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Considering Covariance in Reference Flowmeter-based Calibration Facilities

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Abstract

Given their simplicity, flow calibration facilities using flowmeters as reference standards are very common. The present paper introduces a method to estimate calibration facility uncertainty considering empirically determined correlation effects under working conditions. This improvement strongly increases the confidence in flowmeter-based calibration facilities of the particular type used, making third-party recognition of calibration services more reliable and transparent.

A special type of calibration facility having multiple reference flowmeters installed both in-series and in-parallel, and fully automated scale calibration systems using weights was used. No human intervention is required for the calibration of the gravimetric system.

When no covariance is considered, the expanded uncertainty of the facility can be expressed by the single flowmeter uncertainty reduced by the factor $1/m^2$, m being the number of flowmeters used in parallel or in series; for two devices in n lines, by the factor $1/(2n)^2$. This approach has its limitations and does not address possible unknown systematic effects during the calibration of each reference flowmeter. A more conservative approach assumes full correlation between the flowmeters used.

The tested facility was designed to make covariance between two simultaneously calibrated reference devices measurable. By knowing the covariance, better understanding is given to the real performance of the reference flowmeters under working conditions and no full correlation needs to be assumed.

In this paper we present firstly theoretical considerations and discuss the assumptions for modelling this particular type of facility. Secondly historical data is presented and analyzed, and finally, the facility performance of 0.03% up to 100 kg/s is validated through flow comparisons using a highly accurate reference flowmeter.

1. Introduction

The calibration of a flowmeter, either new or already in use, is most accurately performed in dedicated stationary calibration laboratories. However, having devices calibrated on-site, i.e. under real working conditions, can bring enormous advantages; influence quantities that might produce unknown systematic errors are minimized by including their effects implicitly in the calibration factor.

This is common practice when calibration services are offered; the flowmeter is calibrated under reference conditions that are known by the customer.

Modern flowmeters are able to make corrections independently. When these corrections are based on physical models, a very robust flowmeter response can be achieved. The models presented in [1] are available for sensors up to DN250. On the other hand, when corrections are based on empirical models, their robustness depends strongly on the assumptions made during design.

The resources needed to realize a calibration that minimizes the effects of influence quantities empirically and simultaneously reaches the high accuracy of dedicated flow calibration laboratories are very high.

The results presented in this paper were generated in a calibration facility that allows the reference flowmeter to be calibrated to a high degree of precision, fully automated and without need for removal, allowing also determination of covariance.

Correlation and covariance are recommended [2] to be used with caution and to be replaced by validated or well-founded physical models whenever this is possible. In contrast to [3] where the use of correlation is applied under conditions very difficult to control across different countries, our facility design allows for better controllable influences without the risks of transport or removal.

2. Description of the facility

The FCP-25 is a flow rate calibration facility type that uses water as calibration fluid for volume or mass flow rates between 0.5 kg/s and 100 kg/s at ambient temperatures between 8°C and 35°C. A schematic representation is given in Figure 1.

Two different calibration principles are implemented; the first is a conventional open loop gravimetric system that uses a 400 kg capacity premium weighing scale and a diverter system to reach flow rates from 0.5 kg/s up to 11 kg/s. The second system (Figure 2) is a closed and pressurized loop comprising nine DN50 Promass Coriolis flowmeter lines with two instruments in series in each as well as a single line of two DN25 Promass Coriolis in series that is used to reach flow rates from 0.5 kg/s to 100 kg/s.

Each master is traced back to the weighing scale. The facility configuration strives to activate as many reference lines as early as possible as the flow rate increases.

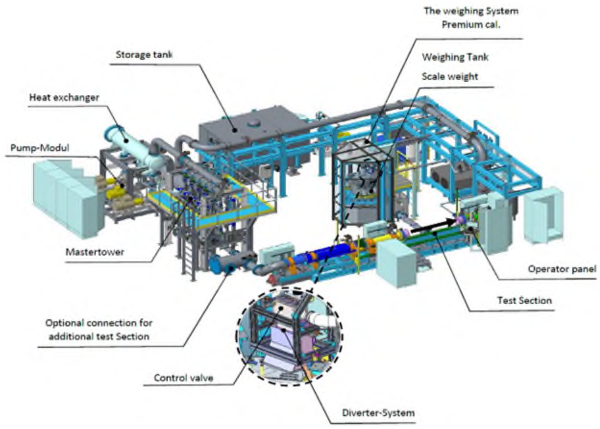

Figure 1 Schematic diagram of the FCP-25

The closed loop scheme, as shown in Figure 2, is used for most calibrations. The device under test (DUT) is installed in the measurement section. The system has been designed in such a way that reference lines and DUT work under the same conditions of flow, pressure and temperature as in the open loop configuration.

