Basic errors of Coriolis flowmeters in liquid-gas two-phase flow

Xiao-Zhang ZHANG[1]

[1]Department of Engineering Physics, Tsinghua University, Beijing， P R China
E-mail zhangxzh@tsinghua.edu.cn

Abstract

Liquid-Gas two-phase flow causes measurement error in Coriolis flow meters. The topic has been studied by researchers including working out mathematical models, doing real flow tests in laboratories or industry.
In this paper, we study the 'bubble model' again, which is often referred to in two-phase flow model for Coriolis flow meters. Using the concept of Archimedes buoyant principle, we obtain a different result from Landau's for acceleration ratio of a sphere bubble in a vibrating ideal fluid. Thus measurement error model of two-phase flow for Coriolis flow meters is re-discussed. On the other hand, data of flowrate measurement errors ($\varepsilon_{\dot{m}}$) abstracted from a number of references are plotted. It was found that most of the data falling within a region of between $\varepsilon_{\dot{m}} = 0$ and $\varepsilon_{\dot{m}} = -\frac{\alpha}{1\text{-}\alpha}$, which is in agreement with the theory deduced in this paper.

1. Introduction

A Coriolis flowmeter (CFM) measures Coriolis force in a vibrating pipe conveying fluids. It relies on vibration of each fluid element in measurement tube. Any density difference among the fluid elements will cause different effect.

People began to study CFMs errors in multiphase flows in 90s of last century (Wang and Baker 2014). Hemp and Hoi (2003) suggested a 'bubble model' and predicted a negative error of CFMs for liquid-gas flowrate. Later, researchers have done more works to further extend the model (Basse 2014，2016）.

It is a key issue to understand the behavior of gas bubbles in two-phase flow. This knowledge will help us to understand mechanism of measurement error of a CFM caused by multiphase flow. When a gas bubble moves in a flowing water, the bubble will move faster than the liquid under same pressure gradient. This is because the bubble is lighter than the water. The relative speed of the bubble will cause the bubble change its shape and move in a complicated trajectory (see figure 1). Under a vibrating liquid, which occurs in a CFM measuring tube, such behavior of bubbles will cause measurement errors of both density and mass flowrate.

As the gas void fraction increases, interaction between gas and liquid will be more complicated （Brennen 2016）. We limit the following discussion only to bubble flow regime with small void fraction. Bubbles are assumed to evenly spread in the liquid and no interaction among them. Also, the bubbles' shape is assumed to be spherical and will not change.

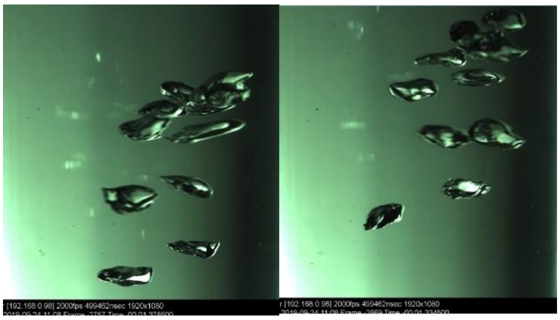

Figure 1: High speed shots of bubbles rising in water

2. Ratio of accelerations between gas and liquid

As mentioned above, the velocity of the gas is not as the same as that of the liquid under same pressure gradient. There exist relative velocities between gas bubbles and the surrounding liquid. The relative velocity of bubbles causes 'induced mass' (or added mass) of the bubbles (Hemp and Young 2003). For a spherical bubble, the 'induced mass' equals to 1/2 of the liquid mass filling the sphere.

Assume there is a sphere gas bubble in a transverse vibrating liquid. The angular frequency of the vibration is ω, equation of the bubble motion is described as

$$m_{in}\frac{d^2x}{dt^2}+c\frac{d(x-y)}{dt}=m_L y_0\omega^2 e^{i\omega t} \qquad (1)$$

where m_{in} is the 'induced mass' , m_L is liquid mass in the same volume of the bubble, *c* is the damping coefficient to the bubble because of relative motion to liquid， *x* and *y* are the displacements of the bubble and the liquid respectively, y_0 is the maximum displacement of the liquid and the pipe.

