. However, we could not show any clear evidence or any physical explanation to support the suggested flow models. In that sense, this paper is considered to be incomplete as an academic paper. Furthermore, the unfortunate thing is difficult for us to confirm these phenomena experimentally. We expect that CFD may give some correct explanation about these phenomena.

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