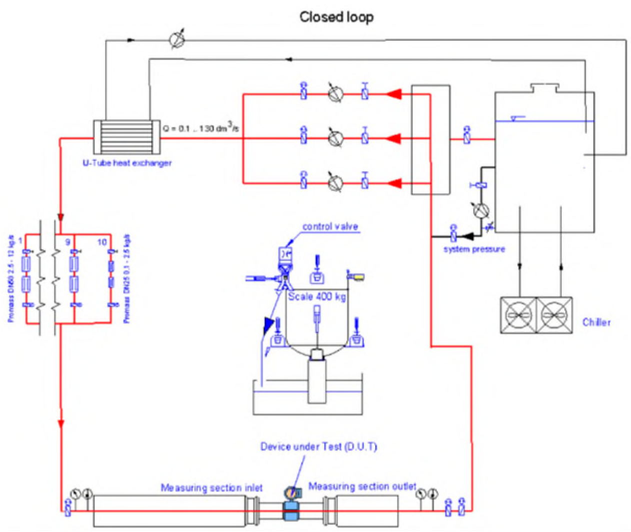

Figure 2 Closed Loop and DUT Calibration

Fehler! Verweisquelle konnte nicht gefunden werden. shows the open loop calibration scheme. As can be observed, calibration of the DUT using the weighing system is also possible, but only up to 11 kg/s. The gravimetric system can perform at the lowest possible uncertainty, but this comes at some cost: using an open system implies reducing pressure down to ambient conditions before the diverter, making it necessary to raise the pressure again as can be seen schematically in **Fehler! Verweisquelle konnte nicht gefunden werden.**. Additionally, enrichment of the air content in the water cannot be avoided. This can only be solved with proper design, i.e. using air separators, applying longer passing times through the tank, among others.

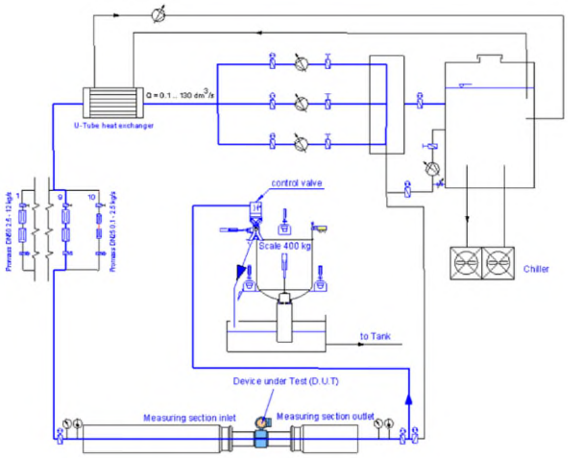

Figure 3 Open Loop for calibrations using the scale

2.1 Uncertainty Estimation

For the purposes of simplicity, we shall consider the following model:

Equation 1

$$m_{total} = \frac{1}{2} \sum_{i=1}^{2n} \left(m_i + \delta m_i\right)$$

Where the total mass, m_{total}, at a specific flow rate is the sum of all the masses measured by all flowmeters, each corrected according to its calibration. The number of flowmeters used is given by *n;* since they are installed pairwise in series the factor ½ is needed. Additional contributions have been presented thoroughly in [3].

The facility and the surrounding building have been designed to minimize thermal gradients. Sensors have been installed to automatically detect small temperature differences and pressure drops caused by leakage; doubly redundant valve concepts are applied to all valves involved in the measurement.

Automated plausibility tests are performed constantly during calibration.

The present paper focuses on the performance of the reference flowmeters. The presented model considers the contributions of the gravimetric system in the correction term. The contribution of the DUT is not part of this study.