A solution of steady state for equation (1) is:

$$x(t)=x_0 e^{i\omega t+i\varphi} \qquad (2)$$

where x_0 is the maximum displacement of the bubble, *t* is the time. The phase lag φ caused by damping is

$$\mathrm{tg}\varphi=\frac{(m_{in}+m_L)c\omega}{m_{in}m_L\omega^2-c^2} \qquad (3)$$

The maximum displacement (or the bubble's vibration module) of the bubble

$$x_0=y_0\frac{\sqrt{(m_{in}+m_L)^2(c\omega)^2+(m_{in}m_L\omega^2-c^2)^2}}{m_{in}^2\omega^2+c^2} \qquad (4)$$

where, x_0/y_0 is the ratio of the maximum displacements of the bubble when liquid in vibrating. Further deduction of equation (4) results:

$$\frac{x_0}{y_0}=\sqrt{\frac{\left(\frac{m_L}{m_{in}}\right)^2+\left(\frac{c}{m_{in}\omega}\right)^2}{1+\left(\frac{c}{m_{in}\omega}\right)^2}} \qquad (5)$$

where $\frac{m_L}{m_{in}}$ is the ratio of liquid mass and the induced mass. According to theory of vibration, $\frac{c}{m_{in}\omega}$ is 2 times of the damping ratio coefficient.

If *c*=0 (no damping), then equation (5) reduces to

$$x_0/y_0=m_L/m_{in} \qquad (6)$$

This means $x_0/y_0=2$ because of $m_{in}=0.5m_L$ for spherical bubble.

Velocity and acceleration of the bubble are expressed as $v=\frac{dx(t)}{dt}$ and $a=\frac{d^2x(t)}{dt^2}$ respectively, thus $x_0\omega$ and $x_0\omega^2$ will be the maximum speed and maximum acceleration of the bubble respectively. Similarity is for the liquid vibration of *y*(t). So that, ratio of x_0/y_0 also stands for ratio of velocity or ratio of acceleration. This means the maximum acceleration of the spherical bubble under no damping is 2 times of the liquid. The result is not the same as those in previous references, where the acceleration of the sphere gas bubble was 3 times the value of the liquid , which seemingly was from the book of Landau. On page 36 of the book (Landau 1979), relation between velocities of liquid and gas under ideal flow assumption is:

$$v_g=\frac{3\rho_L}{\rho_L+2\rho_g}v_L \qquad (7)$$

From equation (7), when the density of gas ρ_g is small enough to be neglected, velocity of the gas bubble v_g is 3 times of the liquid.

It is well known that any object immersed in a fluid will experience an upward force equaling to the weight of the fluid displaced. This is the Archimedes principle. The force is $\rho_L V g$, where ρ_L is the fluid density, *V* the volume of the object (gas bubble) and *g* is the gravitational acceleration. Now, let us assume there is a non-gravity space where the water moves upward with an acceleration of *g*. Owing to the pressure gradient produced by the acceleration, the gas bubble will experience mathematically the identical force as $\rho_L V g$. The same view is used to a vertical pipe filled with water and doing transverse acceleration. If the transverse acceleration is a , then the force on the gas bubble is $\rho_L V a$. The induced mass for spherical bubble is $\frac{1}{2}\rho_L V$. Newton's second law leads to its acceleration to be $2a$. This is the start point of equations (1).

The parameters relation in equation (5) is plotted in figure 2, where *x*-axis is 2 times of damping ratio coefficient, i.e. $\frac{c}{m_{in}\omega}$, y-axis is the ratio of maximum displacement of the gas bubble to maximum displacement of the water, i.e. $\frac{x_0}{y_0}$. It shows that $\frac{x_0}{y_0}$ decreases as $\frac{c}{m_{in}\omega}$ increases for a given $\frac{m_L}{m_{in}}$. This is because damping reduces the relative speed of bubbles to the liquid. When $\frac{m_L}{m_{in}}=2$ (the case of spherical bubble), $\frac{x_0}{y_0}$ becomes the largest for zero damping. This value is 2, i.e. the maximum displacement of the bubble is 2 times of the liquid. For $\frac{m_L}{m_{in}}=1$, the induced mass of the bubble gas is the same as the liquid. In this situation, the bubble's density is the same as the liquid. There will be no relative displacement for such a bubble.