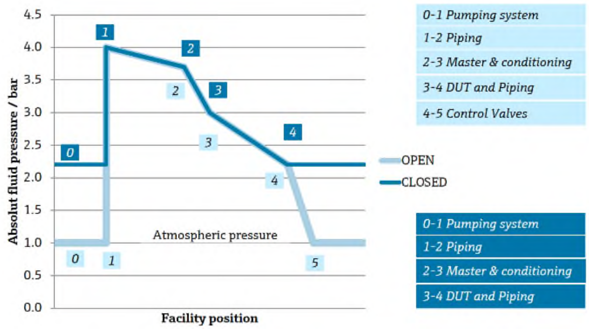

Figure 4 Fluid pressure within the calibration facility

For modelling the uncertainty of equation 1, it is very important to take into account unknown systematic errors. If the flowmeters are exposed to the same conditions, all related unknown systematic errors can be assumed to be correlated. Given that the corrections are measured by the same calibration system, their values must also be correlated.

When applying these assumptions, the uncertainty including covariances can be expressed following the recommendation of the GUM [2] as:

Equation 2

$$u_{total}{}^2 = \left(\frac{1}{2}\right)^2 \sum_{i=1}^{2n} \left(u_{mi}{}^2 + u_{\delta mi}{}^2\right) +$$

$$+2 \cdot \left(\frac{1}{2}\right)^2 \sum_{i=1}^{2n-1} \sum_{j=i+1}^{2n} u\left(m_i, m_j\right) + 2 \cdot \left(\frac{1}{2}\right)^2 \sum_{i=1}^{2n-1} \sum_{j=i+1}^{2n} u\left(\delta m_i, \delta m_j\right)$$

It is assumed that the flowmeters are correlated, and that the corrections given by the gravimetric system are also correlated. A correlation between the gravimetric system and the reference master meter can be neglected.

Assuming the worst case, to simplify notation and estimation, we consider the largest found values of variance and covariance to be representative. By doing this we can express the correlation terms as:

Equation 3

$$2\cdot\left(\frac{1}{2}\right)^{2}\sum_{i=1}^{2n-1}\sum_{j=i+1}^{2n}u\left(m_i,m_j\right)=2\cdot\left(\frac{1}{2}\right)^{2}\frac{4\,n^{2}-2\,n}{2}\cdot u\left(m_i,m_j\right)$$

and applying to equation 2:

Equation 4

$$u_{total}^{2}=\frac{n}{2}\cdot\left(u_{mi}^{2}+u_{\delta mi}^{2}\right)+\frac{n}{2}\cdot(2\,n-1)\cdot\left(u\left(m_i,m_j\right)+u\left(\delta m_i,\delta m_j\right)\right)$$

Expressing uncertainty relatively, i.e. related to the mass flow rate we can write:

Equation 5

$$\left(\frac{u_{total}}{n\cdot m_i}\right)^{2}=\left(\frac{1}{n\cdot m_i}\right)^{2}\cdot\left(\frac{1}{2}\right)^{2}2\,n\cdot\left(u_{mi}^{2}+u_{\delta mi}^{2}\right)+\left(\frac{1}{n\cdot m_i}\right)^{2}\cdot\frac{n\cdot(2\,n-1)}{2}\cdot\left(u\left(m_i,m_j\right)+u\left(\delta m_i,\delta m_j\right)\right.$$

Simplifying and expressing relative terms with index R we have

Equation 6

$$u_{totalR}^{2}=\left(\frac{1}{n}\right)^{2}\cdot\left(\frac{1}{2}\right)^{2}\cdot 2\,n\cdot\left(u_{miR}^{2}+u_{\delta miR}^{2}\right)+\left(\frac{1}{n}\right)^{2}\cdot\frac{n\cdot(2\,n-1)}{2}\cdot\left(u_R\left(m_i,m_j\right)+u_R\left(\delta m_i,\delta m_j\right)\right)$$

Simplifying equation 6 we finally obtain a description of the uncertainty contributions due to correlation.

Equation 7

$$u_{totalR}^{2}=\frac{1}{2\,n}\cdot\left(u_{miR}^{2}+u_{\delta miR}^{2}\right)+\frac{2\,n-1}{2\,n}\cdot\left(u_R\left(m_i,m_j\right)+u_R\left(\delta m_i,\delta m_j\right)\right)$$

All contributions are known or can be determined experimentally. As can be seen in Figure 9, the model results have been validated using a numerical approach.

3. Considering covariance

All reference flowmeters are calibrated line-wise, i.e. two at a time, every two weeks, typically at four flow rates. Calibration takes place after the scale and the diverter have been also calibrated and characterized. All these processes run fully automatically. The generated files contain all results and additional information needed to evaluate the performance and establish traceability.

The results over a six-month period have been considered for all evaluations. A special characteristic of this facility is the use of in-parallel flowmeters. How this contribution is estimated depends on correlation.

Considering the correlation as a main contributor, we start by observing the correlation coefficients across all 18 reference flowmeters.

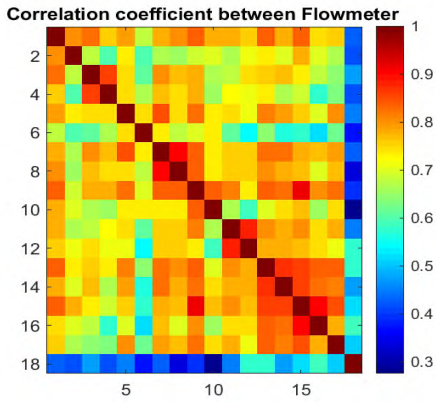

Figure 3 Correlation coefficient between all possible reference flowmeter pair combinations.

The correlation coefficients, defined as the ratio of covariance to the product of the two corresponding standard uncertainties, gives us the strength and the direction of correlation, but without considering covariance no further analysis can be made about the contribution to uncertainty itself.

Figure 5 shows the value of the correlation factor. It is for most devices about 0.7 which might - but not necessarily - imply a large contribution. We can also observe the unique behaviour of reference flowmeter Nr. 18. The correlation is lowest in comparison to all other devices.

Consider Figure 6, drawn using the same data.