In reality, bubbles moving in liquid may not be spherical because of the pressure gradient around them, so $\frac{m_L}{m_{in}}$ would be a value of less than 2 (Brennen 2005). For damping coefficient, we have done some experiments (Zhang 2016). The data were obtained by vibrating a pipe with water-air flow and by measuring the pipe's response under various frequencies. Results are in figure 3 which shows that the damping ratio coefficients are largely in a linear relation with gas void fraction. Its values are between 0.009 and 0.014 for the void fraction from 0 to 0.32. The order of magnitude of the data is the same as experimental results from Carreton et al (2015). Considering the values of damping to be small, then the main factor affecting the relative displacement of the bubbles is only $\frac{m_L}{m_{in}}$, which depends on the shape of gas bubbles.

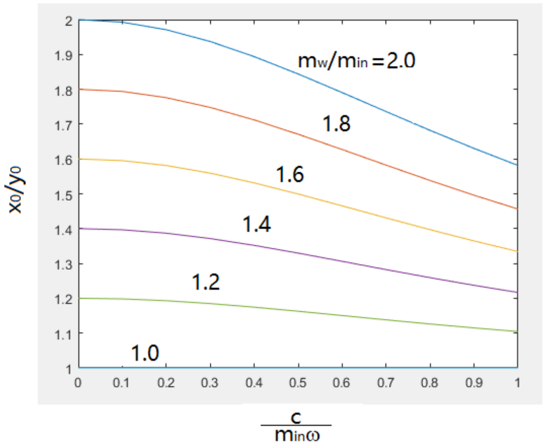

Figure 2: Effect of induced mass and damping on flowrate measurement error

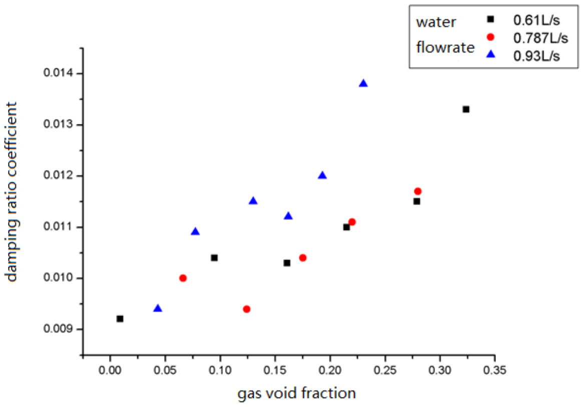

Figure 3: Damping ratio coefficient verse gas void fraction in water-air bubble flow

Assume gas void fraction in a pipe of volume *V* filled with liquid, mass of the liquid in static state is

$$(1\text{-}\alpha)V\rho_w \qquad (8)$$

Now assume the pipe is accelerating at a , then the spherical gas bubbles will do absolute acceleration of $2a$. Apart from the inertial force the liquid experiences, there is an additional force of $a\alpha V\rho_w$ on the liquid because of the relative motion of the bubbles. The total force acting on the liquid is

$$aV(1-\alpha)\rho_W - a\alpha V\rho_w = aV(1-2\alpha)\rho_w \qquad (9)$$

By the relation of Newton's second law, one can obtain an effective mass of

$$m_{eff} = (1-2\alpha)V\rho_w \qquad (10)$$

m_{eff} is the effective mass. Compare m_{eff} to equation (8) we know that the pipe senses less density than reality. For the same reason, the relative motion of the gas bubbles make the Coriolis force smaller than normal by $2\alpha V\rho_w\omega v$ (Coriolis force *F*=2*mωv*). The measurement errors of both density and mass flowrate caused by existence of gas bubbles in Coriolis flowmeters are the same as:

$$\varepsilon_\rho = \varepsilon_{\dot{m}} = -\frac{\alpha}{1-\alpha} \qquad (11)$$

where ε_ρ and $\varepsilon_{\dot{m}}$ are the measurement errors of density and mass flowrate respectively by the Coriolis flowmeters, α is the void fraction of gas. Equation (11) is obtained under simplified conditions, such as potential flow, spherical bubbles and small α, no interaction among bubbles and neglecting damping. As is mentioned above, main factor affecting the relative displacement of the bubbles is the shape. So errors equation (11) are called the basic errors.