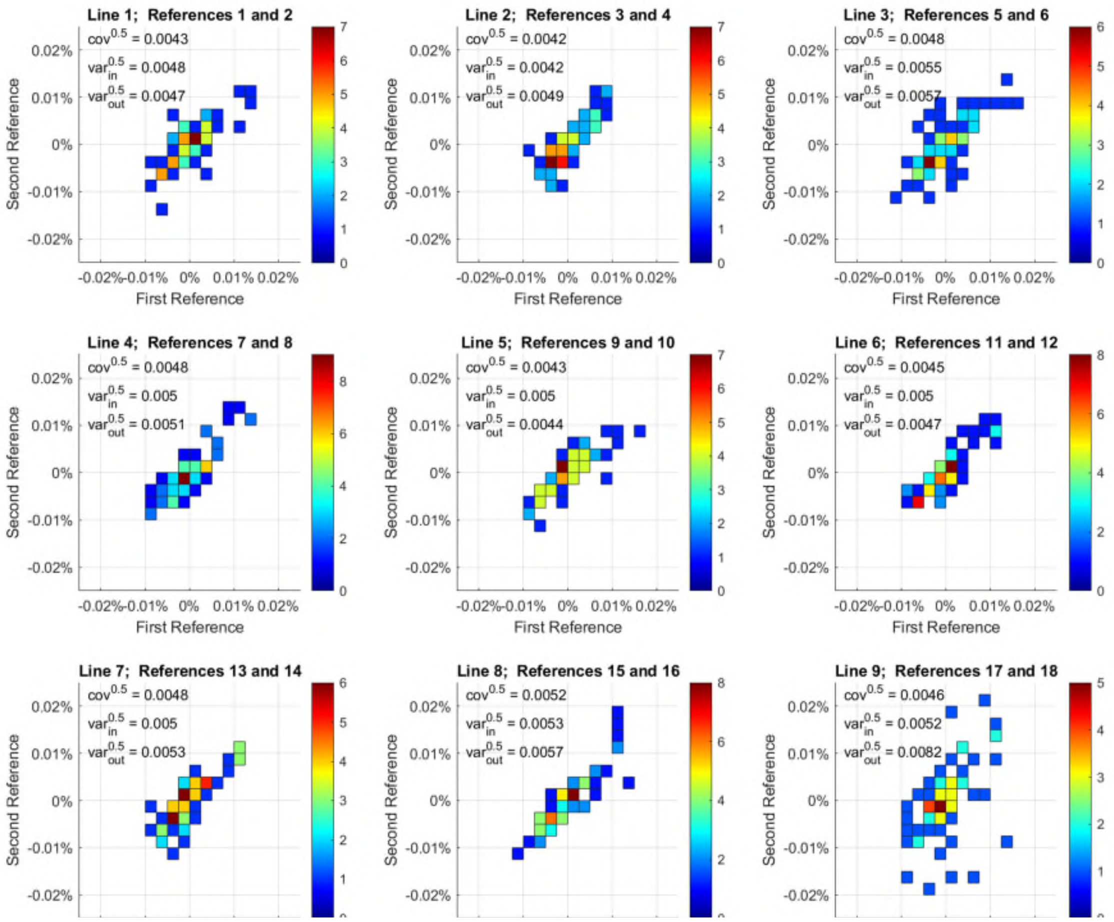

Figure 5 Calibration results for all flowmeter pairs. The colourmap represents the occurrences of the result in each box. In contrast to a Youden plot, this representation attempts to represent the frequency distribution.

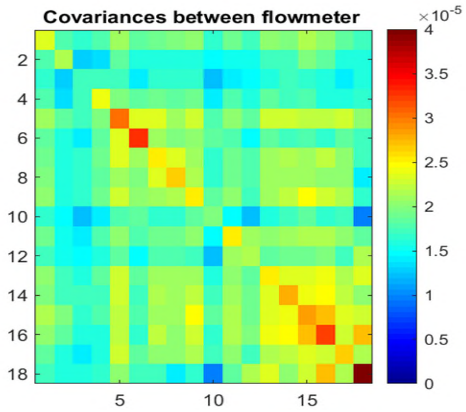

Figure 4 Covariance among all possible reference flowmeter pairs

Here we can clearly see that despite having a strong correlation, the actual values of covariance are very small. The variances given in the diagonal are higher, but still small.

We can also observe that the strongest values of covariance are given for the pair 15/16. 5/6 and 7/8 also have a slightly larger covariance. In general, we can observe that for each flowmeter the maximum covariance is given for consecutive odd and even numbers i.e., flowmeters installed in the same line.

Those flowmeter pairs can be analysed in the Figure 7. The axis represents each flowmeter installed in first or second position. In order to have the same units, the values are the square root of covariance. The colour map represents the frequency of each value pair occurring in each square.

In this representation we can easily see the correlation as a 45° line. This can be interpreted as the occurring unknown systematic errors having approximately the same magnitude and direction in both flowmeters installed in the same line. Even if correlation is obvious, its corresponding contribution is limited, in this case to about ±0.02% peak-to-peak for more than 99% of the results.

The same analysis is presented in Figure 8 but considering all flowmeter lines and all flow rates in a single plot.

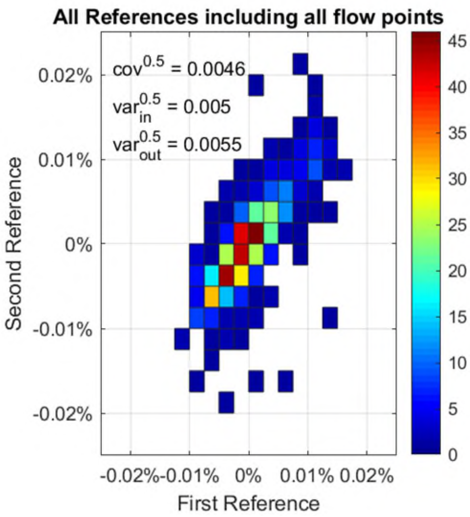

Figure 6 Calibration results given for all flowmeter pairs and all flow points

The performance of the flowmeters can be seen to be very homogeneous; covariances are in the same ranges and evenly distributed. Only few points stand out - the reason for this can be found by considering each flow rate separately as shown in Figure 9.

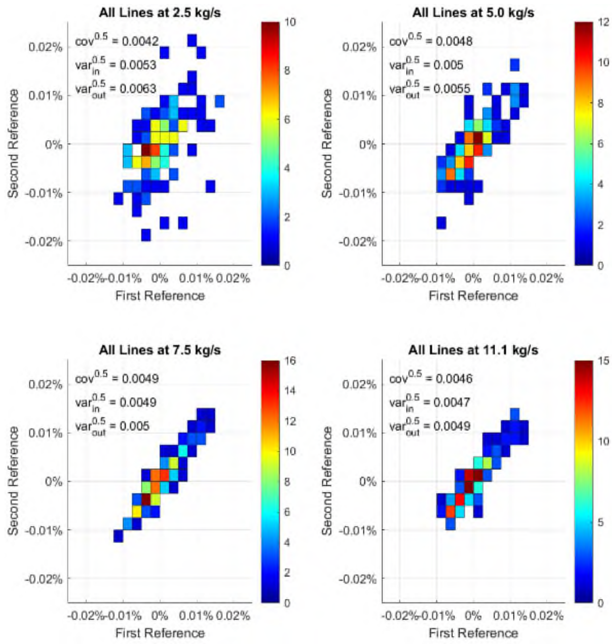

Figure 7 Calibration results for all lines given for each of the four flow rates.

For a flow rate of 2.5 kg/s we can observe a slightly larger variance. The performance is still very good, but one flowmeter is sporadically giving results independent of its partner. Among all flow rates we can also clearly see that the performance becomes better at higher flow rates.
Having plotted all the values of covariance we can select the most representative and conservative values. The largest values of covariance are observed for the flowmeter pairs. This can be also physically explained since flowmeters installed in the same line are more likely to experience the same influences and to be affected by the same factors. These values will be used to estimate the total uncertainty.

Table 1: Variance and covariance contributions

Contribution Eq. 7	Symbol	Value
Flowmeter specification	u_{miR}	0.008 %
Correction due to calibration	$u_{\delta miR}$	0.015 % /2
Flowmeter covariance	$u_R\left(m_i, m_j\right)$	(0.005 %)²
Calibration covariance	$u_R\left(\delta m_i, \delta m_j\right)$	(0.015 % / 2)²

The results depend on the number of lines used as can be observed in Figure 10. The small black circles represent the estimated uncertainty based on the six-month period. The red line with at the top shows that even if the worst possible performance values of the technical specification in the worst-case condition of full correlation are used, the expanded uncertainty is still under 0.03%.

For the uncertainty related to the gravimetric system used for calibrating the reference flowmeter lines we consider full correlation, implying that variance and covariance have the same magnitude. This is very conservative, but greatly simplifies the understanding and analysis of the results. Moreover, given the excellent performance of the gravimetric system, as can be seen in Figure 12, it does not significantly affect the results.

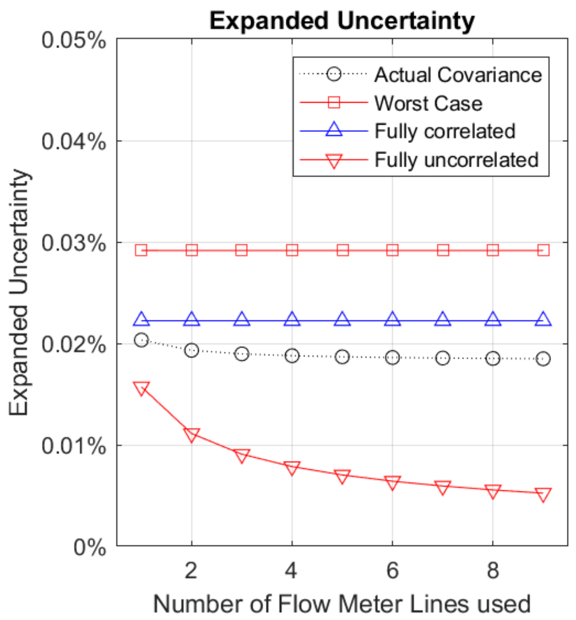

Figure 8 Expanded uncertainty under different conditions. The presented worst case represents the maximum value that can be assumed for the used flowmeter. Fully correlated and uncorrelated values are given considering the actual determined variances.

The uncertainty declared for the flow facility is 0.03% when using two lines or more. This declaration is very easily fulfilled, even if in a very unlikely event unexpected deviations arise. Any possible malfunction caused accidentally by human errors for example, will be reduced by a factor of at least 4 and up to 18 depending on the number of lines used. For example, even an error of 0.05% will cause only 0.013% offset, being still within the declared 0.03% uncertainty.