3. Measurement errors abstracted from published works

There are different types of Coriolis flowmeters, different installation orients as well as different flowrates and flow entrance conditions. Flowrate measurement errors of Coriolis flowmeters in published papers for multiphase flow are rather diverse. We did some statistic job by collecting data from published papers. Totally 37 pairs of data were obtained from 12 papers which reported experimental works on measurement of multiphase flow by Coriolis flowmeters. The error data were chosen to be original and uncorrected. Some data were obtained by estimation from the published figures, so there might be deviations. The final results were plotted in figure 4. From the figure one can see that 35 data are almost in a region between lines of $\varepsilon_{\dot{m}} = \pm\frac{\alpha}{1\text{-}\alpha}$, while 28 of the 37 data are almost in a region by lines between $\varepsilon_{\dot{m}} = 0$ and $\varepsilon_{\dot{m}} = -\frac{\alpha}{1\text{-}\alpha}$.

Equation (11) estimates that flowrate measurement error to be $-\frac{\alpha}{1\text{-}\alpha}$. In reality, bubbles might not be spherical. $\frac{m_L}{m_{in}}$ in equation (6) will be less than 2. This will make the absolute value of errors less than $\left|\frac{\alpha}{1\text{-}\alpha}\right|$. For those positive error data in the figure, the reason is not known.

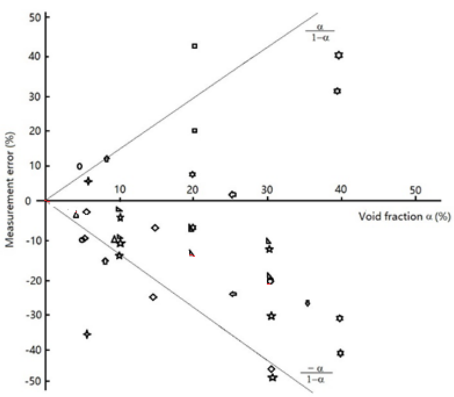

Figure 4: Flowrate errors by Coriolis flowmeters under multiphase flows（abstracted from 12 published papers）

7. Conclusion

We deduced an equation of motion for a bubble’s induced mass under vibrating liquid. Solution of the equation shows that for spherical bubble the maximum displacement of the bubble is 2 times of the liquid. This result is supported by the concept of pressure gradient field of liquid under acceleration, which is related to Archimedes principle. Expressions for measurement errors of both density and mass flowrate are obtained. In discussing influence factors, damping was found to less affect the errors than the induced mass. Work of using statistic result from published papers to investigate the flowrate measurement errors was done. The result is in agreement with the theory deduced in this paper.

References

[1] Wang T, Baker RC, Coriolis flowmeters: a review of developments over the past 20 years, and an assessment of the state of the art and likely future directions, Flow Meas. Instrum. 40 (2014) 99–123.

[2] Hemp J and Yeung H. 2003. Coriolis meter in two phase condition, Computer and Control Engineering. 14:36-36.

[3] Basse NT: A review of the theory in Coriolis flowmeter errors due to entrained particles, Flow. Meas. Instrum. 2014，v37:107-118.

[4] Basse NT. Coriolis flowmeter damping for two-phase flow due to decoupling，Flow. Meas. Instrum. 2016;52:40-52.

[5] Brennen C E. Fundamentals of multiphase flows, Cambridge University Press, 2005

[6] Landau LD, Lifshitz EM. Fluid mechanics. 2nd ed. Pergamon Press; 1979.

[7] Zhang XZ: Damping of vibrating pipe conveying water-air flow and its effect on Coriolis flow meter, SICE2016, Sept. 20-23, 2016, Tsukuba, Japan

[8] Charreton C et al: Two phase damping for internal flow: physical mechanism and effect of excitation parameters, J. Fluid and Structures, V56(2015), 57-74here.