The advantage of having two flowmeters in line also pays off when consistency is proved, i.e., when the results given by both reference flowmeters installed in line are tested statistically during each measurement, avoiding undetected malfunctions.

Figure 11 shows the simulation results using Equation 1 and the Monte Carlo Method as presented in [5]. The experimentally determined values have been used as a basis. For the uncertainty of the reference, full correlation has been assumed.

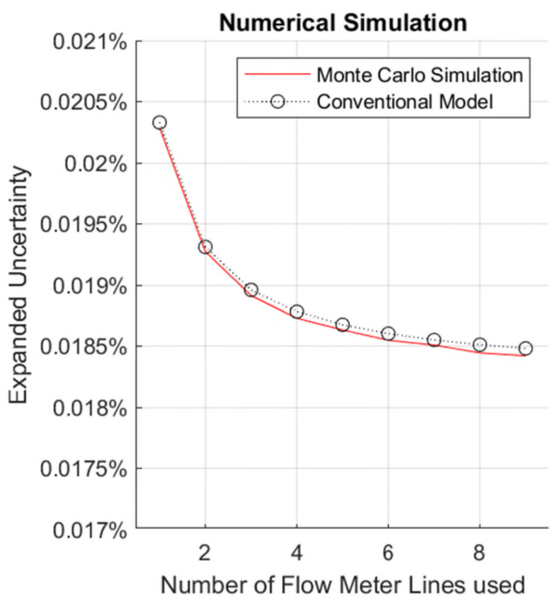

Figure 9 Results of a multivariate correlated Monte Carlo Simulation using experimentally determined covariances.

Figure 10 shows all raw calibration and characterization results of the scale and the diverting system. The performance of both systems is excellent and confirms the declarations given in [3]. Even if full correlation is assumed, the influence of the weighing systems is clearly limited to ±0.005%.

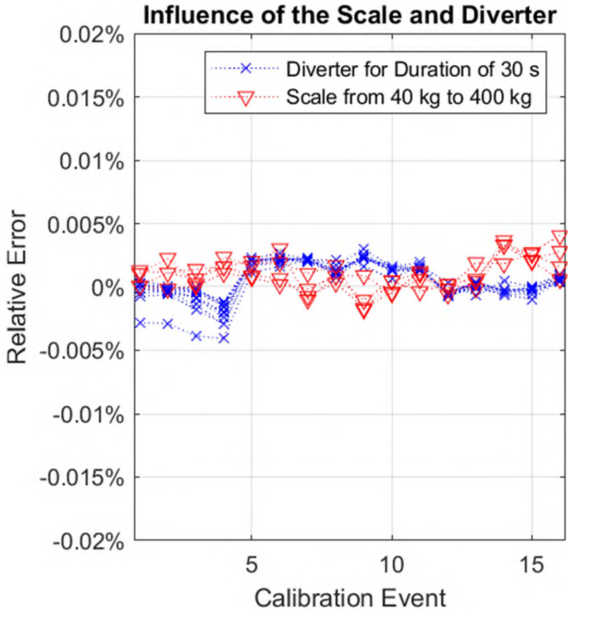

Figure 10 Performance of the diverter system and the weighing scale for the analysed 6-month period. The errors between calibrations are very low. No influence on the master calibration is expected

4. Internal and external comparisons

Interlaboratory comparisons are a well-established procedure to determine the conformity of the measurement results of a calibration facility. Comparisons have been performed at this facility type in two ways. Internally, considering different reference line groups (configurations) as different facilities, and externally, using a flowmeter to compare with a reference flow facility.

In Figure 11 we can see the results for four different configurations compared together against the scale at 10 kg/s. The flowmeter lines run at 10 kg/s, 5 kg/s, 3.3 kg/s and 2.5 kg/s. Independent of the working points of the reference flowmeter, the consistency of the results can be easily demonstrated.

As we cannot use the scale for flows above 11 kg/s, the same exercise has been performed using a Promass Coriolis flowmeter as a transfer standard instead of the weighing scale.

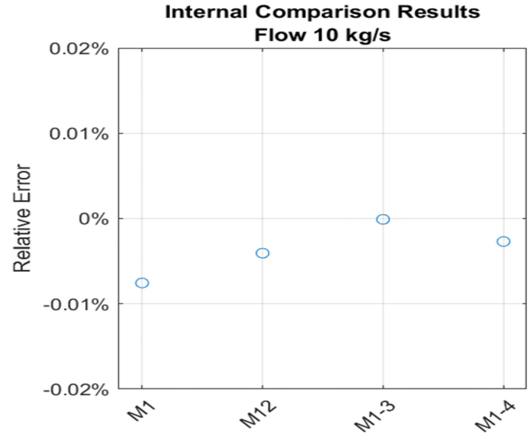

Figure 11 Internal comparison between different reference flowmeter configurations using 1, 2, 3 and 4 lines.

Fehler! Verweisquelle konnte nicht gefunden werden. shows various reference line configurations used at a different flow rate. In order to realize 50 kg/s, lines 5, 6, 7, 8 and 9 are used simultaneously. Depending on the configuration, each line measures between 5.5 kg/s and 10 kg/s. Again, consistency can be easily demonstrated.

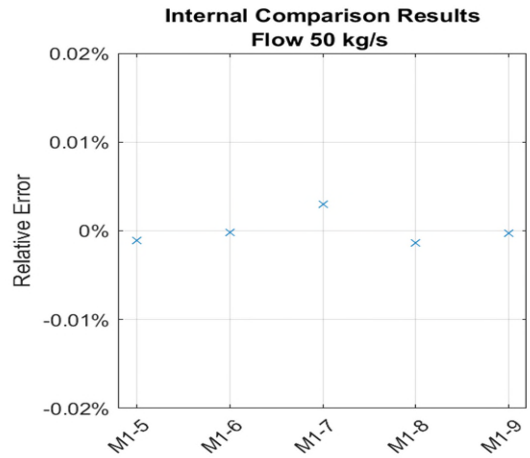

Figure 12 Internal comparison of different reference flowmeter number from 5 to 9 lines.

The FCP-25 has been also compared to an external primary facility with an expanded uncertainty of 0.015%, itself described in [3] and using a Promass Coriolis flowmeter [1]. Figure 13 shows the results as the difference to the results from the pilot laboratory. Again, independent of the used configuration, the results fall well within expanded uncertainty limits.

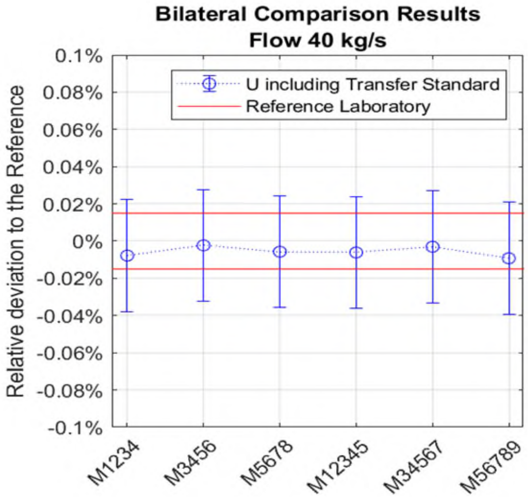

Figure 13 Bilateral comparison at 40 kg/s compared to FCP-7.1.5 described under [3]. The error bars include the Transfer Standard. Consistency is clearly given.

5. Conclusions

The model used could be confirmed through numerical simulations and by performing inter-laboratory comparisons with very high agreement.

It has been shown that by considering covariance and correlation, but more importantly understanding their relation correctly, consistent results can be obtained when this type of facility is used.

The very good performance of the gravimetric system enables the accurate determination of variance and covariance. The periodical tests performed every two weeks, gave us enough data to robustly estimate the performance of the reference flowmeter under working conditions.

It is also important to consider the thorough engineering design that minimizes influence factors as a key to obtain a highly accurate calibration facility.

As in this case, when tests are performed under extremely well controlled conditions without making changes in the system due to transport or installation, correlation can be interpreted as the sum of very small unknown influence factors. However, when transportation or different conditions occur during assembly, or even when different calibration principles, such as those described in [4] are used, careful evaluations must be performed to still consider correlation as a valid modelling approach.

Acknowledgments

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